# Actuarial Cost Methods
# A Review

**Third Edition**

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# Guide to Problems

Many of the problems herein were taken from prior EA-1(B) Examinations. For those students wishing to use this material in conjunction with the prior examinations, the following cross reference list should be helpful. Note that solutions to the May, 1998 examination have been published in the **1999 Supplement to Actuarial Cost Methods, A Review.**

| | Examination | | | | |
|---|---|---|---|---|---|
| **Problem** | **May, 1984** | **May, 1985** | **May, 1986** | **May, 1987** | **May, 1988** |
| 1 | 7 - 1 | 4 - 12 | 2 - 12 | 4 - 26 | 5 - 13 |
| 2 | 6 - 5 | 6 - 7 | 2 - 13 | 2 - 18 | 3 - 11 |
| 3 | 2 - 8 | 7 - 2 | 4 - 19 | 8 - 10 | 7 - 5 |
| 4 | 1 - 14 | 4 - 6 | 1 - 15 | 8 - 11 | 6 - 15 |
| 5 | 2 - 11 | 6 - 10 | 6 - 12 | 8 - 9 | 4 - 28 |
| 6 | 8 - 2 | 7 - 3 | 8 - 5 | 5 - 10 | 1 - 22 |
| 7 | 4 - 13 | 4 - 15 | 4 - 20 | 2 - 16 | 6 - 16 |
| 8 | 4 - 18 | 8 - 3 | 7 - 4 | 3 - 10 | 2 - 19 |
| 9 | 4 - 11 | 2 - 10 | 4 - 21 | 1 - 17 | 6 - 17 |
| 10 | 5 - 5 | 2 - 9 | 2 - 14 | 2 - 15 | 2 - 20 |
| 11 | 6 - 6 | 4 - 17 | 8 - 6 | 2 - 17 | 4 - 29 |
| 12 | 4 - 16 | 5 - 8 | 1 - 16 | 1 - 18 | 7 - 6 |
| 13 | 3 - 8 | 4 - 14 | 5 - 9 | 1 - 9 | 4 - 30 |
| 14 | 5 - 6 | 1 - 9 | 4 - 22 | 8 - 8 | 1 - 23 |
| 15 | 1 - 11 | 6 - 8 | 6 - 13 | 1 - 21 | 6 - 18 |
| 16 | 1 - 12 | 3 - 7 | 4 - 23 | 5 - 11 | 6 - 19 |
| 17 | 8 - 1 | 8 - 4 | 4 - 24 | 1 - 20 | 7 - 7 |
| 18 | 6 - 11 | 6 - 9 | 8 - 7 | 4 - 27 | 8 - 12 |
| 19 | 1 - 13 | 1 - 10 | 3 - 9 | 6 - 14 | 1 - 24 |
| 20 | 5 - 7 | 5 - 4 | 4 - 25 | 5 - 12 | 1 - 25 |

| | Examination | | | | |
|---|---|---|---|---|---|
| Problem | May, 1989 | May, 1990 | May, 1991 | May, 1992 | May, 1993 |
| 1 | 4 - 31 | 2 - 24 | 7 - 11 | 2 - 32 | 4 - 46 |
| 2 | 7 - 8 | 2 - 25 | 8 - 19 | 2 - 33 | 6 - 26 |
| 3 | 8 - 13 | 2 - 26 | 4 - 41 | 8 - 24 | 8 - 28 |
| 4 | 1 - 26 | 4 - 36 | 2 - 30 | 4 - 43 | 4 - 47 |
| 5 | 2 - 21 | 1 - 28 | 8 - 20 | 4 - 44 | 4 - 48 |
| 6 | 4 - 32 | 4 - 37 | 8 - 21 | 5 - 20 | 5 - 22 |
| 7 | 4 - 33 | 6 - 20 | 1 - 31 | 2 - 34 | 8 - 29 |
| 8 | 7 - 9 | 1 - 29 | 1 - 32 | 5 - 21 | 4 - 49 |
| 9 | 5 - 14 | 6 - 21 | 8 - 22 | 1 - 34 | 8 - 30 |
| 10 | 4 - 34 | 2 - 27 | 1 - 33 | 8 - 25 | 4 - 50 |
| 11 | 1 - 27 | 4 - 38 | 4 - 42 | 2 - 35 | 2 - 36 |
| 12 | 4 - 35 | 4 - 39 | 6 - 22 | 8 - 26 | 1 - 40 |
| 13 | 5 - 15 | 8 - 16 | 6 - 23 | 6 - 25 | 4 - 51 |
| 14 | 3 - 12 | 8 - 17 | 2 - 31 | 1 - 35 | 5 - 23 |
| 15 | 2 - 22 | 8 - 18 | 6 - 24 | 4 - 45 | 6 - 27 |
| 16 | 7 - 10 | 2 - 28 | 3 - 13 | 1 - 36 | 4 - 52 |
| 17 | 5 - 16 | 1 - 30 | 7 - 12 | 1 - 37 | 7 - 13 |
| 18 | 8 - 14 | 4 - 40 | 5 - 18 | 8 - 27 | 1 - 41 |
| 19 | 8 - 15 | 5 - 17 | 8 - 23 | 1 - 38 | 3 - 14 |
| 20 | 2 - 23 | 2 - 29 | 5 - 19 | 1 - 39 | 5 - 24 |

| | Examination | | | |
|---|---|---|---|---|
| **Problem** | **May, 1994** | **May, 1995** | **May, 1996** | **May, 1997** |
| 1 | 6 - 28 | 2 - 39 | 1 - 49 | 4 - 67 |
| 2 | 4 - 53 | 8 - 33 | 2 - 44 | 2 - 48 |
| 3 | 2 - 37 | 3 - 16 | 4 - 62 | 3 - 19 |
| 4 | 1 - 42 | 1 - 47 | 3 - 18 | 1 - 52 |
| 5 | 1 - 43 | 5 - 26 | 5 - 27 | 5 - 29 |
| 6 | 1 - 44 | 2 - 40 | 7 - 15 | 8 - 39 |
| 7 | 4 - 54 | 8 - 34 | 2 - 45 | 4 - 68 |
| 8 | 4 - 55 | 4 - 58 | 8 - 36 | 1 - 53 |
| 9 | 4 - 56 | 3 - 17 | 8 - 37 | 7 - 16 |
| 10 | 1 - 45 | 6 - 31 | 4 - 63 | 6 - 35 |
| 11 | 8 - 31 | 2 - 41 | 6 - 34 | 2 - 49 |
| 12 | 4 - 57 | 4 - 59 | 1 - 50 | 3 - 20 |
| 13 | 7 - 14 | 2 - 42 | 5 - 28 | 8 - 40 |
| 14 | 6 - 29 | 6 - 32 | 4 - 64 | 1 - 54 |
| 15 | 3 - 15 | 1 - 48 | 8 - 38 | 7 - 17 |
| 16 | 5 - 25 | 2 - 43 | 4 - 65 | 4 - 69 |
| 17 | 2 - 38 | 8 - 35 | 2 - 46 | 8 - 41 |
| 18 | 6 - 30 | 4 - 60 | 1 - 51 | 8 - 42 |
| 19 | 8 - 32 | 6 - 33 | 2 - 47 | 6 - 36 |
| 20 | 1 - 46 | 4 - 61 | 4 - 66 | 6 - 37 |

# Glossary of Abbreviations

Since many actuarial terms are lengthy, we will use abbreviations in equations and in the text for the sake of brevity. Listed below are many of the abbreviations which will be seen here:

| | |
|---|---|
| $\ddot{a}_x$ | Present value of an annuity due at age x |
| AA, aa or x | Attained Age |
| Assets | Actuarial Value of Assets |
| AL | Accrued Liability |
| ATA | Average Temporary Annuity |
| B(x) | Benefit at age x |
| BP | Benefit Payments |
| $C_t$ | Contributions for year t |
| CV | Cash Value |
| DB | Death Benefit |
| *e*Assets | Expected Value of Assets |
| *e*AL | Expected Accrued Liability |
| *e*PVFB | Expected Present Value of Future Benefits |
| *e*PVFNC | Expected Present Value of Future Normal Cost |
| *e*Salary | Expected Salary |
| *e*UAL | Expected Unfunded Accrued Liability |
| *e*UL | Expected Unfunded Liability |
| EA or ea | Entry Age |
| EAN | Entry Age Normal |
| ERB | Early Retirement Benefit |
| FIL | Frozen Initial Liability |
| $I_{BP}$ | Interest on Benefit Payments |
| $I_c$ | Interest on Contributions |
| ILP | Individual Level Premium |
| J&S or J+S | Joint and Survivor |
| NC | Normal Cost |
| NRA | Normal Retirement Age |
| NRD | Normal Retirement Date |
| PVAB | Present Value of Accrued Benefits |
| PVFB | Present Value of Future Benefits |
| PVFEC | Present Value of Future Employee Contributions |
| PVFNC | Present Value of Future Normal Costs |
| PVFS | Present Value of Future Salary |
| PVFY | Present Value of Future Years |
| RA, ra or y | Retirement Age |
| S, Salary | Salary or Compensation |
| TA | Temporary Annuity |
| UAL | Unfunded Accrued Liability |
| UC | Unit Credit |
| UL | Unfunded Liability |

# Introduction

The purpose of this writing is to assist students who are preparing for the EA-1(B) Examination, Basic Pension Mathematics, which is offered jointly by the American Society of Pension Actuaries, the Society of Actuaries and the Joint Board for the Enrollment of Actuaries. The emphasis is placed on the solution of problems, most of which are taken from previous examinations. Only enough theory is offered to introduce the student to the problems and their solutions. Material is arranged according to the recognized actuarial cost methods with the last section dealing with those principles common to all cost methods.

The purpose of any pension plan, and hence any actuarial cost method, is to provide some degree of security to those real live employees whom we call participants. With that in mind, meet Homer Jones. Homer is married, happily most of the time, with two children, one in high school and the other in the eighth grade. He attends church regularly and usually takes his family out to dinner every Sunday after the service. He works hard to provide for the current needs of his family, to save for his children's education and hopes he will have enough at retirement so as notto burden his family. He is grateful for the pension plan provided by his employer.

The information pertinent to his pension plan and to our discussion is that Homer is forty-five years of age, and has worked at the Flea Flicker Manufacturing Company for nineteen years. Last year, the personnel manager announced that the Company was adopting a pension plan for the benefit of its employees that would provide a retirement benefit of $20 per month for each year of service. This income would commence at a participant's sixty-fifth birthday and would continue for life. Homer calculated that by the time he reached sixty-five, he would have thirty-nine years of service which would entitle him to a pension of $780 per month.

There were other provisions in the plan, such as what happens in case of death or what happens in the event of termination of employment, but since Homer was not contemplating doing either, he did not ask for the details. Homer was concerned, however, in the assurance of knowing that the money that would provide this retirement benefit would indeed be there when he walked away from his job for the last time. The personnel manager explained to him that the company would make contributions each year into a trust fund and these contributions plus the investment earnings would be sufficient to pay Homer's monthly benefit so long as he lived. The personnel manager used the term "funding" to explain this procedure and said that an actuary would be employed to compute the amount of contribution each year.

Of course, there were many other variables that Homer neither knew nor cared about in regard to the funding of the plan. For example, the plan actuary would select one of several actuarial cost methods; he would also make certain assumptions regarding investment earnings, future salaries, employee turnover from death, termination and disability. Generally, these assumptions will change over the years but the particular cost method will not. The selection of the cost method and assumptions will always reflect the basic purpose which is to insure that Homer and the other participants in the plan will receive the benefits that are promised.

The variety of actuarial cost methods provides a great deal of flexibility that enables the plan actuary to tailor his calculations to the needs of the particular plan and employee characteristics. Indeed, by using simple assumptions of six percent investment earnings and annuity rate of $120 per one dollar of retirement income, the Normal Cost for Homer at plan inception can vary between $748 and $2,400 depending only upon the choice of fundingmethods; and the Accrued Liability can be as much as $14,218 andas small as nothing at all. How this is accomplished is the subject of this paper as we walk through this wonderland of actuarial cost methods.

# Chapter 1

# Unit Credit Cost Method (Accrued Benefit)

## 1.1 Accrued Liability and Normal Cost

Any cost method requires the maintenance of a certain amount of trust assets to cover current as well as
future benefits. The distinguishing characteristic of the Unit Credit or the Accrued Benefit method is that the desired value of assets at any particular time is equal to the present value of accrued benefits of all participants. We begin our study of this method by first defining the Accrued Liability:

> Definition: The Accrued Liability on any valuation date is the Sum of the Present Values of Accrued Benefits of all participants.

For example, Homer (refer to introduction) has 19 years of past service, so he has accrued a benefit of \$20 per year of service or \$380 per month. Assuming no pre-retirement mortality and investment return of 6% and $12\ddot{a}_{65}^{(12)} = 120$, the present value of his accrued benefit as well as the Accrued Liability is:

$$AL(\text{Homer}) = 380 \times 120 \times (1.06)^{-20} = 14{,}218$$

From the definition of Accrued Liability, it follows that the Normal Cost is the present value of the increase in accrued benefit between the year $t$ and $t + 1$. In Homer's case, he is accruing \$20 per month per year of service so his Normal Cost is computed as follows.:

$$NC(\text{Homer}) = 20 \times 120 \times (1.06)^{-20} = 748$$

Technically, the Normal Cost under any type of plan formula (flat benefit, unit benefit etc.) is the difference in the Present Values of the Accrued Benefit from time t to (t + 1). The benefit that Homer earned prior to the time the Plan was adopted will need to be amortized over a period of time, but let's take that up later on.

The Accrued Liability and Normal Cost of the Plan is the summation of these quantities for all Participants. Assuming no pre-retirement mortality:

$$AL_t = \sum B(x)\,(\ddot{a}_m^{(12)})(1+i)^{n-m}$$

$$NC_t = \sum \triangle B(x)\,(\ddot{a}_m^{(12)})(1+i)^{n-m}$$

where:

B(x) represents the Accrued Benefit of Participant x;
$\triangle$B(x) represents the change in Accrued Benefit from time t to (t + 1)
m represents retirement age, and
n represents attained age at time t.

We can prove mathematically that the relationship between the Accrued Liability at time t and the Accrued Liability at time (t + 1) is:

$$AL_{t+1} = [AL_t + \sum \triangle B(x)(\ddot{a}_m^{(12)})(1+i)^{n-m}] \times (1+i)$$

less the Present Value of Accrued Benefits for terminated participants and retirees. The above formula is logical as it reflects our procedure of simply applying one year's interest to (1) the Accrued Liability at time t and (2) the total change in Accrued Benefit and then totaling the two.

The Actuarial Value of Plan Assets is the value placed upon the assets of the plan by the Plan Actuary. It could be the market value or market value with some adjustment or it could be some value related to both cost and market values. Any difference between the Actuarial Asset Value and the Accrued Liability is called the Unfunded Accrued Liability (UAL). Thus:

$$UAL_t = AL_t - Assets_t$$

where Assets represents the Actuarial Asset Value. The Unfunded Accrued Liability may be positive or negative depending upon whether the Accrued Liability is larger than the value of assets (positive) or smaller than the value of assets (negative). Nevertheless, we shall use the term Unfunded Accrued Liability to describe either case.

Actuarial Gain or Loss is the difference between the expected Unfunded Accrued Liability and the actual Unfunded Accrued Liability. Thus, the Actuarial Gain/Loss at time t is:

$$\text{Gain/Loss} = \text{Expected } UAL_{t+1} - \text{Actual } UAL_{t+1}$$

It can also be expressed as:

$$\text{Gain/Loss} = (UAL_t + NC_t)(1+i) - C_t - I_t - UAL_{t+1}$$

where $C_t$ represents the contribution made for year t and $I_t$ represents the interest earnings on contributions at the assumed rate from the date made to the end of the Plan Year. A negative value implies an Actuarial Loss.

## 1.2 Unit Credit Method with Interest and Other Assumptions

Now that you are an expert on the Unit Credit Method of funding using an assumption of interest only, let us move ahead and consider the changes if we add pre-retirement mortality, withdrawals and salary scales to our assumptions repertoire. Another assumption we are making is that you are familiar with Commutation and Service Tables. If so, you will understand the following formulas for Accrued Liability and Normal Cost:

$$AL(t) = \sum B(x) \times \ddot{a}_y^{(12)} \times (D_y/D_x)$$

$$NC(t) = \sum \Delta B(x) \times \ddot{a}_y^{(12)} \times (D_y/D_x)$$

where D is the Commutation Function constructed with the probabilities of dying and termination of employment, using an assumed interest rate *i*. Values of D at various ages can be obtained from a Service Table.

For example, if the Flea Flicker Pension Plan did not provide for any death benefits or if the death benefits were computed separately, the Normal Cost and Accrued Liability could be determined using 6% interest as before. However, with the additional consideration of mortality, we would refer to a mortality table where $l_{45} = 961{,}208$ and $l_{65} = 771{,}684$. Computation would be as follows:

$$D_x = l_x v^x$$
$$D_{65} = l_{65} v^{65} = 771{,}684 \times .02265 = 17{,}479$$
$$D_{45} = l_{45} v^{45} = 961{,}208 \times .07265 = 69{,}832$$

$$\begin{aligned} AL(\text{Homer}) &= 380 \times 120 \times (D_{65}/D_{45}) \\ &= 45{,}600 \times 17{,}479 \div 69{,}832 \\ &= 45{,}600 \times .2503 = 11{,}414 \\ \text{and } NC(\text{Homer}) &= 20 \times 120 \times .2503 = 601 \end{aligned}$$

The relationship between the Accrued Liability at times t and (t + 1) follows the same pattern as that which exists for the Accrued Liability and Normal Cost <u>except</u> that if the death of any Participant occurs, then the Accrued Liability of the deceased Participant must be subtracted from the total at time t. However, since the probability of dying has already been taken into account in our calculations, this probability of dying must be applied to the Accrued Liability of the deceased and added back in. Symbolically, this adjustment can be shown as follows where z represents age at death:

$$B(z+1)(\ddot{a}_y^{(12)})(D_y/D_z) - q_z^{(d)} B(z+1) \ddot{a}_y^{(12)} (D_y/D_z)$$

where $q_z^{(d)}$ is the probability of dying at age z. The adjustment described above is shown in the second part of the formula.

In a similar manner, when a Participant withdraws from the Plan because of termination of employment, an adjustment must be made to the Accrued Liability by subtracting the following:

$$B(x + 1)\,(\ddot{a}_y^{(12)})(D_y/D_x) - q_x^{(w)}B(x + 1)\,(\ddot{a}_y^{(12)})(D_y/D_x)$$

### 1.3 Characteristics of Unit Credit Method

1. Accrued Liability is always equal to the Present Value of Accrued Benefit.

2. The Normal Cost at each year's valuation date is the Present Value of the increase in the Accrued Benefit for the plan year.

3. Normal Costs tend to increase as time goes by even though benefits remain the same.

4. Withdrawals due to terminations do not generate actuarial gains for fully vested participants.

5. This method usually requires the smallest initial outlay of cash.

6. This method is suitable in those plans that have a high turnover rate.

### 1.4 Problems

**Problem 1 - 1**

Homer's friend Jake is 52 and has a projected pension of \$300. Compute NC(Jake) and AL(Jake). Assume $12\ddot{a}_{65}^{(12)} = 120$; $i = 6\%$.

**Problem 1 - 2**

Assuming that the Flea Flicker Plan provides a benefit of \$20 per month per year of Service, calculate Homer's Normal Cost and Accrued Liability for each of the first five Plan Years. Can you observe a pattern for each of these quantities?

**Problem 1 - 3**

A certain plan provides a pension that is a flat benefit for each year of Service. The Accrued Liability on 1/1/84 was \$20,049 and on 1/1/85, it was \$24,627. The Plan has three Participants who have been in the Plan since its inception. Their ages on 1/1/84 were 39, 44 and 52. Assuming interest only of 6% and $\ddot{a}_{65}^{(12)} = 10$, determine the Plan's Pension formula.

**Problem 1 - 4**

Assume that the Accrued Benefit increases each year at the rate of B dollars. If A represents the accrued benefit for service prior to the effective date, x is attained age on the valuation date and y is the retirement age, write the formula for the Accrued Liability after n years.

**Problem 1 - 5**

A Plan provides a pension of 2% of compensation for each year of Service. Compute the Actuarial Gain as of 1/1/86 using the following selected specifications:

| | 1/1/85 | 1/1/86 |
|---|---|---|
| Actuarial Value of Assets | \$12,000 | \$14,940 |
| Contribution made on 7/1/85 | 2,000 | |

Selected Employee data:

| | Employee A | Employee B |
|---|---|---|
| Date of Birth | 1/1/45 | 1/1/40 |
| Date of Hire | 1/1/75 | 1/1/79 |
| Compensation in 1985 | \$18,000 | \$12,000 |

Assumptions: $i = 7\%$ $\quad \ddot{a}_{65}^{(12)} = 12$

**Problem 1 - 6**

Compute NC(Jake) and AL(Jake).
Valuation Date: 1/1/85
Date of Hire: 1/1/83
Age at Valuation date: 52

$$12\ddot{a}_{65}^{(12)} = 120 \qquad D_{52} = 46{,}259$$
$$i = 6\% \qquad D_{65} = 17{,}480$$

**Problem 1 - 7**

A Plan provides a Pension of $50 per month at age 65 per year of Service for its sole Participant, age 38. Using 5% interest and a mortality assumption, the Actuary computed the Normal Cost to be $1,533. He then decided to recompute the Normal Cost using a preretirement interest assumption of 6% while using the same mortality table. What is the new Normal Cost as a result of the new interest assumption? ($\ddot{a}_{65}^{(12)} = 12$)

**Problem 1 - 8**

A Plan provides a pension of $50 per month at age 65 for each year of service. There are three participants whose ages are 25, 30, and 35 on 1/1/85. The total Accrued Liability on 1/1/85 is $10,626. No death benefits are provided under the Plan. The oldest Participant, who had 3 years of service on 1/1/85, dies on 12/31/85. Determine the Accrued Liability as of 1/1/86. Selected data items are as follows:.

$p_{35}$ (probability of living for one year at age 35) = .97

$\ddot{a}_{65}^{(12)} = 10$ $\qquad$ $i = .07$

| | | |
|---|---|---|
| $D_{65}/D_{25} = .059$ | $D_{65}/D_{30} = .083$ | $D_{65}/D_{35} = .117$ |
| $D_{65}/D_{26} = .063$ | $D_{65}/D_{31} = .089$ | $D_{65}/D_{36} = .125$ |

**Problem 1 - 9**

| | |
|---|---|
| Normal retirement benefit: | $10 per month for each year of service. |
| Vesting: | 0% before 10 years of service and 100% thereafter. |
| Actuarial cost method: | Accrued benefit (unit credit). |
| Assumed interest rate: | 5% |
| Assumed retirement age: | 65 |

Data for the sole participant as of 1/1/85: Date of birth: 1/1/22
Date of hire : 1/1/74

Selected values and commutation functions are:

| Age x | $q_x^{(T)}$ | $q_x^{(m)}$ | $D_x^{(T)}$ |
|---|---|---|---|
| 63 | .057 | .027 | 1,000 |
| 64 | .059 | .029 | 898 |
| 65 | .038 | .038 | 805 |

Withdrawals are assumed to occur at the end of the year.

$$q_x^{(T)} = q_x^{(m)} + q_x^{(w)}$$

$$\ddot{a}_{65}^{(12)} = 10$$

In what range is the normal cost as of 1/1/85?

(A) Less than $980
(B) $980 but less than $1,000
(C) $1,000 but less than $1,020
(D) $1,020 but less than $1,040
(E) $1,040 or more.

**Problem 1 - 10**

Plan effective date: 1/1/84
Normal retirement benefit: $12 per month for each year of service.
Actuarial cost method: Accrued benefit (unit credit).

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant: Date of birth: 1/1/39
Date of hire: 1/1/69

Contribution for 1984 paid at 1/1/84: $1,200

Effective 1/1/85 the plan is amended to increase the normal retirement benefit to \$15 per month for each year of service. The unfunded accrued liability as of 1/1/85 based on the amended plan is \$8,100.

$\ddot{a}_{65}^{(12)} = 10$

In what range is the experience gain or loss for 1984 attributable to interest?

(A) Loss of \$100 or more.
(B) Loss of less than \$100, or no gain or loss.
(C) Gain of less than \$100.
(D) Gain of \$100 but less than \$200.
(E) Gain of \$200 or more.

**Problem 1 - 11**

Actuarial cost method: Accrued benefit (unit credit method)
Assumed interest rate: 7%

| Valuation results: | 1/1/83 | 1/1/84 |
|---|---|---|
| Normal cost as of 1/1 | \$6,000 | \$7,500 |
| Accrued Liability | 70,000 | 75,000 |
| Actuarial Value of Assets | 45,000 | |

Contribution for 1983 paid at 12/31/83: \$9,000
Net experience gain for 1983: \$4,500
Benefits paid at 7/1/83: \$2,000

In what range is the actuarial value of assets as of 1/1/84?

(A) Less than \$52,000
(B) \$52,000 but less than \$53,500
(C) \$53,500 but less than \$55,000
(D) \$55,000 but less than \$56,500
(E) \$56,500 or more

**Problem 1 - 12**

Actuarial cost method: Accrued benefit (unit credit) method.
Assumed interest rate: 6%.
Normal cost for 1982 as of 12/31/82: $50,000

1/1/83 valuation results:

| | |
|---|---|
| Actuarial value of assets as of 1/1/83 | $425,000 |
| Unfunded Accrued Liability as of 1/1/83 | 450,000 |
| Experience gain for 1982 attributable to investment return | 42,600 |
| Experience gain for 1982 attributable to factors other than investment return | 13,400 |

Contribution for 1982: $75,000 paid at 12/31/82
Benefit payments for 1982: $10,000 paid at 1/1/82

In what range is the accrued liability as of 1/1/82?

(A) Less than $800,000
(B) $800,000 but less than $820,000
(C) $820,000 but less than $840,000
(D) $840,000 but less than $860,000
(E) $860,000 or more

**Problem 1 - 13**

Normal retirement benefit: $20 per month for each year of service.
Vesting: 100% after 10 years of service.
Actuarial cost method: Accrued benefit (unit credit) method.
Assumed interest rate: 7%.
Assumed retirement age: 65

The service table is a double-decrement table based on separate single-decrement mortality and withdrawal tables, as follows (l's are from single-decrement tables):

| Age x | $l_x^{(m)}$ | $l_x^{(w)}$ |
|---|---|---|
| 30 | 1,000 | 1,000 |
| 65 | 900 | 400 |

Data on sole participant as of 1/1/84:

Age 30
10 years of past service

$\ddot{a}_{65}^{(12)} = 9$

In what range is that portion of the accrued liability as of 1/1/84 which is attributable to benefits for vested terminations before age 65?

(A) Less than \$720
(B) \$720 but less than \$840
(C) \$840 but less than \$960
(D) \$960 but less than \$1,080
(E) \$1,080 or more

**Problem 1 - 14**

Type of plan: Contributory.
Accrued benefit: \$50 per month for each year of service.
Actuarial cost method: Accrued benefit (unit credit) method
Assumed interest rate: 6%

| | |
|---|---|
| Pre-retirement death and termination benefit: | Refund of employee contributions with 6% interest (no vesting) |

Data on sole participant as of 1/1/83:

| | |
|---|---|
| Date of birth | 1/1/43 |
| Date of hire | 1/1/78 |

Employee Contributions:

| | |
|---|---|
| Accumulated with interest to 12/31/82: | \$2,500.00 |
| Expected to be contributed for 1983 at 12/31/83: | \$475.00 |
| Rate of interest credited: | 6.0% |

Selected commutation functions:

| Age x | $D_x$ | $C_x$ | $M_x$ |
|---|---|---|---|
| 35 | 2,522 | 174 | 1,226 |
| 36 | 2,205 | 149 | 1,052 |
| 40 | 1,314 | 80 | 576 |
| 41 | 1,160 | 68 | 496 |
| 64 | 127 | 2 | 45 |
| 65 | 118 | 1 | 43 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the employer normal cost as of 1/1/83?

(A) Less than \$100
(B) \$100 but less than \$200
(C) \$200 but less than \$300
(D) \$300 but less than \$400
(E) \$400 or more

**Problem 1 - 15**

Actuarial Cost Method: Accrued Benefit (Unit Credit).
Assumed interest rate: 8%.

Valuation data as of 1/1/85:

| | |
|---|---|
| Accrued liability | \$850,000 |
| Actuarial value of assets | 420,000 |
| Normal cost as of 1/1 | 70,000 |

Actuarial value of assets as of 1/1/86: \$573,000

1985 experience gains/(losses):

| | |
|---|---|
| Investment | \$600 |
| All other sources | 0 |

Benefit payments during 1985: \$500 paid at the beginning of each month.

In what range is the unfunded accrued liability as of 1/1/86?

(A) Less than $414,000
(B) $414,000 but less than $414,500
(C) $414,500 but less than $415,000
(D) $415,000 but less than $415,500
(E) $415,500 or more

**Problem 1 - 16**

Normal Retirement Age: 62
Normal Retirement Benefit: $10 per month for each year of service.
Postponed Retirement Benefit: $10 per month for each year of service.
Actuarial Cost Method: Accrued benefit (Unit Credit).

Actuarial assumptions:

Interest: 7%.
Pre-retirement deaths and terminations: none.
$q_x^{(r)}$ represents the probability of retirement at age x.

| Age x | $q_x^{(r)}$ | $\ddot{a}_x^{(12)}$ |
|---|---|---|
| 60 or less | 0 | -- |
| 61 | 0 | 10.8 |
| 62 | 0 | 10.6 |
| 63 | 0.2 | 10.4 |
| 64 | 0.5 | 10.2 |
| 65 | 1.0 | 10.0 |

Retirements are assumed to occur at the beginning of year.

Data for sole participant as of 1/1/86:

| | |
|---|---|
| Date of birth | 1/1/26 |
| Date of hire | 1/1/66 |

In what range is the accrued liability as of 1/1/86?

(A) Less than $17,500
(B) $17,500 but less than $18,500
(C) $18,500 but less than $19,500
(D) $19,500 but less than $20,500
(E) $20,500 or more

**Problem 1 - 17**

Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest: 8%
Retirement age: 65

| Selected valuation results: | 1/1/86 | 1/1/87 |
|---|---|---|
| Normal cost as of 1/1 | $ 10,000 | $ 11,000 |
| Accrued liability - actives | 135,100 | 156,000 |
| Accrued liability - inactives | 0 | 0 |

Actuarial value of assets as of 1/1/86: $0
Contribution for 1986: $25,000, paid at 7/1/86.
Contribution for 1987: $27,500, paid at 7/1/87.

No benefits were payable for 1986.
Rate of return on the actuarial value of assets for 1986: 10%

In what range is the actuarial gain for 1986 excluding the investment gain?

(A) Less than $300
(B) $300 but less than $500
(C) $500 but less than $700
(D) $700 but less than $900
(E) $900 or more

**Problem 1 - 18**

Normal retirement benefit: $20 per month for each year of service.
Early retirement benefit: Accrued benefit with no reduction for early retirement.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest: 8%
Preretirement deaths and terminations: None.
Retirement:

| Age x | Probability of Retirement at Age x |
|---|---|
| 63 | 20% |
| 64 | 30% |
| 65 | 100% |

Retirements occur at the beginning of the year.

Data for sole participant as of 1/1/87:

| | |
|---|---|
| Date of birth: | 1/1/32 |
| Date of hire: | 1/1/77 |

Selected annuity values:

$\ddot{a}_{63}^{(12)} = 8.56$ $\quad\quad$ $\ddot{a}_{64}^{(12)} = 8.35$ $\quad\quad$ $\ddot{a}_{65}^{(12)} = 8.14$

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than $775
(B) $775 but less than $950
(C) $950 but less than $1,125
(D) $1,125 but less than $1,300
(E) $1,300 or more

**Problem 1 - 19**

Normal Retirement Benefit: $25 per month for each year of service.
Early Retirement Benefit: Accrued benefit reduced by 1/15 for each of the first 5 years preceding age 65 and 1/30 for each of the next 5 years.

Actuarial cost method: Unit credit.

Actuarial assumptions:

> Interest: 8%
> Preretirement terminations other than deaths: None.
> Retirement age: 65

Data for participant:

| | |
|---|---|
| Date of birth: | 1/1/28 |
| Date of hire: | 1/1/63 |
| Date of retirement: | 12/31/86 |

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 58 | 440 | 4,160 |
| 59 | 400 | 3,740 |
| 60 | 365 | 3,355 |
| 65 | 230 | 1,880 |

In what range is the loss in 1986 due to the participant's early retirement?

(A) Less than \$3,000
(B) \$3,000 but less than \$5,000
(C) \$5,000 but less than \$7,000
(D) \$7,000 but less than \$9,000
(E) \$9,000 or more

**Problem 1 - 20**

Normal Retirement Benefit: \$20 per month per year of service.
Actuarial Cost Method: Unit credit.

Actuarial Assumptions:

> Interest: 7%
> Preretirement deaths and terminations: None.
> Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/23 |
| Date of hire | 1/1/81 |

Unfunded accrued liability as of 1/1/86: \$2,000
Contribution for 1986: Normal cost as of 1/1 paid at 1/1/86.
Investment return for 1986: 11%

As of 1/1/87, the assumed interest rate is changed to 8%.

| | 7% | 8% |
|---|---|---|
| $\ddot{a}_{65}^{(12)}$ | 10 | 9 |

In what range is the unfunded accrued liability as of 1/1/87?

(A) Less than \$350
(B) \$350 but less than \$700
(C) \$700 but less than \$1,050
(D) \$1,050 but less than \$1,400
(E) \$1,400 or more

**Problem 1 - 21**

Normal Retirement Benefit: 1% of final year's salary for each year of service.
Actuarial cost method: Projected unit credit (service prorated).

Actuarial assumptions:

Interest: 8%
Salary increases: 6% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

| Name | Date of Birth | Date of Hire | Salary for 1987 |
|---|---|---|---|
| Smith | 1/1/27 | 1/1/67 | \$72,000 |
| Brown | 1/1/27 | 1/1/60 | \$24,000 |

$\ddot{a}_{65}^{(12)} = 8.33$

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than \$5,500
(B) \$5,500 but less than \$6,000
(C) \$6,000 but less than \$6,500
(D) \$6,500 but less than \$7,000
(E) \$7,000 or more

**Problem 1 - 22**

Plan year: Calendar year.
Valuation date: 12/31
Actuarial cost method: Unit credit.
Assumed interest rate: 6%

Valuation results as of 12/31/86:

| | |
|---|---|
| Normal cost as of 12/31 | \$ 20,000 |
| Unfunded accrued liability (excluding normal cost for 1986) | \$250,000 |

Contributions for 1986: \$7,000 paid on 3/31/86, \$7,000 paid on 6/30/86, and \$20,000 paid on 12/31/86.

In what range is the expected unfunded accrued liability as of 12/31/87?

(A) Less than \$235,000
(B) \$235,000 but less than \$240,000
(C) \$240,000 but less than \$245,000
(D) \$245,000 but less than \$250,000
(E) \$250,000 or more

**Problem 1 - 23**

Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest: 6%
Preretirement terminations other than deaths: None.
Retirement age: 65

Participant data as of 1/1/88: 1,000 active participants, all born on 1/1/43 and hired on 1/1/81.

During 1988, 12 participants died and there were no new participants.

Selected commutation functions:

| Age x | $D_x$ |
|---|---|
| 44 | 322 |
| 45 | 303 |
| 46 | 283 |
| 65 | 77 |

$\ddot{a}_{65}^{(12)} = 9.35$

In what range is the mortality gain for 1988 as of 1/1/89?

(A) Less than $4,200
(B) $4,200 but less than $4,400
(C) $4,400 but less than $4,600
(D) $4,600 but less than $4,800
(E) $4,800 or more

**Problem 1 - 24**

Normal retirement benefit: $20 per month for each year of service.

Early retirement benefit:

Before 1988: Accrued benefit reduced by 6% for each year preceding age 65.

After 1987: Unreduced accrued benefit, plus a monthly supplement payable to age 65 of $6.67 per month for each year of service.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest: 8%
Preretirement terminations other than deaths: None.
Normal Retirement Age:

| | |
|---|---|
| Before 1988 | 65 |
| After 1987 | 60 |

Data for sole participant as of 1/1/88:

| | |
|---|---|
| Date of birth | 1/1/38 |
| Date of hire | 1/1/68 |

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 50 | 202 | 2,162 |
| 55 | 133 | 1,330 |
| 60 | 86 | 787 |
| 65 | 54 | 440 |

In what range is the combined increase in the accrued liability as of 1/1/88 due to the plan amendment and the assumption change?

(A) Less than $9,250
(B) $9,250 but less than $10,750
(C) $10,750 but less than $12,250
(D) $12,250 but less than $13,750
(E) $13,750 or more

**Problem 1 - 25**

Normal retirement benefit: $10 per month for each year of service.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest: 6%
Preretirement terminations other than deaths: None.
Retirement age: 65

Participants as of 1/1/87: 100 active employees, all age 60.
Normal cost for 1987 as of 1/1/87: $100,000.
There were no experience gains or losses during 1987.
There were no new participants during 1987.

Selected mortality value:

$q_{60} = .04$

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than $95,000
(B) $95,000 but less than $100,000
(C) $100,000 but less than $105,000
(D) $105,000 but less than $110,000
(E) $110,000 or more

**Problem 1 - 26**

Normal retirement benefit: 2.5% of final year's compensation for each year of service.
Preretirement death benefit: None.
Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/69 |
| 1988 compensation | $100,000 |

After the valuation was done, it was discovered that the assumed mortality rate at age 45 had been incorrectly coded as follows:

| | Original Value | Correct Value |
|---|---|---|
| $q_{45}$ | .034 | .0034 |

Another valuation was performed to correct this error.
Original accrued liability as of 1/1/89: $300,000

In what range is the correct accrued liability as of 1/1/89?

(A) Less than $275,000
(B) $275,000 but less than $295,000
(C) $295,000 but less than $315,000
(D) $315,000 but less than $335,000
(E) $335,000 or more

**Problem 1 - 27**

Normal retirement benefit: $25 per month for each year of service.
Early retirement benefit: Accrued benefit reduced by 1/15 for each year by which commencement of payments precedes age 65.

Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 6%
Preretirement deaths and terminations: None.
Retirement:

Before 1989: 100% retire at age 65.
After 1988: 40% retire at age 62; remainder retire at age 65.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/27 |
| Date of hire | 1/1/79 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 10.10 \qquad \ddot{a}_{65}^{(12)} = 9.35$

In what range is the increase in the accrued liability as of 1/1/89 due to the change in the assumed retirement age?

(A) Less than \$2,000
(B) \$2,000 but less than \$5,000
(C) \$5,000 but less than \$8,000
(D) \$8,000 but less than \$11,000
(E) \$11,000 or more

**Problem 1 - 28**

Normal retirement benefit: \$10 per month for each year of service.
Vesting eligibility: 100% after 5 years of service.
Preretirement death benefit: None.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: Occur at end of year.
Retirement age: 65
$q_x^{(d)} = q_x'^{(d)}$

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/27 |
| Date of hire | 1/1/85 |
| Status as of 1/1/90 | Active |

Selected probabilities and annuity value:

| Age x | $q_x^T$ | $q_x^{(d)}$ |
|---|---|---|
| 63 | .069 | .019 |
| 64 | .081 | .021 |
| 65 | .023 | .023 |

$q_x^T = q_x^{(d)} + q_x^{(w)}$

$\ddot{a}_{65}^{(12)} = 8.736$

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than \$872
(B) \$872 but less than \$877
(C) \$877 but less than \$882
(D) \$882 but less than \$887
(E) \$887 or more

**Problem 1 - 29**

Plan effective date: 1/1/89
Normal retirement age: 63
Normal retirement benefit: 2.5% of final year's compensation for each year of service.
Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 8%
Compensation increases: 4%
Preretirement deaths and terminations: None.
Retirement age: 63

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/55 |
| Date of hire | 1/1/80 |
| 1988 compensation | \$40,000 |
| Status as of 1/1/90 | Active |

Value of assets as of 1/1/89: \$0
Contribution for 1989: \$6,000 paid on 4/1/89.
Noninvestment experience gain or loss for 1989: \$0
Unfunded accrued liability as of 1/1/90: \$25,000

$\ddot{a}_{63}^{(12)} = 8.582$

In what range is the investment experience gain or loss for 1989?

(A) Gain of $400 or more
(B) Gain of $200 but less than $400
(C) Gain or loss of less than $200
(D) Loss of $200 but less than $400
(E) Loss of $400 or more

**Problem 1 - 30**

Normal retirement benefit: 1% of final year's compensation for each year of service up to 30 years.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/80 |
| 1989 compensation | $40,000 |
| Status as of 1/1/90 | Active |

Valuation results as of 1/1/89:

| | |
|---|---|
| Normal cost as of 1/1 | $ 2,000 |
| Accrued liability | 21,000 |
| Value of assets | 12,000 |

Contribution for 1989: $2,500 paid on 12/31/89.
Investment experience gain or loss for 1989: $0.

Selected commutation functions:

$D_{49} = 344 \qquad D_{50} = 320 \qquad N_{65}^{(12)} = 849$

In what range is the experience gain for 1989?

(A) Less than $2,000
(B) $2,000 but less than $2,500
(C) $2,500 but less than $3,000
(D) $3,000 but less than $3,500
(E) $3,500 or more

**Problem 1 - 31**

Normal retirement benefit: $500 per month.
Postretirement death benefit: $5,000 payable at the end of the month of death.

Accrued benefit: The normal retirement benefit and the postretirement death benefit are accrued and prorated on service.

Actuarial cost method: Projected unit credit (prorated on service).

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/31 |
| Date of hire | 1/1/66 |
| Status as of 1/1/91 | Active |

Present value at age 65 of postretirement death benefit: $2,100

In what range is the normal cost for 1991 as of 1/1/91?

(A) Less than $1,250
(B) $1,250 but less than $1,300
(C) $1,300 but less than $1,350
(D) $1,350 but less than $1,400
(E) $1,400 or more

**Problem 1 - 32**

Normal retirement benefit: 1% of final three year average compensation for each year of service.

Preretirement death benefit: None.
Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole plan participant:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Status as of 1/1/90 and 1/1/91 | Active |

The sole participant received an 8% increase in compensation from 1990 to 1991.

Selected probability of death:

$q_{55} = .009$

In what range is the percentage increase in the normal cost for 1991 as of 1/1/91 over the normal cost for 1990 as of 1/1/90?

(A) Less than 6.5%
(B) 6.5% but less than 8.5%
(C) 8.5% but less than 10.5%
(D) 10.5% but less than 12.5%
(E) 12.5% or more

**Problem 1 - 33**

| | |
|---|---|
| Normal retirement benefit: | 2% of final three year average compensation for each year of service. |
| Early retirement eligibility: | Age 60 |
| Early retirement benefit: | Accrued benefit, reduced by 1/15 for each year by which the benefit commencement date precedes the normal retirement date. |

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age:

| | |
|---|---|
| Before 1991 | 65 |
| After 1990 | 62 |

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/51 |
| Date of hire | 1/1/89 |
| 1991 compensation | $21,000 |
| Status as of 1/1/91 | Active |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.40$ $\qquad$ $\ddot{a}_{65}^{(12)} = 8.75$

In what range is the change in the accrued liability as of 1/1/91 due to the change in the assumed retirement age?

(A) Decrease of $370 or more
(B) $0 or decrease of less than $370
(C) Increase of more than $0 but less than $370
(D) Increase of $370 but less than $740
(E) Increase of $740 or more

**Problem 1 - 34**

| | |
|---|---|
| Early retirement eligibility: | Age 55 |
| Early retirement benefit: | Accrued benefit, reduced by 1/15 for each of the first 5 years and 1/30 for each of the next 5 years by which the benefit commencement date precedes the normal retirement date. |
| Actuarial cost method: | Unit credit. |

Actuarial assumptions:

Interest rate: 6% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

On 12/31/91, there are 3 active participants aged 56, 59, and 63 who retire and elect to commence receiving benefits on 1/1/92.

Selected annuity values:

| Age x | $\ddot{a}_x^{(12)}$ |
|---|---|
| 56 | 11.50 |
| 57 | 11.33 |
| 58 | 11.17 |
| 59 | 11.00 |
| 60 | 10.83 |
| 61 | 10.67 |
| 62 | 10.50 |
| 63 | 10.33 |
| 64 | 10.17 |
| 65 | 10.00 |

Which, if any, of the elections will result in an experience gain?

(A) The election by the age 59 participant only.
(B) The election by the age 63 participant only.
(C) The election by the age 56 and age 59 participants only.
(D) The election by the age 56 and age 63 participants only.
(E) The correct answer is not given by (A), (B), (C), or (D) above.

**Problem 1 - 35**

Normal retirement benefit: $10 per month for each year of service.

Preretirement death benefit: Lump sum equal to 100 times the projected monthly normal retirement benefit, payable at the end of the year of death.

Actuarial cost method: Unit credit for retirement benefits; term cost for preretirement death benefits.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/47 |
| Date of hire | 1/1/72 |

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 44 | 4,885 | 63,045 |
| 45 | 4,548 | 58,163 |
| 46 | 4,236 | 53,615 |
| 64 | 1,054 | 9,926 |
| 65 | 965 | 8,872 |
| 66 | 881 | 7,907 |

In what range is the total normal cost for 1992 as of 1/1/92?

(A) Less than $205
(B) $205 but less than $255
(C) $255 but less than $305
(D) $305 but less than $355
(E) $355 or more

**Problem 1 - 36**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service up to 10 years. |
| Actuarial cost method: | Projected unit credit, with benefits prorated over the years of service during which benefits are expected to accrue. |

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: 4% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/37 |
| Date of hire | 1/1/87 |
| 1991 compensation | \$31,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.50$

In what range is the normal cost as of 1/1/92?

(A) Less than \$1,500
(B) \$1,500 but less than \$2,000
(C) \$2,000 but less than \$2,500
(D) \$2,500 but less than \$3,000
(E) \$3,000 or more

**Problem 1 - 37**

Normal retirement benefit:

| | |
|---|---|
| Before 1992: | \$10.00 per month for each year of service. |
| After 1991: | \$10.50 per month for each year of service; applicable to active and inactive participants. |

Actuarial cost method: Unit credit.
Assumed interest rate: 7% per year.

Selected valuation results:

| | 1/1/91 | 1/1/92 |
|---|---|---|
| Normal cost as of 1/1 | \$ 1,000 | \$ 1,500 |
| Accrued liability | 20,000 | 22,500 |
| Value of assets | 10,000 | 13,500 |

Contribution for 1991: \$2,000 paid on 7/1/91.
Benefits payments for 1991: \$500 paid on 7/1/91.

In what range is the experience gain for 1991?

(A) Less than \$500
(B) \$500 but less than \$1,000
(C) \$1,000 but less than \$1,500
(D) \$1,500 but less than \$2,000
(E) \$2,000 or more

**Problem 1 - 38**

Plan effective date: 1/1/87
Normal retirement benefit: \$100 per month for each year of service.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/32 |
| Date of hire | 1/1/87 |

Value of assets as of 1/1/91: \$19,000
Contribution for 1991: \$8,000 paid on 12/31/91.
Value of assets as of 1/1/92: \$26,000

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the absolute value of the experience gain or loss for 1991 as of 12/31/91?

(A) Less than $2,000
(B) $2,000 but less than $2,500
(C) $2,500 but less than $3,000
(D) $3,000 but less than $3,500
(E) $3,500 or more

**Problem 1 - 39**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service up to 30 years. |
| Early retirement eligibility: | Age 60. |
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |
| Actuarial cost method: | Projected unit credit, with benefits prorated over the years of service during which benefits are expected to accrue. |

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: 6% per year.
Preretirement deaths and terminations: None.
Probability of retirement:

Age 62: 15% at the beginning of the year.
Age 65: 100% at the beginning of the year.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/57 |
| Date of hire | 1/1/90 |
| 1991 compensation | $25,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 8.77 \qquad \ddot{a}_{65}^{(12)} = 8.20$

In what range is the accrued liability as of 1/1/92?

(A) Less than \$4,430
(B) \$4,430 but less than \$4,500
(C) \$4,500 but less than \$4,570
(D) \$4,570 but less than \$4,640
(E) \$4,640 or more

**Problem 1 - 40**

Normal retirement benefit: \$15 per month for each year of service.
Postponed retirement benefit: \$15 per month for each year of service.

Early retirement benefit: Accrued benefit, reduced by 6% for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement:

| Age x | $q_x^{(r)}$ |
|---|---|
| 64 | 0.20 |
| 65 | 0.50 |
| 66 | 1.00 |

Retirements are assumed to occur at the beginning of the year.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/33 |
| Date of hire | 1/1/73 |
| Status as of 1/1/93 | Active |

Selected annuity values:

| Age x | $\ddot{a}_x^{(12)}$ |
|---|---|
| 64 | 10.0 |
| 65 | 9.8 |
| 66 | 9.6 |

In what range is the accrued liability as of 1/1/93?

(A) Less than \$24,500
(B) \$24,500 but less than \$25,000
(C) \$25,000 but less than \$25,500
(D) \$25,500 but less than \$26,000
(E) \$26,000 or more

**Problem 1 - 41**

Actuarial cost method: Unit credit.

Assumed preretirement deaths and terminations:

Before 1993: None.
After 1992: Mortality only.

Assumed retirement age: 65

Date of birth for sole participant (active as of 1/1/93): 1/1/40

Accrued liability (before change in assumptions) as of 1/1/93: \$20,000

Selected values:

| Age x | $l_x$ |
|---|---|
| 53 | 8,980,994 |
| 65 | 7,673,269 |

In what range is the decrease in the accrued liability as of 1/1/93 due to the change in actuarial assumptions?

- (A) Less than $1,000
- (B) $1,000 but less than $1,500
- (C) $1,500 but less than $2,000
- (D) $2,000 but less than $2,500
- (E) $2,500 or more

**Problem 1 - 42**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service up to 20 years, plus 1% of final 3-year average compensation for each additional year of service.

Preretirement death benefit: None.

Actuarial cost method: Projected unit credit (based on actual accrual percentages as of valuation date).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data for only participants (both active as of 1/1/94):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/44 | 1/1/44 |
| Date of hire | 1/1/69 | 1/1/84 |
| 1994 valuation compensation | $50,000 | $50,000 |

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 50 | 310,647 | 3,752,218 |
| 65 | 94,414 | 868,052 |

In what range is the accrued liability as of 1/1/94?

(A) Less than $140,000
(B) $140,000 but less than $150,000
(C) $150,000 but less than $160,000
(D) $160,000 but less than $170,000
(E) $170,000 or more

**Problem 1 - 43**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.

Early retirement eligibility: Age 55

Early retirement benefit: Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

Before 1994: 100% at age 65.
After 1993: 50% at age 62, 0% at ages 63 and 64, and 100% at age 65.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/74 |
| 1994 valuation compensation | $50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.394$ $\ddot{a}_{65}^{(12)} = 8.736$

In what range is the change in the accrued liability as of 1/1/94 due to the change in the retirement age assumption?

(A) Less than \$2,150
(B) \$2,150 but less than \$2,200
(C) \$2,200 but less than \$2,250
(D) \$2,250 but less than \$2,300
(E) \$2,300 or more

**Problem 1 - 44**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.
Early retirement eligibility: Age 60.
Early retirement benefit: Unreduced accrued benefit.
Actuarial cost method: Projected unit credit.

Optional form of payment: 95% of accrued benefit, payable until the last death of the participant and his surviving spouse.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65
Form of payment: Life annuity.

Valuation data for sole participant Smith:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/74 |
| 1993 valuation compensation | \$40,000 |
| Actual annual compensation: | |
| 1991 | \$36,281 |
| 1992 | 38,095 |
| 1993 | 40,000 |
| Spouse's date of birth | 1/1/34 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 9.815$ $\ddot{a}_{\overline{60:60}}^{(12)} = 8.094$

$\ddot{a}_{65}^{(12)} = 8.736$ $\ddot{a}_{\overline{65:65}}^{(12)} = 6.896$

On 12/31/93, Smith retires and elects to receive his annuity under the optional form of payment commencing 1/1/94.

In what range is the experience loss as of 1/1/94 due to Smith's retirement?

(A) Less than $30,000
(B) $30,000 but less than $40,000
(C) $40,000 but less than $50,000
(D) $50,000 but less than $60,000
(E) $60,000 or more

**Problem 1 - 45**

Normal retirement benefit: $20 per month for each year of service.

Postponed retirement benefit: Actuarial equivalent of normal retirement benefit, based on postretirement valuation assumptions.

Preretirement death benefit: None.

Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probabilities of retirement (retirements are assumed to occur at beginning of year):

| Age x | Probability at Retirement |
|---|---|
| 65 | 60% |
| 66 | 80% |
| 67 | 100% |

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/29 |
| Date of hire | 1/1/70 |

Selected commutation functions based on postretirement valuation assumptions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 94 | 825 |
| 66 | 86 | 734 |
| 67 | 79 | 651 |

In what range is the accrued liability as of 1/1/94?

(A) Less than \$50,000
(B) \$50,000 but less than \$51,000
(C) \$51,000 but less than \$52,000
(D) \$52,000 but less than \$53,000
(E) \$53,000 or more

**Problem 1 - 46**

Normal retirement benefit: \$20 per month for each year of service.
Termination benefit: Accrued benefit payable at normal retirement age.
Vesting: Full and immediate.
Preretirement death benefit: None.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Pretermination deaths: None.
Post-termination deaths: Included in commutation functions below.
Preretirement terminations: 10% at age 50 only (terminations are assumed to occur at beginning of year).
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/54 |
| Date of hire | 1/1/84 |

Selected commutation functions based on post-termination assumptions:

| Age x | $D_x$ |
|---|---|
| 30 | 1,262 |
| 40 | 632 |
| 50 | 311 |
| 65 | 94 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.7$

In what range is the accrued liability as of 1/1/94?

(A) Less than $3,700
(B) $3,700 but less than $3,800
(C) $3,800 but less than $3,900
(D) $3,900 but less than $4,000
(E) $4,000 or more

**Problem 1 - 47**

Normal retirement benefit: $20 per month for each year of service.
Preretirement death benefit: None.
Termination benefit: Vested accrued benefit, payable at normal retirement date.
Vesting eligibility: 100% after 5 years.
Actuarial cost method: Unit credit.

Actuarial assumptions:
Interest rate: 6% per year.
Preretirement terminations other than deaths and withdrawals: None.
Retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/62 |
| Date of hire | 1/1/92 |

Selected values from preretirement single-decrement tables:

| Age x | $l_x^{\prime(d)}$ | $l_x^{\prime(w)}$ |
|---|---|---|
| 30 | 10,000 | 10,000 |
| 31 | 9,985 | 9,500 |
| 32 | 9,969 | 9,000 |
| 33 | 9,952 | 8,500 |
| 34 | 9,934 | 8,000 |
| 35 | 9,915 | 7,500 |
| 65 | 7,900 | 2,000 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 9.439$$

In what range is the accrued liability for preretirement vested termination benefits as of 1/1/95?

(A) Less than \$200
(B) \$200 but less than \$400
(C) \$400 but less than \$600
(D) \$600 but less than \$800
(E) \$800 or more.

**Problem 1 - 48**

| | |
|---|---|
| Normal retirement benefit: | 2% of final year's compensation for each year of service. |
| Early retirement eligibility: | Age 55. |
| Early retirement benefit: | Accrued benefit, reduced by 1/15 for each of the first 5 years and 1/30 for each of the next 5 years by which the benefit commencement date precedes the normal retirement date. |
| Actuarial cost method: | Projected unit credit. |

Actuarial assumptions:

> Interest rate: 9% per year.
> Compensation increases: 3% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65.

Date of birth for selected participants (all active as of 12/31/94):

| Smith | Brown | Green |
|---|---|---|
| 1/1/39 | 1/1/35 | 1/1/32 |

Smith, Brown, and Green all retire and elect to commence receiving benefits as of 1/1/95.

Selected annuity values:

$\ddot{a}_{56}^{(12)} = 9.84$ $\qquad$ $\ddot{a}_{63}^{(12)} = 8.71$

$\ddot{a}_{60}^{(12)} = 9.25$ $\qquad$ $\ddot{a}_{65}^{(12)} = 8.39$

Which, if any, of the participants generated an experience gain as of 1/1/95 due to his retirement?

(A) Smith only
(B) Brown only
(C) Smith and Brown only
(D) Brown and Green only
(E) The correct answer is not given by (A), (B), (C), or (D) above.

**Problem 1 - 49**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service up to 25 years. |
| Early retirement eligibility: | Age 62. |
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |
| Actuarial cost method: | Projected unit credit (based upon actual accrual percentages as of the valuation date). |

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |

Probability of retirement (assumed to occur at beginning of the year):

| | |
|---|---|
| At age 62 | 20% |
| At age 63 | 0% |
| At age 64 | 0% |
| At age 65 | 100% |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/46 |
| Date of hire | 1/1/84 |
| 1996 valuation compensation | $50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.39$ $\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability as of 1/1/96?

(A) Less than $60,000
(B) $60,000 but less than $62,500
(C) $62,500 but less than $65,000
(D) $65,000 but less than $67,500
(E) $67,500 or more

**Problem 1 - 50**

Normal retirement benefit: 2/12% of final 3-year average compensation for each month of service up to 120 months plus 1/12% of final 3-year average compensation for each additional month of service.

Actuarial cost method: Projected unit credit (based upon actual accrual percentages as of the valuation date).

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/43 |
| Date of hire | 10/1/86 |
| 1996 valuation compensation | $50,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $4,000
(B) $4,000 but less than $4,500
(C) $4,500 but less than $5,000
(D) $5,000 but less than $5,500
(E) $5,500 or more

**Problem 1 - 51**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.
Termination benefit: Accrued benefit payable at normal retirement date.
Vesting: Full and immediate.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 4% per year.
Pre-retirement decrements other than withdrawals: None.

Probability of withdrawal (assumed to occur at beginning of year):

| | |
|---|---|
| 1/1/95 valuation: | 40% at age 50 |
| | 25% at age 55 |
| | 20% at age 60 |
| | 0% at all other ages |
| 1/1/96 valuation: | 50% at age 50 |
| | 20% at age 55 |
| | 0% at all other ages |

Retirement age: 65.

As of 1/1/96, all participants were active and under age 45.

In what range is the ratio of the accrued liability as of 1/1/96 under the revised withdrawal assumptions to the accrued liability as of 1/1/96 under the original withdrawal assumptions?

(A) Less than 0.990
(B) 0.990 but less than 1.000
(C) 1.000 but less than 1.010
(D) 1.010 but less than 1.020
(E) 1.020 or more

**Problem 1 - 52**

Normal retirement benefit: 2% of final 3-year average compensation for each of the first 20 years of service plus 1.5% of final 3-year average compensation for each of the next 10 years of service.

Actuarial cost method: Projected unit credit, prorated on actual accrual percentages.

Assumed compensation increases: 4% per year.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/42 |
| Date of hire | 1/1/77 |
| 1996 valuation compensation for 1/1/96 valuation | $50,000 |
| 1997 valuation compensation for 1/1/97 valuation | $53,000 |

Accrued liability as of 1/1/96: $97,000.

Selected commutation functions:

$D_{54} = 23{,}593$

$D_{55} = 21{,}867$

In what range is the normal cost for 1997 as of 1/1/97?

(A) Less than $4,200
(B) $4,200 but less than $4,600
(C) $4,600 but less than $5,000
(D) $5,000 but less than $5,400
(E) $5,400 or more

**Problem 1 - 53**

Actuarial cost method: Unit credit.

Assumed interest rate: 8% per year.

Selected valuation results:

| | 1/1/96 | 1/1/97 |
|---|---|---|
| Normal cost as of 1/1 | $100,000 | |
| Accrued liability | 800,000 | $1,000,000 |
| Value of assets | 400,000 | 600,000 |

Contributions for 1996: $100,000 paid on 7/1/96 and $50,000 paid on 12/31/96.

Benefit payments for 1996: $13,000 paid on 12/31/96.

In what range is the net experience loss for 1996 from all non-investment sources?

(A) Less than \$4,000
(B) \$4,000 but less than \$14,000
(C) \$14,000 but less than \$24,000
(D) \$24,000 but less than \$34,000
(E) \$34,000 or more

**Problem 1 - 54**

Plan effective date: 1/1/82.

Normal retirement benefit: \$25 per month for first year of accrual, with each subsequent year's accrual being 2% greater than the prior year's accrual.

Actuarial cost method: Unit credit.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Value of assets as of 1/1/97: \$17,810

Valuation data for sole participant (active as of 1/1/97):

| | |
|---|---|
| Date of birth | 1/1/47 |
| Date of hire | 1/1/82 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 9.815$

In what range is the unfunded liability as of 1/1/97?

(A) Less than \$500
(B) \$500 but less than \$1,000
(C) \$1,000 but less than \$1,500
(D) \$1,500 but less than \$2,000
(E) \$2,000 or more

## 1.5 Solutions to Problems

**Problem 1 - 1**

$$\begin{aligned}\text{Normal Cost} &= 20 \times 120 \times (1.06)^{-13} \\ &= 2{,}400 \times .4688 = 1{,}125\end{aligned}$$

From formula, Jake will have 15 years total service at retirement. Since he is now 52, his years of service to date must be 2 and his accrued benefit is $40.

$$\text{Accrued Liability} = 40 \times 120 \times (1.06)^{-13} = 2{,}250$$

**Problem 1 - 2**

| Year | Normal Cost | Accrued Liability |
|---|---|---|
| 1 | $748 | $14,218 |
| 2 | 793 | 15,865 |
| 3 | 841 | 17,657 |
| 4 | 891 | 19,608 |
| 5 | 945 | 21,729 |

The Accrued Liability increases each year by the addition of the Normal Cost and the interest on the sum of the Accrued Liability plus Normal Cost. The Normal Cost increases each year at the valuation rate of interest.

**Problem 1 - 3**

Key Concept: With interest applied, the Accrued Liability at time t plus the Normal Cost equals the Accrued Liability at time $(t + 1)$.

Therefore,

$$\begin{aligned}AL_{85} &= (AL_{84} + NC_{84})(1.06) \\ 24{,}627 &= (20{,}049 + NC_{84})(1.06) \\ NC_{84} &= 3{,}184\end{aligned}$$

But, Normal Cost also equals the Present Value of the year's accruals.

If B represents the flat benefit of the formula:

$$\begin{aligned} NC &= (120B)(1.06)^{-26} + (120B)(1.06)^{-21} + (120B)(1.06)^{-13} \\ 3{,}184 &= (120B)[(1.06)^{-26} + (1.06)^{-21} + (1.06)^{-13}] \\ &= (120B)(.2198 + .2942 + .4688) \\ &= (120B)(.9828) \\ &= (117.936)(B) \\ B &= 27 \end{aligned}$$

The formula provides for $27 per year of service.

## Problem 1 - 4

The formula is $(A + nB)(\ddot{a}_y^{(12)})(1 + i)^{x-y+n}$ or $(A + nB)(\ddot{a}_y^{(12)})(v^{y-x-n})$

## Problem 1 - 5

| | Employee A | Employee B |
|---|---|---|
| Accrued Benefit 1/1/85 | $3,600 | $1,440 |
| Accrued Benefit 1/1/86 | 3,960 | 1,680 |
| Accrued Liability 1/1/85 | 7,960 | 4,465 |
| Accrued Liability 1/1/86 | 9,368 | 5,574 |
| Normal Cost 1/1/85 | 796 | 744 |
| Normal Cost 1/1/86 | 852 | 796 |

$$\begin{aligned} UAL_{85} &= 425 \\ NC_{85} &= 1{,}540 \\ C_{85} &= 2{,}000 \\ I_{85} &= 70 \\ UAL_{86} &= 2 \end{aligned}$$

Therefore, by formula:

$$\begin{aligned} \text{Gain} &= (UAL_{85} + NC_{85})(1 + i) - C_{85} - I_{85} - UAL_{86} \\ &= (425 + 1{,}540)(1.07) - 2{,}000 - 70 - 2 = 31 \end{aligned}$$

**Problem 1 - 6**

$$\text{Normal Cost} = 20 \times 12\ddot{a}_{65}^{(12)} \times (D_{65}/D_{52})$$

$$= 20 \times 120 \times (17{,}480/46{,}259) = 907$$

$$\text{Accrued Liability} = 40 \times 120 \times (17{,}480/46{,}259) = 1{,}814$$

**Problem 1 - 7**

$$\text{Normal Cost} = \text{Monthly Accrual} \times 12\ddot{a}_{65}^{(12)} \times (D_{65}/D_{38})$$
$$1{,}533 = 50 \times 144 \times (D_{65}/D_{38})$$

Therefore, $(D_{65}/D_{38}) = 1{,}533/7{,}200 = .2129$

Also, $(D_{65}/D_{38}) = l_{65}/l_{38} \times (1.05)^{-65}/(1.05)^{-38}$
$.2129 = l_{65}/l_{38} \times .2678$

And $(l_{65}/l_{38}) = .2129 \div .2678 = .7950$

For i = 6%, $v^{65}/\, v^{38} = (.02265) \div (.10924) = .2074$

And $(D_{65}/D_{38}) = .2074 \times .7950 = .1649$

$$\text{Normal Cost} = 50 \times 144 \times .1649 = 1{,}187$$

**Problem 1 - 7** (Alternative solution):

Since the normal cost is equal to a series of products, the revised normal cost is directly proportional to the change in the interest rate assumption.

$$NC = \text{Monthly Accrual} \times 12\ddot{a}_{65}^{(12)} \times (l_{65}/l_{38}) \times (v^{65}/v^{38})$$
$$= \text{Monthly Accrual} \times 12\ddot{a}_{65}^{(12)} \times (l_{65}/l_{38}) \times v^{27}$$

$(1.05)^{-27} = .2678$
$(1.06)^{-27} = .2074$

The revised normal cost $= 1533 \times (.2074 \div .2678) = 1{,}187$

**Problem 1 - 8**

Key Concept: When computing the accrued liability in 1986, the value of the death benefit must be added back after deleting the present value of accrued benefit of the deceased. The net effect on the accrued liability is actual liability minus expected liability released.

$$\text{Normal Cost}_{85} = 50 \times 120 \times (.059 + .083 + .117)$$
$$= 6000 \times .259 = 1{,}554$$

Accrued Liability on 1/1/86 before adjusting for death:

$$AL_{86} = (10{,}626 + 1{,}554)(1.07) = 13{,}033$$

Accrued Liability for deceased as of 1/1/86 (released from plan):

$$= 50 \times (4 \text{ years service}) \times 120 \times .125 = 3{,}000$$

Value of death benefit (expected release):

$$= 3{,}000 \times (1 - p_{35}) = 90$$

$$\text{Accrued Liability} = 13{,}033 - 3{,}000 + 90 = 10{,}123$$

**Problem 1 - 9**

Key Concepts:

(1) Normal cost is present value of future benefits attributable to benefits accruing in 1985 (i.e. $10 per month).

(2) Withdrawals are assumed to occur at end of year; however, as a general condition, retirements occur at retirement age (i.e. beginning of the year.)

(3) Total present value of future benefits includes "cost of vesting" and cost of retirement.

Data provided: Participant is 63 and is 100% vested.

1) Cost of Retirement Benefit:

$$\text{Benefit} \times \ddot{a}_{65}^{(12)} \times (D_{65}^{(T)}/D_{63}^{(T)})$$

2) Cost of vesting:

Note: Withdrawal may occur at end of the year of age 63 or 64.

$[(\text{Benefit} \times \ddot{a}_{65}^{(12)}) / (1+i)^2] \times (p_{63}^{(T)} q_{64}^{(w)} p_{64}^{(m)} + q_{63}^{(w)}\,{}_2p_{63}^{(m)})$

3) Total Normal Cost:

$\text{Benefit} \times \ddot{a}_{65}^{(12)}\ [(D_{65}^{(T)} / D_{63}^{(T)}) + (q_{63}^{(w)}\,{}_2p_{63}^{(m)} + p_{63}^{(T)} q_{64}^{(w)}\, p_{64}^{(m)})v^2]$

$= 10 \times 12 \times 10 \times [(805/1000) + [(.03)(.973)(.971) + (.943)(.03)(.971)]/(1.05)^2]$
$= 1200(.8050 + .0506) = 1{,}026.75$

Note: $q_{63}^{(w)} = q_{63}^{(T)} - q_{63}^{(m)} = .03$

$q_{64}^{(w)} = q_{64}^{(T)} - q_{64}^{(m)} = .03$

$p_{63}^{(T)} = 1 - q_{63}^{(T)} = 1 - .057 = .943$

$p_{63}^{(m)} = 1 - q_{63}^{(m)} = 1 - .027 = .973$

$p_{64}^{(m)} = 1 - q_{64}^{(m)} = 1 - .029 = .971$

Since withdrawals occur at the end of the year, we can use $p_x^{(m)}$ as the mortality rate since $p_x'^{(m)} = p_x^{(m)}$.

Answer is D.

**Problem 1 - 9** (Alternative Solution)

Because the participant is 100% vested, the normal cost benefit payable is independent of whether or not he terminates. Therefore, the only contingency for which a "discount" is necessary is mortality.

Therefore, $\text{NC} = \text{Benefit} \times \ddot{a}_{65}^{(12)} \times p_{63}^{(m)} \times p_{64}^{(m)} \times v^2$
$= 1200 \times .973 \times .971 \times (1.05)^{-2} = 1028.34$

**Problem 1 - 10**

Key Concepts: (1) To calculate a gain or loss, the expected UAL is needed. To calculate the expected UAL, the 1/1/84 valuation results must be obtained.

(2) The effect of the plan amendment must also be taken into account.

Data Provided: Participant is age 45 with 15 years of service.

Calculation of 1/1/84 valuation results:

$$AL_{45} = AB_{45} \times 12\ddot{a}_{65}^{(12)} \times v^{20}$$
$$= (12)(15) \times 120 \times .3118 = 6{,}735$$

$$NC_{45} = 12 \times 120 \times .3118 = 449$$

Expected $UAL_{46}$ (before amendment)

$$= (AL_{45} + NC_{45}) \times (1 + i) - \text{Contribution} \times (1 + i)$$
$$= (6{,}735 + 449) \times 1.06 - (1{,}200)(1.06) = 6{,}343$$

Increase in $UAL_{46}$ due to amendment:

$$= 3 \times (\text{years of service}) \times 12\ddot{a}_{65}^{(12)} \times v^{19}$$
$$= 3 \times 16 \times 120 \times .3305 = 1{,}904$$

Expected $UAL_{46}$ (after amendment)

$$= 6{,}343 + 1{,}904 = 8{,}247$$

Total Gain (Loss) = Expected UAL - Actual UAL
= 8,247 - 8,100
= 147 Gain (Because all other assumptions were realized)

Answer is D.

**Problem 1 - 11**

To compute the actuarial Gain or Loss:

$$\begin{aligned}
\text{Gain/Loss} &= (UAL_{83} + NC_{83})(1 + i) - C_{83} - I_c - UAL_{84} \\
UAL_{83} &= 70{,}000 - 45{,}000 = 25{,}000 \\
\text{Gain} &= (25{,}000 + 6{,}000)(1.07) - 9000 - UAL_{84} \\
4{,}500 &= (31{,}000)(1.07) - 9{,}000 - UAL_{84} \\
UAL_{84} &= 24{,}170 - 4{,}500 = 19{,}670 \\
\text{Asset Value} &= 75{,}000 - 19{,}670 = 55{,}330
\end{aligned}$$

Answer is D.

**Problem 1 - 12**

We first compute the expected Accrued Liability as of 1983 considering only those factors other than investment return.

$$\begin{aligned}
eAL_{83} &= (UAL_{83} + Assets_{83}) + \text{non-investment gain} \\
&= 450{,}000 + 425{,}000 + 13{,}400 = 888{,}400
\end{aligned}$$

We use the formula for computing gain/loss again considering non-investment factors only.

$$\begin{aligned}
eAL_{83} &= (AL_{82})(1.06) + NC_{82} - (\text{benefit payments})(1.06) \\
&= [AL_{82}(1.06) + (50{,}000)] - (10{,}000)(1.06) \\
&= 1.06(AL_{82}) + 50{,}000 - 10{,}600 \\
&= 1.06(AL_{82}) + 39{,}400 = 888{,}400
\end{aligned}$$

Solving for $AL_{82}$:

$$AL_{82} = (888{,}400 - 39{,}400)/1.06 = 800{,}943$$

Answer is B.

**Problem 1 - 13**

Key Concepts: The Unit Credit or Accrued Benefit Funding Method has several unique properties. Among them are the following:

(1) Early retirement benefits that are actuarially equivalent cause no gain or loss.

(2) Since the accrued liability for each participant is exactly equal to the present value of his accrued benefit, there is no gain or loss at employment termination, if fully vested.

(3) After 100% vesting, the values of Normal Cost and Accrued Liability are independent of withdrawal decrements used.

We first compute the Accrued Liability ignoring withdrawal tables:

$$\begin{aligned} AL &= (20 \times 10 \text{ years of service}) \times 12\ddot{a}_{65}^{(12)} \times {}_{35}p_{30} \times 1.07^{-35} \\ &= 200 \times 108 \times .9 \times .09366 = 1{,}821 \end{aligned}$$

We next calculate the accrued liability for future retirement benefits (Note: the probability of survival to age 65 = .4 × .9 = .36):

Accrued Liability for future benefits

$$\begin{aligned} &= (20 \times 10 \text{ years}) \times 12\ddot{a}_{65}^{(12)} \times .36 \times 1.07^{-35} \\ &= (36/90) \times 1{,}821 = 728 \end{aligned}$$

Hence, the accrued liability for future vesting benefits is:

$$=1821 - 728 = 1{,}093$$

Answer is E.

**Problem 1 - 14**

Key Concepts: (1) The total normal cost is the present value of the retirement benefit accrued for the year, plus the present value of the refund of the current year's employee contributions. The <u>employer</u> normal cost is equal to the total normal cost, minus the employee contributions (discounted from the end of the year to the valuation date).

(2) Because the employee contributions accumulate at an interest rate equal to the assumed interest rate, the present value of the refund can be determined based on an interest rate of 0%.

Step I: Present value of retirement benefit accrual

$$= 50 \times 12 \times \ddot{a}_{65}^{(12)} \times (D_{65}/D_{40})$$
$$= 600 \times 10 \times (118/1{,}314) = 539$$

Step II: Based on concept 2 above, the present value of refund is the amount of contribution times the probability of termination or death after contribution is made and prior to retirement, discounted to the valuation date.

Probability of survival from age 41 to age 65

$$l_{65}/l_{41} = (D_{65}/D_{41}) \times (1 + i)^{24}$$
$$= (118/1{,}160) \times 4.0489 = .4119$$

Therefore, the probability of decrement = 1 - .4119 = .5881

Present value of refund = Expected Contribution × $(D_{41}/D_{40})$ × .5881

$$= 475 \times (1{,}160/1{,}314) \times .5881 = 247$$

Step III: Total Normal Cost = 539 + 247 = 786

Step IV: Because the employee contribution is assumed to be made at the end of the year, its present value as of 1/1/83 is the appropriate amount to subtract from the total normal cost which has been determined as of 1/1/83.

Present Value of Employee contribution

$$= 475 \times (D_{41}/D_{40})$$
$$= 475 \times (1{,}160/1{,}314) = 419$$

Step V: Employer Normal Cost = 786 - 419 = 367

Answer is D.

**Problem 1 - 14** (Alternative Solution)

Because of concept (2) above, a termination or death has no impact (positive or negative) on the employer's cost of the plan. Therefore, there is an employer cost only for those who remain to retirement. The employer cost for such a participant is the cost of the retirement benefit not provided by the employee's contributions. This is calculated as:

Total value at retirement for current year accrued:

$$= 50 \times 12 \times \ddot{a}_{65}^{(12)} = 6{,}000$$

Accumulated Employee contribution for current year

$$= 475 \times (1 + i)^{24}$$
$$= 475 \times 4.0489 = 1{,}923$$

Value of Employer benefit at retirement

$$= 6{,}000 - 1{,}923 = 4{,}077$$

Present value = Employer Normal Cost

$$= 4{,}077 \times (D_{65}/D_{40})$$
$$= 4{,}077 \times (118/1{,}314) = 366$$

**Problem 1 - 15**

Key Concept: From the definition of actuarial gain, the Unfunded Accrued Liability can be determined as follows:

$$\text{Gain} = (UAL_{85} + NC_{85})(1 + i) - (C_{85} + I_c) - UAL_{86}$$

and $$UAL_{86} = (UAL_{85} + NC_{85})(1 + i) - (C_{85} + I_c) - \text{Gain}$$

All of the above terms are known except the $UAL_{85}$ and the segment $(C_{85} + I_c)$.

Step I: Determine the Unfunded Accrued Liability as of 1/1/85.

$$UAL_{85} = AL_{85} - Assets_{85}$$
$$= 850{,}000 - 420{,}000 = 430{,}000$$

Step II: Determine the value of $(C_{85} + I_c)$.

$$Assets_{86} = (Assets_{85})(1 + i) + (C_{85} + I_c) + Gain - (BP + I_{BP})$$

where BP represents Benefit Payments and $I_{BP}$ represents interest on benefit payments.

$$573{,}000 = (420{,}000)(1.08) + (C_{85} + I_c) + 600 - (6{,}000)(1 + 13i / 24)$$

$$(C_{85} + I_c) = 573{,}000 - 453{,}600 - 600 + (6{,}000)(1.0433) = 125{,}060$$

Step III: From formula above, determine the Unfunded Accrued Liability as of 1/1/86.

$$\begin{aligned} UAL_{86} &= (UAL_{85} + NC_{85})(1 + i) - (C_{85} + I_c) - Gain \\ &= (430{,}000 + 70{,}000)(1.08) - 125{,}060 - 600 \\ &= 540{,}000 - 125{,}060 - 600 = 414{,}340 \end{aligned}$$

Answer is B.

**Problem 1 - 15** (Alternative Solution)

Step I: Use the formula for calculating the gain/loss considering the non-investment factors.

$$\begin{aligned} AL_{86} &= (AL_{85} + NC_{85})(1 + i) - (\text{Benefit Payments})(1 + 13i/24) \\ &= [(850{,}000 + 70{,}000)(1.08)] - 6{,}000(1.0433) = 987{,}340 \end{aligned}$$

Step II:
$$\begin{aligned} UAL_{86} &= AL_{86} - Assets_{86} \\ &= 987{,}340 - 573{,}000 = 414{,}340 \end{aligned}$$

**Problem 1 - 16**

Key Concept: The accrued liability at each retirement age must be calculated by multiplying the accrued benefit by a factor that is the probability of retiring at that age by $\ddot{a}_x^{(12)}$ for that age.

Step I: Determine the Accrued Benefit at age 60.

$$\begin{aligned} \text{Accrued Benefit} &= 10 \times 20 \text{ years of service} \\ &= 200 \text{ per month or } 2{,}400 \text{ per year.} \end{aligned}$$

Step II: Determine the Present Value of the Accrued Benefit (Accrued Liability) for each retirement age.

For those retiring at age 63:

$$\begin{aligned} AL_{63} &= (2{,}400)(\ddot{a}_{63}^{(12)})(\text{probability of retiring at 63})(v^3) \\ &= (2{,}400)(10.4)(.2)(v^3) = 4{,}075 \end{aligned}$$

For those retiring at age 64:

$$\begin{aligned} AL_{64} &= (2{,}400)(\ddot{a}_{64}^{(12)})(\text{probability of not retiring at 63}) \\ &\qquad \times (\text{probability of retiring at 64})(v^4) \\ &= (2{,}400)(10.2)(1 - .2)(.5)(v^4) \\ &= (2{,}400)(10.2)(.8)(.5)(.7629) = 7{,}470 \end{aligned}$$

For those retiring at age 65:

$$\begin{aligned} AL_{65} &= (2{,}400)(\ddot{a}_{65}^{(12)})(\text{probability of } \underline{\text{not}} \text{ retiring at 63 or 64} \\ &\qquad \underline{\text{and}} \text{ retiring at 65})(v^5) \\ &= (2{,}400)(10.0)(1 - .2)(1 - .5)(1.0)(v^5) \\ &= (2{,}400)(10.0)(.8)(.5)(1.0)(.713) = 6{,}845 \end{aligned}$$

Total accrued liability = 4,075 + 7,470 + 6,845 = 18,390

Answer is B.

**Problem 1 - 17**

Key Concept: The total actuarial gain can be determined as well as the investment gain. The answer is the difference the total actuarial gain and the investment gain.

Step I: Determine the gain from investments alone.

$$\begin{aligned} e\text{Assets}_{87} &= (\text{Assets}_{86})(1.08) + \text{Contribution}(1{+}.04) \\ &= (0 \times 1.08) + (25{,}000)(1.04) = 26{,}000 \end{aligned}$$

$$\begin{aligned} \text{Assets}_{87} &= \text{Contribution} \times 1.05 \\ &= 25{,}000 \times 1.05 = 26{,}250 \end{aligned}$$

Gain from investments alone = 26,250 - 26,000 = 250

Step II: Determine expected Unfunded Accrued Liability at 1/1/87.

$$
\begin{aligned}
UAL_{86} &= AL_{86} - Assets_{86} \\
&= 135{,}100 - 0 = 135{,}100
\end{aligned}
$$

$$
\begin{aligned}
eUAL_{87} &= (UAL_{86} + NC_{86})(1.08) - C_{86} - I_c \\
&= (135{,}100 + 10{,}000)(1.08) - 25{,}000 - (25{,}000)(.04) = 130{,}708
\end{aligned}
$$

Step III: Determine total actuarial gain.

$$
\begin{aligned}
UAL_{87} &= AL_{87} - Assets_{87} \\
&= 156{,}000 - 26{,}250 = 129{,}750 \\
\text{Gain} &= eUAL - UAL \\
&= 130{,}708 - 129{,}750 = 958
\end{aligned}
$$

Gain excluding investment gain = 958 - 250 = 708

Answer is D.

**Problem 1 - 17** (Alternative Solution)

Key Concept: In the foregoing solution, we used simple interest. We could just as well have used compound interest since neither is specified. In the solution that follows, compound interest is used.

$$
\begin{aligned}
eAssets_{87} &= (\text{Contribution})(1.08^{.5}) \\
&= 25{,}000 \times 1.0392 = 25{,}980
\end{aligned}
$$

$$
\begin{aligned}
\text{Assets} &= \text{Contribution} \times (1.10^{.5}) \\
&= 25{,}000 \times 1.0488 = 26{,}220
\end{aligned}
$$

Actuarial gain from investments = 26,220 - 25,980 = 240

$$
\begin{aligned}
eUAL_{87} &= (135{,}100 + 10{,}000)(1.08) - 25{,}000(1.08)^{.5} \\
&= 156{,}708 - 25{,}980 = 130{,}728
\end{aligned}
$$

$$\text{Actual } UAL_{87} = 156{,}000 - 26{,}220 = 129{,}780$$

$$\text{Gain} = 130{,}728 - 129{,}780 = 948$$

Gain excluding investment earnings = 948 - 240 = 708

Answer is the same.

We could also solve the problem by ignoring actual investment earnings and use the assumed interest for investment earnings. In this case:

$$\text{Assumed Assets}_{87} = 25{,}000\ (1.04) = 26{,}000$$

$$UAL_{87} = 156{,}000 - 26{,}000 = 130{,}000$$

as before $\quad eUAL_{87} = 130{,}708$

$$\text{Gain} = 130{,}708 - 130{,}000 = 708$$

Answer is the same.

**Problem 1 - 18**

Key Concept: Normal cost must be computed as the sum of (1) normal cost based upon probability of retiring at age 63, (2) normal cost based upon probability of not retiring at age 63 and retiring at age 64, and (3) normal cost based upon the probability of not retiring at age 63 or 64 and retiring at age 65.

Normal cost based upon retirement at age 63:

$$NC_{63} = 20\% \times 12 \times 20 \times \ddot{a}_{63}^{(12)} \times v^{8}$$
$$= 20\% \times 12 \times 20 \times 8.56 \times .5403 = 221.99$$

Normal cost based upon retirement at age 64:

$$NC_{64} = (30\%)(1 - 20\%)(12)(20) \times \ddot{a}_{64}^{(12)} \times v^{9}$$
$$= (.30)(.80)(12)(20)(8.35)(.5002) = 240.58$$

Normal cost based upon retirement at age 65:

$$NC_{65} = (1 - 30\%)(1 - 20\%)(12)(20) \times \ddot{a}_{65}^{(12)} \times v^{10}$$
$$= (.70)(.80)(12)(20)(8.14)(.4632) = 506.75$$

Total Normal Cost = 221.99 + 240.58 + 506.75 = 969.32

Answer is C.

**Problem 1 - 19**

Key Concepts: As with all problems dealing with actuarial gains, the expected Accrued Liability must be compared with the actual Accrued Liability.

In this problem, we must calculate the expected Accrued Liability by projecting ahead to 12/31/86. Hence, commutation functions at age 59 should be used.

Step I: Determine the expected Accrued Liability at 12/31/86.

$$eAL_{87} = (25 \times 12)(24 \text{ years})(N_{65}^{(12)}/D_{59})$$
$$= 300 \times 24 \times 1{,}880 \div 400 = 33{,}840$$

Step II: Determine actual Accrued Liability at 12/31/86.

$$\text{Benefit} = (25 \times 24 \text{ years})(1 - 5/15 - 1/30)$$
$$= 600 \times 19 \div 30 = 380$$

$$AL_{87} = (\text{Benefit} \times 12)(N_{59}^{(12)}/D_{59})$$
$$= 4{,}560 \times 3{,}740 \div 400 = 42{,}636$$

$$\text{Gain/(Loss)} = 33{,}840 - 42{,}636 = (8{,}796) \text{ Loss}$$

Answer is D.

**Problem 1 - 20**

Key Concept: To determine the Unfunded Accrued Liability at 1/1/87, the Actuarial Asset Value and Accrued Liability must be determined. To determine these quantities, the Normal Cost and Actuarial Asset value at 1/1/86 must be calculated.

| | |
|---|---|
| Age of participant at 1/1/86: | 63 |
| Years of past service: | 5 |
| Years of future service: | 2 |

Step I: Determine Accrued Liability at 1/1/86:

$$AL_{86} = (5 \text{ years} \times 20) \times (12 \times \ddot{a}_{65}^{(12)} \times (1.07^{-2}))$$
$$= (1{,}200 \times 10) \times .8734 = 10{,}481$$

Step II: Determine Normal Cost at 1/1/86:

$$NC_{86} = 20 \times (12 \times \ddot{a}_{65}^{(12)}) \times (1.07)^{-2}$$
$$= 20 \times (12 \times 10) \times .8734 = 2{,}096$$

Step III: Determine Actuarial Value of Assets at 1/1/86:

$$UAL_{86} = AL_{86} - Assets_{86}$$

or $$Assets_{86} = AL_{86} - UAL_{86}$$
$$= 10{,}481 - 2{,}000 = 8{,}481$$

Step IV: Determine Actuarial Value of Assets at 1/1/87:

$$Assets_{87} = (Assets_{86} + NC_{86})\,(1 + .11)$$
$$= (8{,}481 + 2{,}096)(1.11) = 11{,}740$$

Step V: Determine Accrued Liability at 1/1/87:

$$AL_{87} = (6 \text{ years} \times 20) \times (12 \times \ddot{a}_{65}^{(12)}) \times (1.08)^{-1}$$
$$= (120) \times (12 \times 9) \times .9259 = 12{,}000$$

(Note that $\ddot{a}_{65}^{(12)}$ is valued using the new 8% interest assumption.)

Step VI: Determine Unfunded Accrued Liability at 1/1/87:

$$UAL_{87} = AL_{87} - Assets_{87}$$
$$= 12{,}000 - 11{,}740 = 260$$

Answer is A.

**Problem 1 - 21**

Key Concept: Under the Projected Unit Credit Method, salaries are projected to the retirement date under the salary increase assumption. The projected salaries are then used to calculate the retirement benefit with annual accruals based upon this projected benefit spread evenly over the total years of service.

Remember, when using a salary increase assumption, count the number of salary changes to normal retirement. In this problem, each participant is age 60 but salary is given for 1987, so there are only 4 projected salary changes.

| | | Smith | Brown |
|---|---|---|---|
| (1) | Attained Age | 60 | 60 |
| (2) | Past years of service | 20 | 27 |
| (3) | Future years of service | 5 | 5 |
| (4) | Total years of service | 25 | 32 |
| (5) | Projected salary at NRD (Salary × $1.06^4$) | 90,898 | 30,299 |
| (6) | Projected Pension (5) × 1% × (4) | 22,724 | 9,696 |
| (7) | Annual Accrual (6) ÷ (4) | 909 | 303 |
| (8) | Normal Cost (7) × 8.33 × $1.08^{-5}$ | 5,153 | 1,718 |

(9) Total Normal Cost = 5,153 + 1,718 = 6,871

Answer is D.

**Problem 1 - 22**

Key Concept: Expected Unfunded Accrued Liability at 12/31/87 is equal to the unfunded accrued liability at 12/31/86 plus normal cost less contributions all adjusted for interest. Interest on 1986 contributions should be accumulated to 12/31/87.

$$eUAL_{87} = (UAL_{86} + NC)(1 + i) - \text{Contribution} - I_c$$
$$I_c = (21/12)(7{,}000)(.06) + (18/12)(7{,}000)(.06) + (20{,}000)(.06) = 2{,}565$$
$$eUAL_{87} = (250{,}000 + 20{,}000)(1.06) - 34{,}000 - 2{,}565$$
$$= 286{,}200 - 34{,}000 - 2{,}565 = 249{,}635$$

Answer is D.

**Problem 1 - 23**

Key Concept: In this problem, any actuarial gain could only arise from mortality or earnings. Hence, the mortality gain would be the actuarial gain less any investment gain.

| | |
|---|---|
| Ages on 1/1/88 | 45 |
| Years of past service | 7 |
| Accrued Benefit per participant | \$70 per month |

$$\text{PV of Accrued Benefits} = (70)(1{,}000)(12\ddot{a}_{65}^{(12)})(D_{65}/D_{45})$$
$$= (70)(1{,}000)(12)(9.35)(77/303) = 1{,}995{,}901$$

$$\text{Normal Cost as of 1/1/88} = (10)(1{,}000)(12\ddot{a}_{65}^{(12)})(D_{65}/D_{45})$$
$$= (10)(1{,}000)(12)(9.35)(77/303) = 285{,}129$$

$$\text{Expected AL} = (1{,}995{,}901 + 285{,}129)(D_{45}/D_{46})$$
$$= (2{,}281{,}030)(303/283) = 2{,}442{,}234$$

$$\text{Expected AL (from interest only)} = (2{,}281{,}030)(1.06) = 2{,}417{,}892$$

$$\text{Actual AL} = (80)(988 \text{ participants})(12\ddot{a}_{65}^{(12)})(D_{65}/D_{46})$$
$$= (80)(988)(12)(9.35)(77/283) = 2{,}412{,}926$$

$$\text{Mortality Gain} = e\text{AL (Interest)} - \text{Actual AL}$$
$$= 2{,}417{,}892 - 2{,}412{,}926 = 4{,}966$$

Answer is E.

**Problem 1 - 24**

Key Concept: This problem calls for valuation of benefits that are paid for a limited period of time, namely for five years. The value of this benefit at normal retirement age 60 is determined by the formula:

$$PVFB_{60} = (\text{Benefit})[(N_{60}^{(12)} - N_{65}^{(12)})/D_{60}]$$

| Prior to amendment and assumption change | Normal Retirement Age 60 | Normal Retirement Age 65 |
|---|---|---|
| Attained Age | 50 | 50 |
| Past years of service | 20 | 20 |
| Future Years of service | 10 | 15 |
| Total years of service | 30 | 35 |

Step I: Determine Accrued Liability before amendment and assumption change. Since Normal Retirement age is 65:

$$\text{Benefit at age 65} = (20)(35 \text{ years}) = 700$$

$$AL = (20 \text{ years}/35 \text{ years})(700)(12)(N_{65}^{(12)}/D_{50})$$
$$= (20/35)(700)(12)(440/202) = 10{,}455$$

Step II: Determine Accrued Liability after plan amendment and assumption change.

The benefit consists of two components:

(1) Usual benefit of $20 per month for each year of service
(2) Monthly supplement for five years of $6.67 per month for each year of service.

Consider the benefit of $20 per month per year of service.

Benefit at age 60 = (20)(30 years) = 600

$$AL = (20 / 30)(600)(12)(N_{60}^{(12)} / D_{50})$$
$$= (20 / 30)(600)(12)(787 / 202) = 18{,}701$$

Now consider the benefit of $6.67 per month for each year of service payable from age 60 to age 65.

Benefit at age 60 = (6.67)(30) = 200

$$AL = (20 \text{ years} / 30 \text{ years})(200)(12)[(N_{60}^{(12)} - N_{65}^{(12)})/ D_{50}]$$
$$= (20 / 30)(200)(12)[(787 - 440) / 202] = 2{,}748$$

Total AL = 18,701 + 2,748 = 21,449

Increase in Accrued Liability due to plan amendment and assumption change:

Change = 21,449 - 10,455 = 10,994

Answer is C.

**Problem 1 - 25**

Key Concept: Commutation functions are not given, however, the quantity $N_{65}^{(12)}/D_{60}$ can be determined from information given.

Step I: From data provided, calculate $N_{65}^{(12)}/D_{60}$.

$$NC_{60} = (10)(100 \text{ participants})(12)(N_{65}^{(12)}/D_{60}) = 100{,}000$$
$$= (12{,}000)(N_{65}^{(12)}/D_{60}) = 100{,}000$$

or $\quad N_{65}^{(12)}/D_{60} = 100{,}000/12{,}000 = 8.3333$

Step II: Determine the value of $N_{65}^{(12)}/D_{61}$

$$D_{61} = (l_{61})(v^{61}) = (p_{60}l_{60})(v^{60} \times v)$$

$$= (p_{60} \times v)(l_{60}v^{60})$$

$$= (p_{60} \times v)(D_{60})$$

and $\quad N_{65}^{(12)}/D_{61} = N_{65}^{(12)}/[(p_{60} \times v)(D_{60})]$

$$= (N_{65}^{(12)}/D_{60})/(p_{60} \times v)$$

$$= 8.3333(1.06)/p_{60} = 8.3333(1.06)/.96 = 9.2014$$

Step III: Determine Normal Cost at 1/1/88. Since there were no experience gains or losses during 1987, there must have been four deaths and the number of participants on 1/1/88 was 96.

$$NC_{88} = (10)(96 \text{ participants})(12)(N_{65}^{(12)}/D_{61})$$
$$= (11{,}520)(9.2014) = 106{,}000$$

Answer is D.

Shortcut: The answer can be obtained very simply by increasing the normal cost by interest only ($100{,}000 \times 1.06 = 106{,}000$). This works since four deaths occurred and hence, no experience gain or loss.

**Problem 1-26**

$$\text{Final salary} = 100{,}000 \times (1.05)^{20} = 265{,}330$$

$$\text{Projected Benefit} = .025 \times (265{,}330)(40 \text{ years}) = 265{,}330$$

From the original valuation, we can calculate the Present Value of Future Benefits (PVFB) as follows:

$$AL = \frac{\underline{PVFB \times (\text{years of service as of } 1/1/89)}}{\text{Years of service at retirement}}$$

$$300{,}000 = PVFB \times (20/40)$$
$$PVFB = 600{,}000$$

Note that,

$$PVFB = {}_{20}p_{45} \times v^{20} \times \ddot{a}_{65}^{(12)} \times \text{Benefit}_{65}$$

$$= p_{45} \times {}_{19}p_{46} \times v^{20} \times \ddot{a}_{65}^{(12)} \times \text{Benefit}_{65}$$

The original value of $p_{45}$ was:

$$p_{45} = 1 - q_{45} = 1 - .034 = .966$$

The correct value of $p_{45}$ is:

$$p_{45} = 1 - q_{45} = 1 - .0034 = .9966$$

With the revised mortality at age 45, the new Present Value of Future Benefits can be calculated as

$$PVFB' = (600{,}000) \times \frac{p'_{45}}{p_{45}} = 600{,}000 \times (.9966/.966) = 619{,}006$$

Therefore, $AL = 619{,}006 \times (20/40) = 309{,}503$

Answer is C.

**Problem 1 - 27**

Key Concept: The Accrued Liability in the Unit Credit Method is the present value of the accrued benefit as of the first day of the plan year.

Step I: Calculate the Accrued Liability under the old retirement assumptions.

$$\text{Accrued Benefit} = (25)(10 \text{ Years of Service})$$

$$\text{Accrued Liability} = (250)(12\ddot{a}_{65}^{(12)})(v^3) = 23{,}551$$

Step II: Calculate the Accrued Liability under the new retirement assumptions.

Under the new assumption, $q_{62}^{(r)} = .4$ and $q_{65}^{(r)} = 1.0$.

The probability of retirement at age 62 is $q_{62}^{(r)} = .4$, and the probability of retirement at age 65 is $(1 - q_{62}^{(r)})(q_{65}^{(r)}) = 0.6$.

The total Accrued Liability is the sum of the present value of the accrued benefits if retirement occurs at each of ages 62 and 65, each multiplied by the probability of retirement at that age.

If retirement occurs at age 62,

$$\begin{aligned}\text{Accrued Benefit} &= (250)(\text{early retirement adjustment})\\ &= (250)(1 - 3/15) = 200\end{aligned}$$

$$\text{PVAB} = (200) \times 12\ddot{a}_{62}^{(12)} = 24{,}240$$

If retirement occurs at age 65, the present value of the accrued benefit is \$23,551, as in Step I. Therefore,

$$\text{AL} = (24{,}240)(.4) + (23{,}551)(.6) = 23{,}827$$

Step III: Calculate the increase in the Accrued Liability.

$$\text{Increase} = 23{,}827 - 23{,}551 = 276$$

Answer is A.

**Problem 1 - 28**

Key Concept: The sole participant is fully vested as of 1/1/90. Under the Unit Credit Method, withdrawal decrements can be ignored for fully vested participants. Normal Cost is the present value of the accrual for the current year.

$$\text{NC} = 10 \times 12\ddot{a}_{65}^{(12)} \times v^2 \times p_{63}^{\prime(d)} \times p_{64}^{\prime(d)}$$

$$= 10 \times 12\ddot{a}_{65}^{(12)} \times v^2 \times (1 - .019) \times (1 - .021) = 879$$

Answer is C.

Note that since terminations occur at the end of the year,

$$q_x'^{(d)} = q_x^{(d)}$$

**Problem 1 - 29**

First, find the Final Year's Projected Compensation:

$$40{,}000\,(1.04)^{29} = 124{,}746$$

Then the benefit can be calculated as follows:

$$\text{Benefit at retirement} = .025 \times 124{,}746 \times 38 \text{ years of service} = 118{,}509$$

The Projected Unit Credit Normal Cost is the present value of that portion of the projected benefit at retirement which is accrued for the year.

$$NC_{89} = \text{Benefit} \times v^{29} \times \ddot{a}_{63}^{(12)}/38 = 2{,}872$$

The accrued liability is the number years of past service times the normal cost:

$$UL_{89} = AL_{89} = NC_{89} \times 9 \text{ years} = 25{,}848$$

The expected unfunded accrued liability as of 1/1/90 is based on the 1/1/89 unfunded accrued liability and normal cost as follows:

$$e\text{UAL}_{90} = (UL_{89} + NC_{89})\,(1.08) - C_{89} - I_c$$

Since the \$6,000 contribution was deposited on 4/1/89, the interest is:

$$I_c = (6{,}000) \times (.08) \times (3/4) = 360$$

$$e\text{UAL}_{90} = (25{,}848 + 2{,}872)(1.08) - 6{,}000 - 360 = 24{,}658$$

$$\text{Actual } UL_{90} = 25{,}000$$

$$\text{Loss} = 25{,}000 - 24{,}658 = 342$$

Answer is D.

**Problem 1 - 30**

Key Concept: Since there is no experience gain or loss due to investments, the experience gain is equal to the excess of the expected Accrued Liability over the Actual Accrued Liability.

Step I: Calculate the expected Accrued Liability.

$$\begin{aligned} e\mathrm{AL}_{90} &= (\mathrm{AL}_{89} + \mathrm{NC}_{89}) \times (1 + i) \\ &= (21{,}000{+}2{,}000) \times (1.07) = 24{,}610 \end{aligned}$$

Step II: Calculate the actual Accrued Liability

$$\text{Final salary} = 40{,}000 \times (1.05)^{15} = 83{,}157$$

$$\text{Projected Past Service Benefit as of 1/1/90} = 83{,}157 \times (.01) \times 10 \text{ years} = 8{,}316$$

$$\begin{aligned} \text{Actual Accrued Liability} &= \text{Present Value of Projected Past Service benefit} \\ &= 8{,}316 \times N_{65}^{(12)} / D_{50} \\ &= 8{,}316 \times 849/320 = 22{,}063 \end{aligned}$$

Step III: Calculate the experience gain.

$$\text{Gain} = 24{,}610 - 22{,}063 = 2{,}547$$

Answer is C.

**Problem 1 - 31**

Key Concept: Normal cost for Projected Unit Credit (prorated on service) is equal to the present value of the projected benefit at retirement divided by accrual service.

The Normal Cost for the retirement benefit at 1/1/91 can be calculated:

$$\mathrm{NC}_r = (500 \times 12\ddot{a}_{65}^{(12)} \times v^5) \div 30$$

Normal cost for death benefit at 1/1/91:

$$\mathrm{NC}_d = 2{,}100 \times v^5 \div 30 = 50$$

To determine the Normal Cost for the retirement benefit, we must calculate the value of $\ddot{a}_{65}^{(12)}$.

Recall from Life Contingencies that:

$$A_x^{(m)} = 1 - d^{(m)}\ddot{a}_x^{(m)}$$

An approximation to $d^{(m)}$ is: $\dfrac{i}{(1+i)^{½}}$ which will be used in the solution.

Present value at age 65 of post-retirement death benefit:

$$\begin{aligned} 2{,}100 &= 5{,}000 \times A_{65}^{(12)} \\ &= 5{,}000\,(1 - d^{(12)}\ddot{a}_{65}^{(12)}) \\ &= 5{,}000\,(1 - .07/(1.07)^{½} \times \ddot{a}_{65}^{(12)}) \\ &= 5{,}000 - (338.36 \times \ddot{a}_{65}^{(12)}) \end{aligned}$$

Therefore, $\ddot{a}_{65}^{(12)} = 8.571$

The Normal Cost for retirement benefit at 1/1/91 can now be calculated:

$$NC_r = 500 \times 12 \times 8.571 \times v^5 / 30 = 1{,}222$$

$$NC = 1{,}222 + 50 = 1{,}272$$

Answer is B.

Note: Using the exact value for $d^{(12)}$ gives a value of 8.597 for $\ddot{a}_{65}^{(12)}$, which yields a Normal Cost for the retirement benefit of \$1,226. Then the total Normal Cost is \$1,276, which is still Answer B.

**Problem 1 - 32**

Step I: Determine the projected benefit as of 1/1/90.

$$\begin{aligned} \text{Benefit}_{90} &= \text{Salary}_{90} \times (1.05^7 + 1.05^8 + 1.05^9)/3 \times .01 \times \text{Years of Service} \\ &= \text{Salary}_{90} \times 1.4786 \times .01 \times \text{Years of Service} \end{aligned}$$

Step II: Determine the Normal Cost at 1/1/90.

$$NC_{90} = \text{Benefit} \times \ddot{a}_{65}^{(12)} \times v^{10} \times {}_{10}p_{55} \div \text{Years of Service}$$

Step III: Calculate the projected benefit as of 1/1/91.

$$\begin{aligned}\text{Benefit}_{91} &= \text{Salary}_{90} \times 1.08 \times (1.05^6 + 1.05^7 + 1.05^8)/3 \times .01 \times \text{Years of Service}\\ &= \text{Salary}_{90} \times 1.08 \times 1.4082 \times .01 \times \text{Years of Service}\end{aligned}$$

Step IV: Calculate the Normal Cost as of 1/1/91.

$$\text{NC}_{91} = \text{Benefit} \times \ddot{a}_{65}^{(12)} \times v^9 \times {}_9p_{56} \div \text{Years of Service}$$

Step V: We can now calculate the ratio of the Normal Costs.

$$\frac{\text{NC}_{91}}{\text{NC}_{90}} = \frac{\text{Benefit}_{91} \times \ddot{a}_{65}^{(12)} \times v^9 \times {}_9p_{56} \div \text{Years of Service}}{\text{Benefit}_{90} \times \ddot{a}_{65}^{(12)} \times v^{10} \times {}_{10}p_{55} \div \text{Years of Service}}$$

Since $\text{Benefit}_{91} = \text{Benefit}_{90} \times 1.08 \div 1.05$, we can cancel factors to result in:
$= (1.08 \times 1.4082)/(1.4786 \times vp_{55})$

$\Delta\text{NC}\% = 11.058\%$

Note that $p_{55} = 1 - q_{55} = 1 - .009 = .991$

Answer is D.

**Problem 1 - 32** (Alternative Solution)

Without the compensation increase, the normal cost will increase from one year to the next by interest and the probability of survivorship. Since salaries increased 8% (instead of the assumed 5%), the normal cost increases by an additional factor of (1.08/1.05).

$$\Delta\text{NC}\% = 1.07 \times (1.08/1.05)/p_{55}$$

$$= 1.07 \times (1.08/1.05)/(.991) = 11.057\%$$

**Problem 1 - 33**

Key Concept: The Accrued Liability under the Projected Unit Credit method is equal to the present value of the portion of the projected retirement benefit attributable to past service.

Step I: Calculate Accrued Liability using old retirement age assumption.

$$\text{Benefit}_{65} = .02 \times 21{,}000 \times \left( \frac{(1.05)^{22} + (1.05)^{23} + (1.05)^{24}}{3} \right) \times 27 \text{ years of service} = 34{,}859$$

$$\text{Normal Cost}_{91} = \frac{(34{,}859)\ v^{25}\ \ddot{a}_{65}^{(12)}}{27} = 2{,}081$$

$$\text{Accrued Liability}_{91} = (\text{Past Service}) \times NC_{91} = 2 \times 2{,}081 = 4{,}162$$

Step II: Calculate Accrued Liability using new retirement age assumption.

$\text{Benefit}_{62}$ (before early retirement adjustment)

$$= (.02) \times 21{,}000 \times \left( \frac{(1.05)^{19} + (1.05)^{20} + (1.05)^{21}}{3} \right) \times 24 \text{ years of service} = 26{,}766$$

Adjusting for retirement earlier than 65,

$$\text{Benefit}_{62} = 26{,}766 \times (12/15) = 21{,}413$$

$$\text{Normal Cost} = \frac{21{,}413 \times v^{22}\ \ddot{a}_{62}^{(12)}}{24} = 1{,}893$$

$$\text{Accrued Liability}_{91} = NC \times \text{Past Service} = 3{,}786$$

Step III: Calculate decrease in Accrued Liability

Accrued Liability = 3,786 - 4,162 = (376)

Answer is A.

**Problem 1 - 34**

Key Concept: An experience gain results when the early retirement reduction exceeds the plan actuarial equivalence reduction.

Step I: Determine the result of the election for the participant whose attained age is age 56.

The plan's early retirement reduction at age 56 is 5/15 + 4/30 = .4667.

Actuarial equivalence at age 56 can be calculated as:

$$X\ \ddot{a}_{56}^{(12)} = AB \times \ddot{a}_{65}^{(12)} \times v^{9}$$

$$X = AB \times \left[ \frac{\ddot{a}_{65}^{(12)}}{\ddot{a}_{56}^{(12)}} \times v^{9} \right]$$

$$= \left[ \frac{10}{11.5} \times .591898 \right] = .5147$$

Therefore, the actuarial equivalence reduction is 1 - .5147 = .4853

Thus, this election results in a loss since the early retirement reduction is less than the actuarial equivalent reduction.

Step II: Determine the result of the election for the participant whose attained age is age 59.

The plan reduction at age 59 is 5/15 + 1/30 = .3667

Actuarial Equivalence at age 59 can be calculated as:

$$X = AB \times \left[ \frac{\ddot{a}_{65}^{(12)}}{\ddot{a}_{59}^{(12)}} \times v^{6} \right] = \left[ \frac{10}{11} \times .70496 \right] = .6409$$

Therefore, the actuarial equivalence reduction is 1 - .6409 = .3591

Thus, this election results in a gain since the early retirement reduction exceeds the actuarial equivalence reduction.

Step III: The participant's attained age is age 63.

The plan's reduction at age 63 is 2/15 = .1333.

Actuarial Equivalence at age 63 can be calculated as:

$$X = AB \times \left[ \frac{\ddot{a}_{65}^{(12)}}{\ddot{a}_{63}^{(12)}} \times v^2 \right] = \frac{10}{10.33} \times .88999 = .8616$$

Therefore, the actuarial equivalence reduction is 1 - .8616 = .1384

Thus, this election results in a <u>loss</u> since the early retirement reduction is less than the actuarial equivalence reduction.

An experience gain results from the age 59 election only.

Answer is A.

**Problem 1 - 35**

Step I: Calculate the Retirement Benefit Normal Cost:

$$NC = 10 \times 12\ddot{a}_{65}^{(12)} \left( \frac{D_{65}}{D_{45}} \right) = 120 \times \frac{N_{65}^{(12)}}{D_{45}}$$

$$\text{Recall, } N_{65}^{(12)} = N_{65} - \frac{11}{24} D_{65}$$

$$8{,}872 - \frac{11}{24}(965) = 8{,}430$$

$$\text{Thus, } NC_{UC} = 120 \times \left( \frac{8{,}430}{4{,}548} \right) = 222$$

Step II: Calculate the One Year Term Cost for the Death Benefit

Death Benefit = 100 × 10 × 40 years of service = 40,000

Term Cost = Death Benefit × $v$ × $q_x$

We now must calculate $q_{45}$:

$$D_{45} = v^{45} \times l_{45} = 4{,}548$$

$$D_{46} = v^{46} \times l_{46} = 4{,}236$$

$$vp_{45} = \frac{4{,}236}{4{,}548}$$

$$p_{45} = .996596$$

$$q_{45} = 1 - p_{45} = .003404$$

The term cost can now be calculated as

$$\text{Term Cost} = 40{,}000 \times .003404 \times v = 127$$

$$\text{Total Cost at } 1/1/92 = 222 + 127 = 349$$

Answer is D.

**Problem 1-36**

Key Concept: The normal cost will equal the present value of 1/10 of the retirement benefit for each of the first 10 years of service. There will be no normal cost for each year after 10.

Annual Projected Benefit

$$= (.02) \times 10 \text{ years of service} \times 31{,}000 \times \left| \frac{(1.04)^8 + (1.04)^9 + (1.04)^{10}}{3} \right|$$

$$= (.02) \times (10) \times (31{,}000) \times \left[ \frac{4.272125}{3} \right] = 8{,}829$$

$$\text{NC} = \frac{8{,}829 \times \ddot{a}_{65}^{(12)} \times v^{10}}{10} = 3{,}476$$

Answer is E.

**Problem 1 - 37**

Key Concept: Experience gains are calculated before the effect of a plan amendment is taken into account.

The Accrued Liability as of 1/1/92, which reflects the plan amendment, is applicable to all participants.

$$AL_{92} \text{ Pre-amendment} = 22{,}500 \times (10.0/10.5) = 21{,}429$$

$$UAL_{92} = AL_{92} - \text{Assets} = 21{,}429 - 13{,}500 = 7{,}929$$

$$eUAL_{92} = (UAL_{91} + NC_{91})(1 + i) - C - I_c$$
$$= (10{,}000 + 1{,}000)(1.07) - 2000(1 + .07/2) = 9{,}700$$

$$\text{Gain} = 9{,}700 - 7{,}929 = 1{,}771$$

Answer is D.

To analyze the components of this gain, information is available to calculate the exact investment gain or loss. The component necessary for this analysis that was not needed for the above determination of the total gain was the amount of benefits paid.

The actual investment earnings can be determined from the equation

$$A_t + C_t - B_t + I_t = A_{t+1}$$

$$10{,}000 + 2{,}000 - 5000 + I_t = 13{,}500$$

$$I_t = 2{,}000$$

where $A_t$ represents the assets at the beginning of the year t, and $C_t$, $B_t$ and $I_t$ are the contributions, benefit payments and actual investment earnings for year t.

The expected investment earnings were

$$10{,}000\ (.07) + 2{,}000 \left(\frac{.07}{2}\right) - 500 \left(\frac{.07}{2}\right) = 753$$

Investment gain was 2,000 - 753 = 1,247.

The remainder of the gain, \$524, is due to experience of the active and inactive plan population (i.e., terminations, deaths, new entrants).

**Problem 1 - 38**

The experience gain/loss is the difference between the expected Unfunded Liability and the actual Unfunded Liability. Since the formula is a unit benefit formula,

$$NC_{91} = 100 \times 12\ddot{a}_{65}^{(12)} \times v^6 = 6{,}989$$

$$AL_{91} = \text{Past Service} \times NC_{91} = 4 \times 6{,}989 = 27{,}956$$

$$\begin{aligned} eUAL_{92} &= (AL_{91} - \text{Assets}_{91} + NC_{91})(1+i) - C - I_c \\ &= (27{,}956 - 19{,}000 + 6{,}989)(1.07) - 8{,}000 = 9{,}061 \end{aligned}$$

Note that the interest on the contribution, $I_c$, is \$0, since the contribution was paid on at the end of the year.

$$AL_{92} = 5 \times 100 \times 12\ddot{a}_{65}^{(12)} \times v^5 = 37{,}389$$

$$UL_{1/92\ \text{Actual}} = 37{,}389 - 26{,}000 = 11{,}389$$

Experience gain/loss = 11,389 - 9,061 = 2,328, a <u>loss.</u>

Answer is B.

**Problem 1 - 38 (Alternative Solution):**

A simpler approach in this case is due to the fact that the <u>only</u> component of experience is investment performance. It was assumed that no deaths or terminations would occur, and none did!

Expected Assets = 19,000 × 1.07 + 8,000 = 28,330

Actual Assets = 26,000

Experience Loss = 28,330 - 26,000 = 2,330

**Problem 1 - 39**

Key Concept: Actual liability equals 15% of AL if retirement occurs at age 62 plus 85% of AL if retirement occurs at age 65.

Step I: Calculate the Accrued Liability for Retirement Age 62.

$$\text{Final Average Earnings} = 25{,}000 \times \left[\frac{(1.06)^{25} + (1.06)^{26} + (1.06)^{27}}{3}\right] = 113{,}863$$

$\text{Benefit}_{62} = (.02)(29 \text{ years of service})(113{,}863)(.91 \text{ early retirement reduction}) = 60{,}097$

$$\text{PVFB}_{92} = \left[60{,}097 \times \ddot{a}_{62}^{(12)} \times v^{27} \times (.15)\right] = 9{,}897$$

$$\text{AL}_{92} = \frac{9{,}897}{29} \times 2 = 682$$

Step II: Calculate Accrued Liability for Retirement Age 65

$$\text{Final Average Earnings} = 25{,}000 \times \left[\frac{(1.06)^{28} + (1.06)^{29} + (1.06)^{30}}{3}\right] = 135{,}613$$

$\text{Benefit}_{65} = (.02)(30 \text{ years of service})(135{,}613) = 81{,}368$

$$\text{PVFB}_{92} = 81{,}368 \times \ddot{a}_{65}^{(12)} \times v^{30} \times .85 = 56{,}360$$

$$\text{AL}_{92} = \frac{56{,}360}{30} \times 2 = 3{,}758$$

$$\text{Total AL} = 682 + 3{,}758 = 4{,}440$$

Answer is B.

**Problem 1 - 40**

Key Concept: When applying probabilities of retirement to the Unit Credit funding method, the Accrued Liability is calculated by the computing the presenting value of prior accruals for each retirement age and multiplying each present value by the probability of retirement at that age.

Accrued Benefit (without regard to retirement age)

$$= (15)(20 \text{ years of service}) = 300$$

If retirement occurs at age 64, there is a 6% reduction in the benefit due to early retirement. Therefore, the Accrued Benefit would be

$$300 \times (1 - .06) = 282$$

The Present Value as of 1/1/93 of the Accrued Benefit at each retirement age is:

$$\text{Age 64: } 282 \times 12\ddot{a}_{64}^{(12)} \times v^4 = 25{,}816$$

$$\text{Age 65: } 300 \times 12\ddot{a}_{65}^{(12)} \times v^5 = 25{,}154$$

$$\text{Age 66: } 300 \times 12\ddot{a}_{66}^{(12)} \times v^6 = 23{,}029$$

The Accrued Liability can now be determined as follows:

$$\begin{aligned} AL &= (25{,}816)(q_{64}^{(r)}) + (25{,}154)(1 - q_{64}^{(r)})(q_{65}^{(r)}) + (23{,}029)(1 - q_{64}^{(r)})(1 - q_{65}^{(r)}) \\ &= (25{,}816)(.2) + (25{,}154)(.8)(.5) + (23{,}029)(.8)(.5) \\ &= 5{,}163 + 10{,}062 + 9{,}211 = 24{,}436 \end{aligned}$$

Answer is A.

**Problem 1 - 41**

The Accrued Liability before the change in assumptions is calculated:

$$\text{Prior } AL_{93} = \text{Accrued Benefit} \times 12\ddot{a}_{65}^{(12)} \times v^{12} = 20{,}000$$

The Accrued Liability after the mortality assumption is added is:

$$\begin{aligned} \text{New } AL_{93} &= \text{Accrued Benefit} \times 12\ddot{a}_{65}^{(12)} \times v^{12} \times (l_{65}/l_{53}) \\ &= \text{Prior } AL_{93} \times (l_{65}/l_{53}) \\ &= (20{,}000)(7{,}673{,}269/8{,}980{,}994) = 17{,}088 \end{aligned}$$

$$\text{Decrease} = 20{,}000 - 17{,}088 = 2{,}912$$

Answer is E.

**Problem 1 - 42**

Key Concept: The Accrued Liability determined using the Projected Unit Credit method is equal to the present value of the accrued benefit using projected salary.

Step I: Calculate the Final Average Salary. Note that since Smith and Brown have the same salary and date of birth, they will have the same Final Average Salary.

$$\text{Final Average Salary} = 50{,}000((1.05)^{14} + (1.05)^{13} + (1.05)^{12})/3$$
$$= 94{,}357$$

Step II: Calculate the accrued benefits using the Final Average Salary.

Smith: (94,357)((.02)(20 years of service) + (.01)(5 years of service)) = 42,461

Brown: (94,357)(.02)(10 years of service) = 18,871

Step III: Since $\ddot{a}_{65}^{(12)}$ was not provided, this factor must be calculated.

$$\ddot{a}_{65}^{(12)} = (N_{65} - \tfrac{11}{24}D_{65})/D_{65}$$
$$= 8.73577$$

Step IV: Calculate the Present Value of the Accrued Benefits.

Smith: $42{,}461 \times \ddot{a}_{65}^{(12)} \times (D_{65}/D_{50}) = 112{,}735$

Brown: $18{,}871 \times \ddot{a}_{65}^{(12)} \times (D_{65}/D_{50}) = 50{,}103$

Total Accrued Liability = 112,735 + 50,103 = 162,838

Answer is D.

**Problem 1 - 43**

Step I: Calculate the Final Average Salary at each retirement age. See Problem 4 - 53 for a discussion of Final Average Salary.

$$\text{Final Average Salary RA} = 65\text{:} \quad (50{,}000)((1.05)^{14} + (1.05)^{13} + (1.05)^{12})/3 = 94{,}357$$

$$\text{Final Average Salary RA} = 62\text{:} \quad (50{,}000)((1.05)^{11} + (1.05)^{10} + (1.05)^{9})/3 = 81{,}509$$

Step II: Calculate the Accrued Liability under the old retirement age assumptions (100% at age 65).

Projected Accrued Benefit = (94,357)(.02)(20 years of service) = 37,743

$$\begin{aligned} AL\ (RA\ 65) = PVAB &= (37{,}743)(\ddot{a}_{65}^{(12)})(v^{15}) \\ &= (37{,}743)(8.736)(.362446) = 119{,}507 \end{aligned}$$

Step III: Calculate the Accrued Liability at retirement age 62.

Projected Accrued Benefit = (81,509)(.02)(20 years of service)(1 - (.03)(3 years))
= 29,669

$$\begin{aligned} AL\ (RA\ 62) = PVAB &= (29{,}669)(\ddot{a}_{62}^{(12)})(v^{12}) \\ &= (29{,}669)(9.394)(.444012) = 123{,}751 \end{aligned}$$

Step IV: Calculate the Accrued Liability under the new retirement age assumptions (50% at age 62, 100% at age 65).

$$AL = (123{,}751)(.5) + (119{,}507)(.5) = 121{,}629$$

Step V: Calculate the increase in the Accrued Liability due to the change in assumptions.

$$\text{Increase} = 121{,}629 - 119{,}507 = 2{,}122$$

Answer is A.

**Problem 1 - 44**

Key Concept: The experience loss due to early retirement is the difference between the actual liability and the Accrued Liability determined using the funding assumptions and cost method.

Step I: Calculate the Projected Unit Credit Accrued Liability as of 1/1/94.

$$\text{Final Average Salary} = (40{,}000)((1.05)^5 + (1.05)^4 + (1.05)^3)/3 = 48{,}659$$

Note that 1993, not 1994, salary is given. See Problem 4 - 53 for a discussion of Final Average Salary.

$$\text{Projected Accrued Benefit} = (48{,}659)(.02)(20 \text{ years of service}) = 19{,}464$$

$$\begin{aligned} AL = PVAB &= (19{,}464)(\ddot{a}_{65}^{(12)})(v^5) \\ &= (19{,}464)(8.736)(.712986) = 121{,}234 \end{aligned}$$

Step II: Calculate the value of the early retirement benefit.

$$\text{Actual Final Average Salary} = (36{,}281 + 38{,}095 + 40{,}000)/3 = 38{,}125$$

$$\text{Actual Accrued Benefit} = (38{,}125)(.02)(20 \text{ years of service}) = 15{,}250$$

The actual liability is the present value of the actual accrued benefit, calculated at age 60 using the optional form of benefit, since that is the benefit which was elected.

$$\begin{aligned} PVAB &= (15{,}250)(.95)(\ddot{a}_{\overline{60:60}}^{(12)}) \\ &= (15{,}250)(.95)(\ddot{a}_{60}^{(12)} + \ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}) \\ &= (15{,}250)(.95)(9.815 + 9.815 - 8.094) = 167{,}128 \end{aligned}$$

Step IV: Calculate the difference between the Accrued Liability and the actual liability, which is the actuarial loss.

$$\text{Actuarial Loss} = 167{,}128 - 121{,}234 = 45{,}894$$

Answer is C.

**Problem 1 - 45**

Key Concept: The Accrued Liability is equal to the sum of the liabilities for each possible retirement age, each multiplied by the probability of retirement at that age.

Step I: Calculate the retirement benefit at each possible retirement age. Since there is no preretirement death benefit, actuarial equivalence includes both interest and mortality.

Benefit at RA 65 = (20)(24 years of service) = 480

$$\text{Benefit at RA } 66 = \text{Benefit at } 65 \times (N_{65}^{(12)}/N_{66}^{(12)})$$
$$= 480 \times (825/734) = 539.51$$

$$\text{Benefit at RA } 67 = \text{Benefit at } 65 \times (N_{65}^{(12)}/N_{67}^{(12)})$$
$$= 480 \times (825/651) = 608.29$$

Step II: Calculate the liability assuming retirement at each age. Since there are no assumed preretirement decrements, the discount is done on an interest only basis.

Retirement at age 65:
$$= (\text{Benefit})(12)(N_{65}^{(12)}/D_{65})$$
$$= (480)(12)(825/94) = 50{,}553$$

Retirement at age 66:
$$= (\text{Benefit})(12)(N_{66}^{(12)}/D_{66})(v)$$
$$= (539.51)(12)(734/86)(.934579) = 51{,}641$$

Retirement at age 67:
$$= (\text{Benefit})(12)(N_{67}^{(12)}/D_{67})(v^2)$$
$$= (608.29)(12)(651/79)(.873439) = 52{,}539$$

Step III: Calculate the Accrued Liability.

$$AL = (q_{65}^{(r)})(50{,}553) + (p_{65}^{(r)})(q_{66}^{(r)})(51{,}641) + (p_{65}^{(r)})(p_{66}^{(r)})(q_{67}^{(r)})(52{,}539)$$
$$= (.6)(50{,}553) + (.4)(.8)(51{,}641) + (.4)(.2)(1)(52{,}539)$$
$$= 30{,}332 + 16{,}525 + 4{,}203 = 51{,}060$$

Answer is C.

**Problem 1 - 46**

Key Concept: The Accrued Liability determined using the Unit Credit method is the sum of the present value of the accrued termination benefit plus the present value of the accrued retirement benefit.

Step I: Calculate the accrued monthly benefit.

$$\text{Accrued Benefit} = (20)(10 \text{ years of service}) = 200$$

Step II: Calculate the present value of the retirement benefit. Note that the only preretirement decrement assumed is the 10% termination assumption at age 50. There is <u>no</u> preretirement mortality assumed for retirement benefits.

$$\begin{aligned} \text{PVFB} &= (200)(12\ddot{a}_{65}^{(12)})(v^{25})({}_{25}p_{40}) \\ &= (200)(12)(8.7)(.18425)(.9) = 3{,}462 \end{aligned}$$

Step III: Calculate the present value of the termination benefit. Note that for the termination benefit, the post-termination mortality assumption is in effect from age 50 to 65.

$$\begin{aligned} \text{PVFB} &= (200)(12\ddot{a}_{65}^{(12)})(v^{10})(q_{50}^{(w)})(D_{65}/D_{50}) \\ &= (200)(12)(8.7)(.50835)(.1)(94/311) = 321 \end{aligned}$$

Step IV: Calculate the Accrued Liability as of 1/1/94.

$$AL_{94} = 3{,}462 + 321 = 3{,}783$$

Answer is B.

**Problem 1 - 47**

The benefit to be used to determine the Accrued Liability in the Unit Credit method is the accrued benefit as of the beginning plan year.

$$\text{Accrued Benefit}_{1/1/95} = 20 \times 3 \text{ years of service} = 60$$

This benefit will not become vested until 1/1/97, when the participant will be 35 years old.

The Accrued Liability to be determined is the present value of the Accrued Benefit at 1/1/95, payable if the participant terminates employment between ages 35 and 65. Note that the participant must still survive to age 65 in order to receive a benefit.

$$\text{AL(due to vested termination)} = (60)(12\ddot{a}_{65}^{(12)})\,({}_{32}p_{33}'^{(d)})({}_{2}p_{33}'^{(w)})({}_{30}q_{35}'^{(w)})(v^{32})$$
$$= (60)(12\ddot{a}_{65}^{(12)})\,\frac{(l_{65}'^{(d)})}{(l_{33}'^{(d)})}\frac{(l_{35}'^{(w)})}{(l_{33}'^{(w)})}\frac{(l_{35}'^{(w)} - l_{65}'^{(w)})}{(l_{35}'^{(w)})}(v^{32})$$

$$= (60)(12)(9.439)(7900/9952)(7500/8500) \times ((7500\text{-}2000)/7500)(.155) = 541$$

Answer is C.

**Problem 1 - 48**

Key Concept: An experience loss is created if the actual liability due to the early retirement election exceeds the Accrued Liability under the funding method. We must look at each participant individually.

Smith: Projected Unit Credit Accrued Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1.03)^9(\ddot{a}_{65}^{(12)})v^9$$
$$= (.02)(\text{Years of service})(\text{Salary})(5.04)$$

Early Retirement Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1 - 5/15 - 4/30)(\ddot{a}_{56}^{(12)})$$
$$= (.02)(\text{Years of service})(\text{Salary})(5.248)$$

Experience Loss, since early retirement yields a larger liability.

Brown: Projected Unit Credit Accrued Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1.03)^5(\ddot{a}_{65}^{(12)})v^5$$
$$= (.02)(\text{Years of service})(\text{Salary})(6.32)$$

Early Retirement Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1 - 5/15)(\ddot{a}_{60}^{(12)})$$
$$= (.02)(\text{Years of service})(\text{Salary})(6.17)$$

Experience Gain, since early retirement yields a smaller liability.

Green: Projected Unit Credit Accrued Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1.03)^2(\ddot{a}_{65}^{(12)})v^2$$
$$= (.02)(\text{Years of service})(\text{Salary})(7.49)$$

Early Retirement Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1 - 2/15)(\ddot{a}_{63}^{(12)})$$
$$= (.02)(\text{Years of service})(\text{Salary})(7.55)$$

Experience Loss, since early retirement yields a larger liability.

Answer is B.

**Problem 1 - 49**

Step I: Calculate the Final Average Salary at each retirement age.

Final Average Salary RA = 65: $(50{,}000)[(1.04)^{14} + (1.04)^{13} + (1.04)^{12}]/3 = 83{,}296$

Final Average Salary RA = 62: $(50{,}000)[(1.04)^{11} + (1.04)^{10} + (1.04)^{9}]/3 = 74{,}050$

Step II: Calculate the Accrued Liability at retirement age 65.

Projected Accrued Benefit = (83,296)(.02)(12 years of service) = 19,991

$$\text{AL (RA 65)} = \text{PVAB} = (19{,}991)(\ddot{a}_{65}^{(12)})(v^{15})$$
$$= (19{,}991)(8.74)(.362446) = 63{,}327$$

Step III: Calculate the Accrued Liability at retirement age 62.

Projected Accrued Benefit = (74,050)(.02)(12 years of service)(1 - (.03)(3 years))
= 16,173

$$\text{AL (RA 62)} = \text{PVAB} = (16{,}173)(\ddot{a}_{62}^{(12)})(v^{12})$$
$$= (16{,}173)(9.39)(.444012) = 67{,}430$$

Step IV: Calculate the Accrued Liability under the actual retirement age assumptions (20% probability at age 62, 100% probability at age 65)

$$AL = (67{,}430)(.2) + (63{,}327)(.8) = 64{,}148$$

Answer is C.

**Problem 1 - 50**

Step I: Calculate the projected Final Average Salary.

$$\text{Final Average Salary} = (50{,}000)[(1.04)^{11} + (1.04)^{10} + (1.04)^{9}]/3 = 74{,}050$$

Step II: Calculate the projected 1996 accrual.

The participant has worked 111 months prior to 1996. So, the benefit to be accrued in 1996 will be based upon 9 months of service at 2/12% of final average salary plus 3 months of service at 1/12% of final average salary.

$$\begin{aligned}\text{Projected 1996 Accrual} &= [(2/12\% \times 9 \text{ months}) + (1/12\% \times 3 \text{ months})](74{,}050)\\ &= 1{,}295.88\end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned}NC &= (1{,}295.88)(\ddot{a}_{65}^{(12)})(v^{12})\\ &= 5{,}029\end{aligned}$$

Answer is D.

**Problem 1 - 51**

Key Concept: The Accrued Liability is equal to the sum of the liability attributable to retirement benefits plus the liability attributable to withdrawal benefits.

Step I: Calculate the Accrued Liability attributable to retirement benefits.

Under the 1/1/95 valuation assumptions,

$$\text{Retirement Benefit } AL_{Old} = PV \times p_{50}^{(w)} \times p_{55}^{(w)} \times p_{60}^{(w)}$$
$$= (PV)(.6)(.75)(.8)$$
$$= .36PV$$

where PV represents the present value of the projected accrued benefit as of 1/1/96, discounting with interest only.

Under the 1/1/96 valuation assumptions,

$$\text{Retirement Benefit } AL_{New} = PV \times p_{50}^{(w)} \times p_{55}^{(w)}$$
$$= (PV)(.5)(.8)$$
$$= .4PV$$

Step II: Calculate the Accrued Liability attributable to withdrawal benefits.

Under the 1/1/95 valuation assumptions,

$$\text{Withdrawal Benefit } AL_{Old} = (PV)(1/1.04)^{15}(q_{50}^{(w)}) + (PV)(1/1.04)^{10}(p_{50}^{(w)})(q_{55}^{(w)})$$
$$+ (PV)(1/1.04)^{5}(p_{50}^{(w)})(p_{55}^{(w)})(q_{60}^{(w)})$$
$$= (PV)(1/1.04)^{15}(.4) + (PV)(1/1.04)^{10}(.6)(.25)$$
$$+ (PV)(1/1.04)^{5}(.6)(.75)(.2)$$
$$= .3974PV$$

Note that since termination would occur, the participant would not have the impact of future salary increases from termination to retirement. This explains the multiplication of the present value by 1/1.04 for the number of years from termination to retirement.

Under the 1/1/96 valuation assumptions,

$$\text{Withdrawal Benefit } AL_{New} = (PV)(1/1.04)^{15}(q_{50}^{(w)}) + (PV)(1/1.04)^{10}(p_{50}^{(w)})(q_{55}^{(w)})$$
$$= (PV)(1/1.04)^{15}(.5) + (PV)(1/1.04)^{10}(.5)(.2)$$
$$= .3452PV$$

Step III: Calculate the ratio of the revised Accrued Liability to the original Accrued Liability.

$$\text{Ratio} = (.4PV + .3452PV)/(.36PV + .3974PV)$$
$$= .7452/.7574$$
$$= .9839$$

Answer is A.

**Problem 1 - 52**

Key concept: Normal Cost under the Projected Unit Credit method increases with interest and preretirement survival each year. In addition, the Normal Cost increases (decreases) by the percentage that the compensation increases (decreases) other than as assumed. Finally, the Normal Cost will increase (decrease) proportionally as the benefit formula increases (decreases).

Step I: Calculate the Normal Cost as of 1/1/96.

The Accrued Liability as of 1/1/96 represents the present value of the benefit accrued during the first 19 years of service. The accrual for the year 1996 is earned at the same rate (2%) as each of the first 19 years. Therefore, the Normal Cost must be equal to one-nineteenth of the Accrued Liability.

$$NC_{1/1/96} = AL_{1/1/96} \times 1/19 = \$97{,}000 \times 1/19 = \$5{,}105$$

Step II: Calculate the Normal Cost as of 1/1/97.

The expected compensation for the 1997 valuation is:

$$e\text{Salary}_{1997} = \$50{,}000 \times 1.04 = \$52{,}000$$

Since the actual compensation is \$53,000, the percentage increase above the expected salary can be represented by 53,000/52,000.

The participant is in his twenty-first year of service in 1997. Therefore, the accrual for the year decreases from 2% of final 3-year average compensation to 1.5% of final 3-year average compensation. The percentage decrease in the benefit can be represented by 1.5/2.

The Normal Cost as of 1/1/97 is:

$$\begin{aligned} NC_{1/1/97} &= 5{,}105 \times (D_{54}/D_{55}) \times (53{,}000/52{,}000) \times (1.5/2) \\ &= 5{,}105 \times (23{,}593/21{,}867) \times (53{,}000/52{,}000) \times (1.5/2) \\ &= 4{,}210 \end{aligned}$$

Answer is B.

**Problem 1 - 53**

Key concept: The noninvestment experience gain or loss is simply the gain or loss in the Accrued Liability.

Step I: Calculate the expected Accrued Liability.

$$
\begin{aligned}
eAL_{1/1/97} &= (AL_{1/1/96} + NC_{1/1/96})(1.08) - BP_{1996} \\
&= (800{,}000 + 100{,}000)(1.08) - 13{,}000 \\
&= 959{,}000
\end{aligned}
$$

Note that the benefit payment is subtracted since it is a release of Accrued Liability at the time it is paid. There is no interest adjustment to the benefit payment since it was made on the last day of the year.

Step II: Calculate the experience loss for 1996 from all non-investment sources.

$$
\begin{aligned}
\text{Loss} &= AL_{1/1/97} - eAL_{1/1/97} \\
&= 1{,}000{,}000 - 959{,}000 \\
&= 41{,}000
\end{aligned}
$$

Answer is E.

**Problem 1-53 (Alternative Solution)**

The noninvestment experience gain or loss can be calculated as the difference between the total experience gain or loss and the investment experience gain or loss.

Step I: Calculate the total experience gain or loss.

The total gain or loss is equal to the difference between the expected and actual unfunded liabilities. The expected unfunded liability is:

$$
\begin{aligned}
eUL_{1/1/97} &= (UL_{1/1/96} + NC_{1/1/96})(1 + i) - \text{Contribution}_{1996} - \text{Contribution Interest}_{1996} \\
&= ([AL_{1/1/96} - \text{Assets}_{1/1/96}] + NC_{1/1/96})(1 + i) - \text{Contribution}_{1996} \\
&\qquad - \text{Contribution Interest}_{1996} \\
&= ([800{,}000 - 400{,}000] + 100{,}000)(1.08) - 150{,}000 - (100{,}000)(.04) \\
&= 386{,}000
\end{aligned}
$$

$$\text{Total Loss} = UL_{1/1/97} - eUL_{1/1/97}$$
$$= 400{,}000 - 386{,}000$$
$$= 14{,}000$$

Step II: Calculate the investment experience gain or loss.

$$e\text{Assets}_{1/1/97} = (\text{Assets}_{1/1/96})(1 + i) + \text{Contribution}_{1996} + \text{Contribution Interest}_{1996}$$
$$- \text{Benefit Payments}_{1996} - \text{Benefit Payment Interest}_{1996}$$
$$= (400{,}000)(1.08) + 150{,}000 + (100{,}000)(.04) - 13{,}000$$
$$= 573{,}000$$

$$\text{Asset Gain} = \text{Assets}_{1/1/97} - e\text{Assets}_{1/1/97}$$
$$= 600{,}000 - 573{,}000$$
$$= 27{,}000$$

Step III: Calculate the noninvestment gain or loss.

$$\text{Noninvestment Loss} = \text{Total Loss} + \text{Asset Gain}$$
$$= 14{,}000 + 27{,}000$$
$$= 41{,}000$$

Answer is E.

**Problem 1 - 54**

Step I: Calculate the Accrued Benefit as of 1/1/97.

The participant has 15 years of past service, and the accrual increased by 2% per year. So, the Accrued Benefit is:

$$\text{Accrued Benefit}_{1/1/97} = 25 + 25(1.02) + 25(1.02)^2 + \ldots + 25(1.02)^{14}$$
$$= 25(1 + 1.02 + 1.02^2 + \ldots + 1.02^{14})$$
$$= 25s_{\overline{15}|.02}$$
$$= (25)(17.2934)$$
$$= 432.34$$

Step II: Calculate the Accrued Liability as of 1/1/97.

$$\begin{aligned} AL_{1/1/97} &= (432.34)(12\ddot{a}_{65}^{(12)})(v^{15}) \\ &= (432.34)(12)(9.815)(.3624) \\ &= 18{,}454 \end{aligned}$$

Step III: Calculate the Unfunded Liability as of 1/1/97.

$$\begin{aligned} UL_{1/1/97} &= AL_{1/1/97} - Assets_{1/1/97} \\ &= 18{,}454 - 17{,}810 \\ &= 644 \end{aligned}$$

Answer is B.

# Chapter 2

# Aggregate Cost Method

## 2.1 Normal Cost with Interest Assumption Only

You will recall from chapter 1 that, under the Unit Credit Cost Method, the Accrued Liability is equal to the Present Value of Accrued Benefits. It is also equal to the desired value of plan assets on any valuation date. From this it follows that the Normal Cost is the difference in Accrued Liability in two successive years with appropriate adjustments for actuarial assumptions and interest. Thus, if Homer and Jake were the only Participants and we were using only a pre-retirement earnings assumption of 6%, the Normal Cost for the first Plan Year would be:

NC(Homer) = \$748
NC(Jake) = \$1,125
NC(total) = \$1,873

Any difference between the expected and actual value of plan assets and accrued liabilities is an actuarial gain or loss and is amortized over a period of years for funding purposes.

Under the Aggregate Method, the accrued liability is defined to be equal to the actuarial value of assets (provided that contributions have been properly made). In other words, no separate Unfunded Accrued Liability is identified. The Normal Cost per participant is then defined as the Present Value of Future Normal Costs divided by the Present Value of remaining years to retirement. The Plan Normal Cost is the Normal Cost per participant multiplied by the number of active participants. Thus, for the first Plan Year when the Plan Assets are zero:

NC(Homer) = NC(Jake) = (PVFB) ÷ (PV of Future Years)

where PVFB is the Present Value of Future Benefits, and PV of Future Years is the sum of the Present Values of the Years to Retirement for all Participants.

Thus, using an interest assumption of 6% and $12\ddot{a}_{65}^{(12)} = 120$:

$$\text{PVFB(Homer)} = 780 \times 120 \times (1.06)^{-20} = 29{,}185$$
$$\text{PVFB(Jake)} = 300 \times 120 \times (1.06)^{-13} = 16{,}878$$
$$\text{PV Future Years(Homer)} = \ddot{a}_{\overline{20}|} = 12.16$$
$$\text{PV Future Years(Jake)} = \ddot{a}_{\overline{13}|} = 9.38$$

$$\text{NC(Homer)} = \text{NC(Jake)} = (29{,}185 + 16{,}878) \div (12.16 + 9.38) = 2{,}138$$

And if Homer and Jake were the only Participants, the Normal Cost for the first year would be 2 × \$2,138 or \$4,276.

In future years, we subtract the actuarial value of Plan Assets from the Present Value of Future Benefits to arrive at the Present Value of Future Normal Costs, which is then divided by the Present Value of Future Years.

$$NC = n(PVFB_t - Assets_t) \div PVFY_t$$

where n is the number of active Participants.

## 2.2 Other Assumptions

Valuing plan liabilities using mortality, turnover assumptions, etc. will require reference to a Service Table. If salary increases are assumed, the procedure described in the preceding section must be altered somewhat. Normal Cost is calculated in the same manner except that the present value of future salaries is substituted for the present value of future service years. The resulting accrual rate is multiplied by total current salaries.

(1) Present Value of Future Benefits (PVFB)
(2) Actuarial Value of Plan Assets (Assets)
(3) Present Value of Future Normal Costs (1) - (2)
(4) Present Value of Future Salaries (PVFS)
(5) Normal Cost Rate (3) ÷ (4)
(6) Current Annual Salaries
(7) Normal Cost (5) × (6)

or, $NC = [(PVFB_t - Assets_t) \div PVFS_t] \times \sum S_t$
where $S_t$ is the annual salary of each Participant,

and $PVFS_t = \sum S_t ({}^sN_{aa} - {}^sN_{ra}) \div {}^sD_{aa}$

As an example, suppose we introduce a salary increase assumption of 4% per year into the valuation of the Flea Flicker Pension Plan, and if Homer's and Jake's salaries were $25,000 and $20,000, the Normal Cost on the first valuation date would be calculated as follows:

Given: $({}^sN_{40} - {}^sN_{65}) / {}^sD_{40} = 18.1942$
$({}^sN_{52} - {}^sN_{65}) / {}^sD_{52} = 12.2414$

| | |
|---|---|
| (1) Present Value of Future Benefits (no change from Section 2.1) | 29,185 + 16,878 = 46,063 |
| (2) Present Value of Future Salaries | |
| PVFS(Homer) | 25,000 × 18.1942 = 454,855 |
| PVFS(Jake) | 20,000 × 12.2414 = 244,828 |
| | Total = 699,683 |
| (3) Normal Cost Rate | (1) ÷ (2) = 6.58% |
| (4) Current Salaries | 25,000 + 20,000 = 45,000 |
| (5) Normal Cost | (3) × (4) = 2,961 |

Note that the Normal Cost Rate actually represents the normal cost as a percentage of payroll.

The student is probably wondering why we sometimes compute the Normal Cost as a dollar amount per participant and sometimes as a percentage of salary. Is one way better than the other? Actually, both methods are acceptable. However, in practice, the approach used is usually related to the nature of the plan's benefit formula. If the plan's benefits are not related to salary (e.g., $20 per month per year of service), Normal Costs are generally calculated under the dollar amount per participant approach; in this way, it is not even necessary to collect salary information on the employees. If plan benefits are based on salary, Normal Costs are generally calculated as a percentage of salary. This in effect weights the temporary annuity by each participant's salary, the result being that more of the Normal Cost is "attributed' to the higher paid participants, who generally receive larger benefits and contributions. Also note, the dollar amount per participant approach can be viewed as a special case of the percentage of salary approach, where each participant is treated as having an annual salary of $1.00.

### 2.3 Characteristics of Aggregate Cost Method

1. The Aggregate Cost Method is a "spread gain" method in that gains or losses are not separately identified and amortized but any deviation from expected results is automatically spread over the remaining working lifetimes of participants.

2. The Aggregate Cost Method determines an "average" Normal Cost per participant and hence is not suitable for small plans with a wide range of ages.

3. The Normal Cost for each participant is not separately identified.

4. Calculations are relatively simple.

5. Where salary increases are assumed, the Normal Cost is generally based upon a percentage of current compensation.

## 2.4 Problems

### Problem 2 - 1

It is now the first day of the third Plan Year and Pete, age 36 with one year of service, becomes eligible. Homer and Jake are still working and there have been no Breaks in Service. Plan Assets are $8,698. Compute the Normal Cost of the Plan as of that Valuation Date under the Aggregate Method.

($12\ddot{a}_{65}^{(12)} = 120$; $i = 6\%$).

### Problem 2 - 2

Census data given to the plan actuary listed ages of the four Participants to be to be 45, 50, 55 and 60. Based on this data, he computed the Normal Cost to be $8,000. It was then learned that these ages were obtained from an old record and all employees are actually one year older than those shown. Compute the corrected Normal Cost.

Normal Retirement Age: 65
Plan Assets: $42,560
Interest rate: 6%
PVFB: $112,000 (as originally calculated)

### Problem 2 - 3

Selected Plan Data:

Form of Benefit: Life Only
Assumed Retirement Age: 65
Assumed age of spouse: Same as participant.

Valuation Results as of 1/1/85 before amendment:

| | |
|---|---|
| Present Value of Benefits - Active Participants | $450,000 |
| Present Value of Benefits - Retired Participants | $100,000 |
| Actuarial Value of Assets | $250,000 |
| Normal Cost as of 1/1/85 | $25,000 |

$\ddot{a}_{65}^{(12)} = 12$ $\qquad\qquad$ $\ddot{a}_{\overline{65:65}}^{(12)} = 14$

The Plan was then amended, effective 1/1/85, to provide the same monthly benefit to the future retirees who so elected on a joint and 100% survivor form. It is estimated that 60% of pensions for future retirees will be paid under the joint-and-survivor form. Recalculate the Normal Cost as of 1/1/85.

**Problem 2 - 4**

Pension Formula: 25% of final year salary
Normal Retirement Age: 65
Valuation Date: 1/1/86
Value of Plan Assets: $22,000

Assumptions: $\ddot{a}_{65}^{(12)} = 10$
Interest Rate: 6%
Salary Scale: 4%
Pre-retirement deaths or terminations: None.

There is one Participant, age 54, whose salary was $2,400 per month during 1985. Calculate the Normal Cost as of 1/1/86.

**Problem 2 - 5**

Pension Formula: 2% of final year's salary per year of service.
Valuation Date: 1/1/86
Normal Retirement Age: 60
Value of Plan Assets: $125,000

Assumptions: Interest Rate: 8%
Salary Scale: 4%
$\ddot{a}_{65}^{(12)} = 12.5$

A plan has two participants with census data as follows:

| Age x | Years of Service | 1985 Annual Salary | $({}^sN_x - {}^sN_{60})/{}^sD_x$ | $D_{60}/D_x$ |
|---|---|---|---|---|
| 54 | 19 | $50,000 | 5.471 | .630 |
| 43 | 2 | 30,000 | 12.806 | .270 |

Compute the Normal Cost for 1986 as of 1/1/86.

**Problem 2 - 6**

A Plan has one Participant, age 55, with 1985 salary of $50,000. Plan data is as follows:

Valuation Date: 1/1/86
Normal Retirement Age: 65
Interest Rate: 8%
Salary Scale: 3%
Pre-retirement deaths or terminations: None

Selected Valuation Results:

| | |
|---|---|
| Present Value of Future Benefits | $75,000 |
| Actuarial Value of Plan Assets | $30,000 |
| Present Value of Future Salaries | $407,777 |
| Normal Cost | $5,518 |

After the valuation, the actuary learned that the Participant's salary had been frozen for the year 1986. He then recalculated by postponing the salary increase assumption until 1987. What effect would this have upon the Present Value of Future Benefits and upon the Present Value of Future Salaries?

**Problem 2 - 7**

The actuarial assumptions include the following absolute rates of decrement:

$q'^{(d)}_{30} = .015 \qquad q'^{(w)}_{30} = .25$

Normal Retirement Age is 65.

The sole Participant is age 30 as of the valuation date and will not be eligible for vesting until next year. There are no pre-retirement death or severance benefits.

The absolute rate of decrement for withdrawal at age 30 is changed to $q'^{(w)}_{30} = .30$, with all other assumptions remaining the same. What effect does this change have upon the Present Value of Future Benefits? Upon the Present Value of Future Salaries?

**Problem 2 - 8**

Effective date: 1/1/84
Normal retirement benefit: 50% of final year's salary.
Normal retirement age: 60
Actuarial cost method: Aggregate method.

Actuarial Assumptions:

Interest: 6%
Salary increases: 3%

Data for the sole participant:

| Valuation Date | Age x | Annual Salary | Projected Annual Benefit | $({}^sN_x - {}^sN_{60})/{}^sD_x$ | $D_{60}/D_x$ |
|---|---|---|---|---|---|
| 1/1/84 | 35 | \$50,000 | \$50,820 | 17.935 | .233 |
| 1/1/85 | 36 | \$60,000 | \$59,208 | 17.445 | .247 |

$\ddot{a}_{60}^{(12)} = 12.5$

Actuarial value of assets as of 1/1/85: \$9,000

In what range is the increase in normal cost as of 1/1/85 resulting from the salary increase in excess of that assumed?

(A) Less than \$750
(B) \$750 but less than \$1,100
(C) \$1,100 but less than \$1,450
(D) \$1,450 but less than \$1,800
(E) \$1,800 or more.

**Problem 2 - 9**

Plan effective date: 1/1/83
Normal retirement benefit: \$50 per month for each year of service.
Actuarial cost method: Aggregate

The actuarial assumptions include the following absolute rates of decrement:

$q'^{(d)}_{25} = .02 \qquad q'^{(w)}_{25} = .20$

The sole plan participant is age 25 as of 1/1/85 and will not be eligible for vesting until after age 26. There are no pre-retirement death or severance benefits.

Selected valuation results as of 1/1/85:

| | |
|---|---|
| Present value of future benefits | \$20,000 |
| Actuarial value of assets | \$2,000 |
| Normal cost as of 1/1 | \$1,080 |

The absolute rate of decrement for withdrawal at age 25 is changed to $q'^{(w)}_{25} = .30$, with all other assumptions remaining the same. The normal cost is then recalculated. In what range is the recalculated normal cost?

(A) Less than \$1,035
(B) \$1,035 but less than \$1,045
(C) \$1,045 but less than \$1,055
(D) \$1,055 but less than \$1,065
(E) \$1,065 or more.

**Problem 2 - 10**

Normal retirement benefit: 50% of salary in the year preceding retirement.
Actuarial cost method: Aggregate

Selected valuation results as of 1/1/85:

| | |
|---|---|
| Total present value of future benefits | \$2,000,000 |
| Present value of future benefits for retired and terminated participants | 400,000 |
| Present value of future salaries | 24,000,000 |
| Actuarial value of assets | 500,000 |
| Annual salaries | 4,000,000 |

There are no participants within one year of the assumed retirement age.

After completing the valuation, it is discovered that all active participants received a 5% salary increase which had not been reported. The normal cost is then recalculated. In what range is the increase in the normal cost as of 1/1/85 due to the increased salaries?

(A) Less than $13,000
(B) $13,000 but less than $14,000
(C) $14,000 but less than $15,000
(D) $15,000 but less than $16,000
(E) $16,000 or more.

**Problem 2 - 11**

Optional form of benefit: Joint and 100% survivor annuity, actuarially equivalent to the normal form on valuation assumptions.

Actuarial cost method: Aggregate method.
Assumed spouse's age: Same as participant's age.
Assumed retirement age: 65

Assumed percent of normal pensions that will be paid under the joint-and-survivor form: 80%

Valuation results as of 1/1/84:

| | |
|---|---|
| Present value of future benefits for active participants | $960,000 |
| Present value of future benefits for retired participants | 700,000 |
| Actuarial value of assets | 800,000 |
| Present value of future salaries | 12,000,000 |
| Total annual salaries | 1,500,000 |

$\ddot{a}_{65}^{(12)} = 10 \qquad \ddot{a}_{\overline{65:65}}^{(12)} = 13$

After the above results were obtained, the plan was amended effective 1/1/84 to provide, for participants retiring after that date, a uniform 10% reduction rather than an actuarially equivalent reduction for the optional form. The 1/1/84 normal cost was then recalculated. In what range is the recalculated normal cost for 1984 as of 1/1/84?

(A) Less than $123,000
(B) $123,000 but less than $127,000
(C) $127,000 but less than $131,000
(D) $131,000 but less than $135,000
(E) $135,000 or more

**Problem 2 - 12**

Plan effective date: 1/1/85
Normal retirement benefit: $1,000 per month.
Actuarial cost method: Aggregate.
Assumed interest rate: 7%
Assumed retirement age: 65
Sole participant's date of birth: 1/1/45

It is assumed that there are no terminations prior to retirement, other than by death.

Normal cost as of 1/1/85: $1,500
Actuarial value of assets as of 1/1/86: $1,675

Selected commutation functions:

| Age x | $D_x$ | $N_x - N_{65}$ |
|---|---|---|
| 40 | 67 | 787 |
| 41 | 62 | 720 |
| 65 | 10 | 0 |

In what range is the normal cost for 1986 as of 1/1/86?

(A) Less than $1,455
(B) $1,455 but less than $1,470
(C) $1,470 but less than $1,485
(D) $1,485 but less than $1,500
(E) $1,500 or more

**Problem 2 - 13**

Normal retirement benefit: 50% of final year's salary.

Actuarial cost method: Aggregate with normal costs determined as a level dollar amount (split funded).

Actuarial assumptions for the side fund:

Interest: 6%
Salary increases: 4% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant as of 1/1/86:

| | |
|---|---|
| Date of birth: | 1/1/51 |
| Annual salary for 1986: | $50,000 |

Life insurance policy data:.

| | |
|---|---|
| Annual premium: | $3,250 |
| Current cash value: | $10,000 |
| Guaranteed cash value at age 65: | $110,000 |

Actuarial value of assets (not including insurance cash value) as of 1/1/86: $20,000

$\ddot{a}_{65}^{(12)} = 10$

In what range is the side-fund normal cost for 1986 as of 1/1/86?

(A) Less than $5,900
(B) $5,900 but less than $6,900
(C) $6,900 but less than $7,900
(D) $7,900 but less than $8,900
(E) $8,900 or more

**Problem 2 - 14**

| | |
|---|---|
| Normal retirement benefit: | $1,000 per month. |
| Pre-retirement death benefit: | 100 times the normal retirement benefit (provided by individual ordinary life policies). |
| Actuarial cost method: | Aggregate (split-funded) |

Actuarial assumptions for the side fund:

Interest: 7%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Participant data as of 1/1/86:

| | Attained Age | Cash Value of Insurance at Age 65 |
|---|---|---|
| Smith | 30 | \$40,000 |
| Brown | 40 | \$30,000 |

Actuarial value of side-fund assets as of 1/1/86: \$5,000

$\ddot{a}_{65}^{(12)} = 10$

In what range is the side-fund normal cost for 1986 as of 1/1/86?

(A) Less than \$1,500
(B) \$1,500 but less than \$1,750
(C) \$1,750 but less than \$2,000
(D) \$2,000 but less than \$2,250
(E) \$2,250 or more

**Problem 2 - 15**

Plan provisions:

Normal retirement benefit: \$10 per month for each year of service.
Normal retirement age: The later of age 65 or age at ten years of participation.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest: 8%
Preretirement deaths and terminations: None.
Retirement age: Normal retirement age.

Participant data as of 1/1/87:

| | Date of Birth | Date of Hire | Date of Participation |
|---|---|---|---|
| Smith | 1/1/37 | 1/1/70 | 1/1/79 |
| Brown | 1/1/22 | 1/1/62 | 1/1/79 |

Actuarial value of assets as of 1/1/87: $10,000

Selected annuity values:

| Age x | $\ddot{a}_x^{(12)}$ |
|---|---|
| 65 | 8.142 |
| 66 | 7.951 |
| 67 | 7.702 |

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than $1,500
(B) $1,500 but less than $2,250
(C) $2,250 but less than $3,000
(D) $3,000 but less than $3,750
(E) $3,750 or more

**Problem 2 - 16**

Retirement benefit: 50% of final year's salary.
Actuarial cost method: Aggregate.

Participant data as of 1/1/87:

| Attained Age | Salary for 1987 |
|---|---|
| 47 | $35,000 |
| 47 | 45,000 |
| 47 | 55,000 |
| 47 | 65,000 |

Original valuation results as of 1/1/87:

| | |
|---|---|
| Present value of future benefits: | \$500,000 |
| Actuarial value of assets: | 100,000 |
| Normal cost as of 1/1: | 40,000 |

After the 1/1/87 valuation, it was discovered that an employee with the following data had been omitted:

| Attained Age | Salary for 1987 |
|---|---|
| 47 | \$25,000 |

The valuation was revised to included the previously omitted employee. In what range is the revised normal cost for 1987 as of 1/1/87?

(A) Less than \$44,500
(B) \$44,500 but less than \$45,000
(C) \$45,000 but less than \$45,500
(D) \$45,500 but less than \$46,000
(E) \$46,000 or more

**Problem 2 - 17**

Normal retirement benefit: 40% of final five-year average salary.
Actuarial cost method: Aggregate.

Actuarial assumptions: Interest: 8%.
Salary increases: 6% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Data for sole participant as of 1/1/87:

| | |
|---|---|
| Date of birth | 1/1/32 |
| Date of hire | 1/1/77 |
| Salary for 1986 | $32,000 |

Actuarial value of assets as of 1/1/87: $30,000

$\ddot{a}_{65}^{(12)} = 8.65$

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than $5,100
(B) $5,100 but less than $5,400
(C) $5,400 but less than $5,700
(D) $5,700 but less than $6,000
(E) $6,000 or more

**Problem 2 - 18**

Normal retirement benefit: 50% of final year's salary.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest: 8%
Salary increases: 6% per year.
Preretirement deaths and terminations: None.

Selected valuation results as of 1/1/86:

| | |
|---|---|
| Present value of future benefits for active participants | $950,000 |
| Present value of future benefits for inactive participants | 0 |
| Actuarial value of assets | 500,000 |
| Annual salaries | 1,000,000 |
| Normal cost as of 1/1 | 60,000 |

Plan experience during 1986:

The rate of return on the actuarial value of assets was 10%.
Salaries increased 5%.
The normal cost for 1986 was paid at 1/2/86.
There were no deaths, terminations, retirements, or new participants.
There were no active participants within two years of the assumed retirement age.

In what range is the normal cost rate (as a percentage of salaries) as of 1/1/87?

(A) Less than 5.75%
(B) 5.75% but less than 5.80%
(C) 5.80% but less than 5.85%
(D) 5.85% but less than 5.90%
(E) 5.90% or more

**Problem 2 - 19**

Normal retirement benefit: \$25 per month for each year of service.

Actuarial cost method:

Before 1988: Unit credit.
After 1987: Aggregate.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/78 |

Valuation results as of 1/1/87:

| | |
|---|---|
| Actuarial value of assets | \$6,300 |
| Unfunded accrued liability | 2,500 |

Contribution for 1987: $1,250 paid on 12/31/87.
There were no noninvestment experience gains or losses during 1987.
The rate of return on the actuarial value of assets during 1987 was 10%.

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than $1,700
(B) $1,700 but less than $1,800
(C) $1,800 but less than $1,900
(D) $1,900 but less than $2,000
(E) $2,000 or more

**Problem 2 - 20**

Normal retirement benefit: $10 per month for each year of service.

Early retirement benefit: Normal retirement benefit reduced by 6% for each year by which commencement of retirement payments precedes age 65.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Pre-retirement terminations other than deaths: None.
Retirement age: 65

Participant data as of 1/1/88:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/33 | 1/1/26 |
| Date of hire | 1/1/58 | 1/1/51 |
| Status | Active | Active |

Actuarial value of assets as of 1/1/88: $15,000

Selected commutation functions and annuity values:

| Age x | $D_x$ | $N_x - N_{65}$ | $\ddot{a}_x^{(12)}$ |
|---|---|---|---|
| 55 | 367 | 2,730 | 11.74 |
| 62 | 224 | 623 | 10.10 |
| 65 | 178 | 0 | 9.35 |

After the 1/1/88 valuation was completed, it was found that Brown retired on 12/31/87.

In what range is the change in the normal cost for 1988 as of 1/1/88 due to Brown's retirement?

(A) Less than \$(2,400)
(B) \$(2,400) but less than \$(1,200)
(C) \$(1,200) but less than \$0
(D) \$0 but less than \$1,200
(E) \$1,200 or more

**Problem 2 - 21**

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: 5% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Valuation results as of 1/1/89:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 31,250 |
| Present value of future benefits: | |
| Active participants | 900,000 |
| Inactive participants | 100,000 |
| Actuarial value of assets | 500,000 |
| Annual compensation | 1,000,000 |

Contribution for 1989: \$31,250 paid on 1/1/89.
During 1989, there were no new entrants to the plan.

There were no experience gains or losses during 1989 other than an experience gain of $30,000 due to the deaths of three retired participants.

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than $30,000
(B) $30,000 but less than $31,000
(C) $31,000 but less than $32,000
(D) $32,000 but less than $33,000
(E) $33,000 or more

**Problem 2 - 22**

Normal retirement age:

Before 1989: 65
After 1988: 64

Normal retirement benefit: $20 per month for each year of service.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 8%
Pre-retirement deaths and terminations: None.
Retirement age: Normal retirement age.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/64 |

Actuarial value of assets as of 1/1/89: $10,000

Selected annuity values:

$\ddot{a}_{64}^{(12)} = 8.35$ $\ddot{a}_{65}^{(12)} = 8.14$

In what range is the increase in the normal cost for 1989 as of 1/1/89 due to the change in the normal retirement age?

- (A) Less than $600
- (B) $600 but less than $650
- (C) $650 but less than $700
- (D) $700 but less than $750
- (E) $750 or more

**Problem 2 - 23**

Normal retirement benefit: 50% of final year's compensation.

Normal form of payment:

Retirements before 1989: Life annuity.
Retirements after 1988:

Single retirees: Life annuity.
Married retirees: Joint and 50% survivor annuity.

Actuarial cost method: Aggregate.

Actuarial assumptions:

| | |
|---|---|
| Retirement age: 65 | |
| Marital characteristics: | 85% of participants are married at the time of retirement; spouses are the same age as participants |

Valuation results as of 1/1/89, before change in normal form of payment:

| | |
|---|---|
| Normal cost as of 1/1 | $ 85,000 |
| Present value of future benefits for retired participants | 400,000 |
| Actuarial value of assets | 600,000 |
| Present value of future compensation | 4,200,000 |
| Annual compensation | 350,000 |

Selected unisex annuity values:

$\ddot{a}_{65}^{(12)} = 9.90 \qquad \ddot{a}_{65:65}^{(12)} = 7.82$

In what range is the normal cost for 1989 as of 1/1/89?

(A) Less than \$90,000
(B) \$90,000 but less than \$95,000
(C) \$95,000 but less than \$100,000
(D) \$100,000 but less than \$105,000
(E) \$105,000 or more

**Problem 2 - 24**

Type of plan: Contributory.
Rate of employee contribution: 1.5% of compensation.
Death benefit: Refund of employee contributions, with interest to date of death.
Actuarial cost method: Aggregate.

Valuation results as of 1/1/90:

| | |
|---|---|
| Present value of future retirement benefits | \$1,500,000 |
| Present value of future death benefits | 30,000 |
| 1990 employee contributions | 15,000 |
| Value of assets | 400,000 |
| Accumulated employee contributions with interest (included in assets) | 60,000 |
| Present value of future compensation | 8,000,000 |
| Annual compensation | 1,000,000 |

In what range is the employer's normal cost for 1990 as of 1/1/90?

(A) Less than \$130,000
(B) \$130,000 but less than \$135,000
(C) \$135,000 but less than \$140,000
(D) \$140,000 but less than \$145,000
(E) \$145,000 or more

**Problem 2 - 25**

Normal retirement benefit:

Before 1991: 40% of final 3-year average compensation.
After 1990: 50% of final 3-year average compensation.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation results as of 1/1/90:

| | |
|---|---|
| Present value of future benefits | |
| Active participants | \$ 800,000 |
| Inactive participants | 0 |
| Value of assets | 300,000 |
| Present value of future compensation | 11,250,000 |
| 1990 compensation | 900,000 |

As of 1/1/90, participants were under age 64.
Contribution for 1990: Normal cost for 1990 as of 1/1/90, paid on 1/1/90.

There were no experience gains or losses due to new participants, deaths, terminations, or retirements during 1990.

There were no new participants as of 1/1/91.
All active participants received an 8% compensation increase from 1990 to 1991.

Normal cost for 1991 as of 1/1/91: \$60,000.

In what range is the value of assets as of 12/31/90?

(A) Less than \$365,000
(B) \$365,000 but less than \$375,000
(C) \$375,000 but less than \$385,000
(D) \$385,000 but less than \$395,000
(E) \$395,000 or more

**Problem 2 - 26**

Valuation date: 12/31
Normal retirement benefit: 50% of final 3-year average compensation.
Actuarial cost method: Aggregate.

Actuarial assumptions:

> Interest rate: 7% per year.
> Compensation increases: None.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Data for the only participants:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/26 | 1/1/45 |
| 1990 compensation | - | $30,000 |
| Status as of 12/31/90 | Retired | Active |
| Monthly benefit | $1,000 | - |

Value of assets as of 12/31/90: $94,650

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1990 as of 12/31/90?

(A) Less than $3,000
(B) $3,000 but less than $3,400
(C) $3,400 but less than $3,800
(D) $3,800 but less than $4,200
(E) $4,200 or more

**Problem 2 - 27**

Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Aggregate.

Actuarial assumptions:

> Interest rate: 7%
> Preretirement terminations other than deaths: None.
> Retirement age: 65

Data for all participants as of 1/1/90:

| Age | Number of Participants | Past Service | Status |
|---|---|---|---|
| 45 | 15 | 20 | Active |
| 55 | 10 | 30 | Active |
| 65 | 5 | 40 | Retired |

Value of assets as of 1/1/90: \$300,000.

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 45 | 4,528 | 58,163 |
| 55 | 2,187 | 24,581 |
| 65 | 965 | 8,872 |

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than \$22,250
(B) \$22,250 but less than \$24,250
(C) \$24,250 but less than \$26,250
(D) \$26,250 but less than \$28,250
(E) \$28,250 or more

**Problem 2 - 28**

Plan effective date: 1/1/90
Normal retirement benefit: 1% of final year's compensation for each year of service.
Early retirement benefit: Payable immediately without reduction.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 7% per year.
Preretirement deaths and terminations: None.

| Age x | Probability of retiring at age x |
|---|---|
| 62 | 50% |
| 63 | 0% |
| 64 | 0% |
| 65 | 100% |

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/89 |
| 1989 compensation | $100,000 |
| Status as of 1/1/90 | Active |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.394 \qquad \ddot{a}_{65}^{(12)} = 8.736$

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than $9,850
(B) $9,850 but less than $10,000
(C) $10,000 but less than $10,150
(D) $10,150 but less than $10,300
(E) $10,300 or more

**Problem 2 - 29**

Normal retirement benefit:

Before 1990: 50% of highest 3-year average compensation.
After 1989: 100% of highest 3-year average compensation.

Normal form of payment:

| | |
|---|---|
| Retirements before 1990: | If single, life annuity. If married, 100% joint and survivor annuity (fully subsidized.) |
| Retirements after 1989: | Life annuity. |

Actuarial cost method: Aggregate.

Actuarial assumptions:

Preretirement deaths and terminations: None.
Retirement age: 65
Marital characteristics: 85% of participants are married at the time of retirement; spouses are the same age as participants.

Valuation results as of 1/1/90, before amendments:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 94,000 |
| Present value of future benefits for inactive participants (all became inactive before 1990) | 500,000 |
| Value of assets | 730,000 |
| Present value of future compensation | 4,850,000 |
| 1990 compensation | 375,000 |

Selected annuity values:

$\ddot{a}_{65}^{(12)} = 10$ $\ddot{a}_{65:65}^{(12)} = 8.2$

In what range is the normal cost for 1990 as of 1/1/90 after the amendments?

(A) Less than \$175,000
(B) \$175,000 but less than \$185,000
(C) \$185,000 but less than \$195,000
(D) \$195,000 but less than \$205,000
(E) \$205,000 or more

**Problem 2 - 30**

Normal retirement benefit: 50% of final year's compensation.
Preretirement death benefit: None.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases:
Assumption before 1991: None.
Assumption after 1990: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/29 |
| 1991 compensation | \$40,000 |
| Status as of 1/1/91 | Active |

Normal cost for 1991 as of 1/1/91, before change in assumed compensation increases: \$23,615

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 62 | 365 | 3,514 |
| 63 | 335 | 3,149 |
| 64 | 306 | 2,814 |
| 65 | 279 | 2,508 |
| 66 | 254 | 2,229 |

In what range is the normal cost for 1991 as of 1/1/91?

(A) Less than $25,250
(B) $25,250 but less than $27,250
(C) $27,250 but less than $29,250
(D) $29,250 but less than $31,250
(E) $31,250 or more

**Problem 2 - 31**

Actuarial cost method: Aggregate.

Selected valuation results as of 1/1/91:

| | |
|---|---|
| Present value of all future benefits | $10,850,000 |
| Present value of future death and termination benefits due to accumulated and future employee contributions (included above) | 850,000 |
| Present value of future employee contributions | 2,000,000 |
| Value of assets, including $1,200,000 of accumulated employee contributions | 4,800,000 |
| Present value of future compensation | 50,600,000 |
| Annual compensation | 3,500,000 |

In what range is the increase in the employer's normal cost for 1991 as of 1/1/91 if future employee contributions were discontinued and all accumulated employee contributions were refunded to participants as of 12/31/90?

(A) Less than $100,000
(B) $100,000 but less than $120,000
(C) $120,000 but less than $140,000
(D) $140,000 but less than $160,000
(E) $160,000 or more

**Problem 2 - 32**

Normal retirement benefit: $25 per month for each year of service.
Early retirement reduction: None.
Actuarial cost method: Aggregate.

Actuarial assumptions:

> Interest rate: 7% per year.
> Preretirement deaths and terminations: None.
> Retirement age:
>
> > Before 1992: 60
> > After 1991: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/80 |

Value of assets as of 1/1/92: $5,000

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 9.82$ $\qquad$ $\ddot{a}_{65}^{(12)} = 8.74$

In what range is the decrease in the normal cost as of 1/1/92 due to the change in the assumed retirement age?

(A) Less than $1,700
(B) $1,700 but less than $1,850
(C) $1,850 but less than $2,000
(D) $2,000 but less than $2,150
(E) $2,150 or more

**Problem 2 - 33**

Plan effective date: 1/1/91
Normal retirement benefit: 30% of final 3-year average compensation.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Normal cost for 1991 as of 1/1/91: $7,750
Value of assets as of 1/1/92: $8,292
Ratio of present value of future compensation to current annual compensation as of 1/1/91: 20.13

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/51 |
| Compensation increase during 1991 | 8% |

In what range is the increase in the normal cost for 1992 as of 1/1/92 due to the actual increase in compensation over the assumed increase in compensation?

(A) Less than $227
(B) $227 but less than $234
(C) $234 but less than $241
(D) $241 but less than $248
(E) $248 or more

**Problem 2 - 34**

Normal retirement benefit: $40 per month for each year of service.

Preretirement death benefit: Lump sum equal to 50 times the monthly projected normal retirement benefit; provided by a whole life insurance policy.

Actuarial cost method: Aggregate, split-funded with life insurance.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/52 |
| Date of hire | 1/1/87 |

Cash value at age 65 per $1,000 of life insurance: $200
Annual premium as of 1/1/92: $800
Value of assets in side fund as of 1/1/92: $5,000

$$\ddot{a}_{65}^{(12)} = 8.74$$

In what range is the normal cost for the side fund for 1992 as of 1/1/92?

(A) Less than $1,200
(B) $1,200 but less than $1,400
(C) $1,400 but less than $1,600
(D) $1,600 but less than $1,800
(E) $1,800 or more

**Problem 2 - 35**

Normal retirement benefit: 40% of final year's compensation.
Actuarial cost method: Aggregate.

Initial valuation results as of 1/1/92:

| | |
|---|---|
| Present value of all future benefits | $ 1,600,000 |
| Present value of future benefits for retired and terminated participants | 320,000 |
| Value of assets | 400,000 |
| Present value of future compensation | 19,200,000 |
| Annual compensation | 3,200,000 |

As of 1/1/92, there are no active participants within one year of the assumed retirement age.

After completing the valuation, it was discovered that all active participants received a 7% increase in annual compensation which had not been reported, and the valuation results were recalculated.

In what range is the recalculated normal cost for 1992 as of 1/1/92?

(A) Less than \$210,500
(B) \$210,500 but less than \$214,500
(C) \$214,500 but less than \$218,500
(D) \$218,500 but less than \$222,500
(E) \$222,500 or more

**Problem 2 - 36**

Type of plan: Contributory.
Employee contributions: 1.5% of annual compensation.
Actuarial cost method: Aggregate.

Employee contributions are assumed to be paid on 1/1 of each year.

Selected valuation results as of 1/1/93:

| | |
|---|---|
| Present value of future benefits: | |
| Retirement benefits | \$2,000,000 |
| Refunds of employee contributions upon death or termination | 70,000 |
| Other death and termination benefits | 200,000 |
| Value of total assets | 500,000 |
| Accumulated employee contributions included in total assets | 80,000 |
| Present value of future compensation | 7,000,000 |
| Annual compensation | 1,000,000 |

In what range is the employer's normal cost for 1993 as of 1/1/93?

(A) Less than \$220,000
(B) \$220,000 but less than \$230,000
(C) \$230,000 but less than \$240,000
(D) \$240,000 but less than \$250,000
(E) \$250,000 or more

**Problem 2 - 37**

Normal retirement benefit: 50% of 5-year final average compensation.
Actuarial cost method: Aggregate (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases:

Before 1994: 5% per year.
After 1993: 6% per year.

Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/69 |
| 1994 valuation compensation | $100,000 |

Selected valuation results as of 1/1/94 before change in assumptions:

| | |
|---|---|
| Normal cost as of 1/1 | $ 38,120 |
| Value of assets | $200,000 |

In what range is the normal cost for 1994 as of 1/1/94 after the change in actuarial assumptions?

(A) Less than $38,800
(B) $38,800 but less than $39,200
(C) $39,200 but less than $39,600
(D) $39,600 but less than $40,000
(E) $40,000 or more

**Problem 2 - 38**

Normal retirement benefit: $2,000 per month.

Actuarial cost method: Aggregate with side fund and life insurance (split-funded).

Actuarial assumptions:

Compensation increases: None.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data as of 1/1/94:

| | |
|---|---|
| Value of assets in side fund | $10,000 |
| Annual premium for life insurance | 2,000 |
| Cash value of life insurance at age 65 | 50,000 |

Date of birth for sole participant (active as of 1/1/94): 1/1/49

Selected commutation functions based on preretirement assumptions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 45 | 445 | 5,691 |
| 55 | 214 | 2,405 |
| 65 | 94 | 868 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.78$

In what range is the total normal cost for 1994 as of 1/1/94?

(A) Less than $4,000
(B) $4,000 but less than $4,300
(C) $4,300 but less than $4,600
(D) $4,600 but less than $4,900
(E) $4,900 or more

**Problem 2 - 39**

Plan effective date: 1/1/95.
Normal retirement benefit: $10 per month for each year of service since date of hire.
Preretirement death benefit: None.
Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65.

Selected valuation data for only participants (all active as of 1/1/95):

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/60 | 1/1/60 | 1/1/50 |
| Date of hire | 1/1/81 | 1/1/81 | 1/1/71 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 35 | 894,190 | 12,364,650 |
| 45 | 445,008 | 5,690,850 |
| 65 | 94,414 | 868,052 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than $1,600
(B) $1,600 but less than $1,700
(C) $1,700 but less than $1,800
(D) $1,800 but less than $1,900
(E) $1,900 or more

**Problem 2 - 40**

Mandatory employee contribution: 0.5% of compensation, paid on 1/1.
Actuarial cost method: Aggregate.

Present value of future benefits (excluding any benefits due to voluntary employee contributions) as of 1/1/95:

| | |
|---|---|
| Retirement benefits | \$1,000,000 |
| Return of mandatory employee contributions at death or termination | 20,000 |
| Employer-provided termination benefits | 80,000 |

Present value of future compensation as of 1/1/95: \$4,800,000

Total valuation compensation for 1995: \$600,000
Mandatory employee contributions for 1995: \$3,000 paid on 1/1/95.
Voluntary employee contributions for 1995: \$2,000 paid on 1/1/95.

Value of assets as of 12/31/94:

| | |
|---|---|
| Mandatory employee contributions with interest | \$30,000 |
| Voluntary employee contributions with interest | 20,000 |
| Other assets | 150,000 |
| Total assets | 200,000 |

In what range is the employer normal cost for 1995 as of 1/1/95?

(A) Less than \$109,500
(B) \$109,500 but less than \$111,000
(C) \$111,000 but less than \$112,500
(D) \$112,500 but less than \$114,000
(E) \$114,000 or more

**Problem 2 - 41**

Normal retirement benefit:

Before 1995: \$30 per month for each year of service.
After 1994: \$35 per month for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

As of 1/1/94, all participants were active and under age 63.

Selected valuation results as of 1/1/94:

| | |
|---|---|
| Present value of future benefits | $900,000 |
| Value of assets | 500,000 |
| Average present value of future working lifetime of active participants | 11 |

The contribution for 1994 was paid on 1/1/94 in an amount equal to the normal cost for 1994 as of 1/1/94.

There were no deaths, terminations, or retirements during 1994, and there are no new participants on 1/1/95.

There were no investment experience gains or losses during 1994.

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than $43,300
(B) $43,300 but less than $47,300
(C) $47,300 but less than $51,300
(D) $51,300 but less than $54,300
(E) $54,300 or more

**Problem 2 - 42**

Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Aggregate.
Assumed compensation increases: 3% per year.

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | $ 149,000 |
| Value of assets | 20,000 |
| Present value of future compensation | 1,200,000 |
| Annual compensation | 150,000 |

Investment gain during 1994: $1,600.
Actual compensation increases during 1994: 10%.
There were no other experience gains or losses during 1994.

There have never been any inactive participants.

In what range is the change in the normal cost for 1995 as of 1/1/95 due to experience gains and losses?

(A) Less than $1,000
(B) $1,000 but less than $1,100
(C) $1,100 but less than $1,200
(D) $1,200 but less than $1,300
(E) $1,300 or more

**Problem 2 - 43**

| | |
|---|---|
| Actuarial cost method: | Aggregate. |
| Assumed interest rate: | 7% per year. |

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | $1,200,000 |
| Present value of future compensation | 9,500,000 |
| Annual compensation | 750,000 |

The plan's assets earned a 4% return in 1994; the resulting investment experience loss increased the normal cost for 1995 as of 1/1/95 by $100.

There were no contributions or disbursements during 1994.

In what range is the value of assets as of 1/1/95?

(A) Less than $34,000
(B) $34,000 but less than $37,000
(C) $37,000 but less than $40,000
(D) $40,000 but less than $43,000
(E) $43,000 or more

**Problem 2 - 44**

Normal retirement benefit:

| | |
|---|---|
| Before 1996: | 60% of final 5-year average compensation. |
| After 1995: | 70% of final 5-year average compensation. |

Actuarial cost method: Aggregate.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | \$1,500,000 |
| Value of assets | 300,000 |
| Normal cost as of 1/1 | 96,000 |

As of 1/1/95, all participants were active and under age 55.

There were no deaths, terminations, retirements, or new participants during 1995, and there are no new participants as of 1/1/96.

For each participant, 1996 valuation compensation is 6% higher than 1995 valuation compensation.

Normal cost for 1996 as of 1/1/96: \$122,650.

In what range is the value of assets as of 1/1/96?

(A) Less than \$200,000
(B) \$200,000 but less than \$300,000
(C) \$300,000 but less than \$400,000
(D) \$400,000 but less than \$500,000
(E) \$500,000 or more

**Problem 2 - 45**

Normal retirement benefit: $50 per month for each year of service.

Normal form of payment: Life annuity with 120 months certain.

Actuarial cost method: Aggregate.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Value of assets as of 1/1/96: $110,000.

Valuation data for all participants as of 1/1/96:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/29 | 1/1/51 |
| Date of hire | 1/1/74 | 1/1/86 |
| Status | Retired on 1/1/94 | Active |

Selected commutation functions:

| $x$ | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 94,414 | 824,780 |
| 67 | 78,601 | 651,367 |
| 75 | 33,855 | 217,236 |
| 77 | 25,541 | 153,955 |

After preparing the 1/1/96 valuation, the actuary was informed that Smith died on 12/15/95. The actuary prepared a revised 1/1/96 valuation based upon this information.

In what range is the difference in the normal cost for 1996 as of 1/1/96 under the two valuations?

(A) Less than $2,450
(B) $2,450 but less than $2,600
(C) $2,600 but less than $2,750
(D) $2,750 but less than $2,900
(E) $2,900 or more

**Problem 2 - 46**

Normal retirement benefit: 1% of final year's compensation for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

> Interest rate: 6% per year.
> Compensation increases: 5% per year.
> Pre-retirement decrements other than deaths: None.
> Retirement age: 65.

Value of assets as of 1/1/96: $60,000.

Valuation data for all participants as of 1/1/96:

| | Smith | Brown | Green |
|---|---|---|---|
| Status | Active | Terminated Vested | Retired |
| Age at hire | 25 | - | - |
| Attained age | 35 | 50 | 65 |
| 1996 valuation compensation | $48,000 | - | - |
| Monthly accrued benefit | $375 | $200 | $300 |

Selected commutation functions and annuity values:

| $x$ | $\dfrac{{}^sN_x - {}^sN_{65}}{{}^sD_x}$ | ${}_{65-x\|}\ddot{a}_x^{(12)}$ |
|---|---|---|
| 25 | 16.7 | 0.4 |
| 30 | 14.3 | 0.7 |
| 35 | 12.5 | 1.0 |
| 50 | 11.1 | 4.0 |
| 65 | 0.0 | 10.0 |

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $5,000
(B) $5,000 but less than $5,200
(C) $5,200 but less than $5,400
(D) $5,400 but less than $5,600
(E) $5,600 or more

**Problem 2 - 47**

Type of plan: Contributory.

Normal retirement benefit: $50 per month for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Pre-retirement decrements: None.
Retirement age: 65.

As of 1/1/95, all participants were active and under age 63.

Selected valuation results:

| | 1/1/95 | 1/1/96 |
|---|---|---|
| Employer's normal cost as of 1/1 | $ 7,200 | |
| Present value of future benefits | 100,000 | |
| Present value of all future employee contributions | 25,000 | $ 24,500 |
| Value of plan assets | 10,000 | 22,000 |

There were no deaths, terminations, retirements, or new participants during 1995, and there are no new participants as of 1/1/96.

In what range is the employer's normal cost for 1996 as of 1/1/96?

(A) Less than $7,000
(B) $7,000 but less than $7,200
(C) $7,200 but less than $7,400
(D) $7,400 but less than $7,600
(E) $7,600 or more

**Problem 2 - 48**

Actuarial cost method: Aggregate.

Mandatory employee contributions: 1.5% of compensation, paid on 1/1.

Voluntary employee contributions for 1997: $10,000, paid on 1/1.

Selected 1/1/97 valuation results:

| | |
|---|---|
| Present value of all future retirement benefits | $2,500,000 |
| Present value of refunds of mandatory employee contributions upon death or termination | 84,000 |
| Present value of other employer-provided death and termination benefits | 25,000 |
| Value of assets as of 12/31/96 | 725,000 |
| Accumulated voluntary employee contributions included in assets as of 12/31/96 | 35,000 |
| Present value of future compensation | 8,400,000 |
| 1997 valuation compensation | 1,200,000 |

In what range is the employer normal cost for 1997 as of 1/1/97?

(A) Less than $250,000
(B) $250,000 but less than $255,000
(C) $255,000 but less than $260,000
(D) $260,000 but less than $265,000
(E) $265,000 or more

**Problem 2 - 49**

Actuarial cost method: Aggregate.

Normal form of payment for single participants: Life annuity.

Normal form of payment for married participants: Joint and 100% survivor annuity with benefit equal to:

| | |
|---|---|
| Before 1/1/97 amendment: | Actuarial equivalent of life annuity benefit. |
| After 1/1/97 amendment: | 100% of life annuity benefit. |

Assumed form of payment:

| | |
|---|---|
| Before 1/1/97 amendment: | Life annuity for all participants. |
| After 1/1/97 amendment: | Life annuity for single participants; joint and 100% survivor annuity for married participants. |

Married participants assumptions: 80% married, with no age difference between spouses.

Selected valuation results as of 1/1/97 (before amendment):

| | |
|---|---|
| Present value of future benefits | $ 2,000,000 |
| Value of assets | 1,050,000 |
| Present value of future compensation | 27,000,000 |
| 1997 valuation compensation | 1,600,000 |

There are no inactive participants as of 1/1/97.

Selected annuity values:

$12\ddot{a}_{65}^{(12)} = 104.83$ $\qquad$ $12\ddot{a}_{\overline{65:65}}^{(12)} = 126.9$

In what range is the increase in the normal cost for 1997 as of 1/1/97 due to the plan amendment?

(A) Less than $19,000
(B) $19,000 but less than $21,000
(C) $21,000 but less than $23,000
(D) $23,000 but less than $25,000
(E) $25,000 or more

## 2.5 Solutions to Problems

**Problem 2 - 1**

| | Pension | Years to NRD | Values at NRD | PVFB | PV Future years |
|---|---|---|---|---|---|
| Homer | $780 | 18 | 93,600 | 32,792 | 11.48 |
| Jake | 300 | 11 | 36,000 | 18,964 | 8.36 |
| Pete | 600 | 29 | 72,000 | 13,288 | 14.41 |
| (30 years of service times $20) | | | | 65,044 | 34.25 |

| | |
|---|---|
| Present Value of Future Benefits | $65,044 |
| Less: Plan assets | (8,698) |
| Present Value of Future Normal Costs | 56,346 |
| Divided by PV Future Years | ÷ 34.25 |
| Normal Cost per Participant | 1,645 |
| For three Participants | 4,935 |

**Problem 2 - 2**

Key Concept: Increasing the ages by one year will affect both the present value of future benefits and the present value of future years. The present value of future benefits will be increased by the amount of interest for one year.

The present value of future years will be reduced by the following:

$$(1.06)^{-19} + (1.06)^{-14} + (1.06)^{-9} + (1.06)^{-4} = .33 + .44 + .59 + .79 = 2.15$$

This can be seen by expanding $\ddot{a}_{\overline{10|}}$

$$\ddot{a}_{\overline{10|}} = 1 + (1.06)^{-1} + (1.06)^{-2} + ... + (1.06\ )^{-9}$$

| | Original Calculation | Revised Calculation |
|---|---|---|
| 1. Pres. Value Benefits | \$112,000 | \$118,720 |
| 2. Plan Assets | 42,560 | 42,560 |
| 3. PV of Future Normal Costs | 69,440 | 76,160 |
| 4. PV of Future Years (3) ÷ (5) | 34.72 | 32.57 |
| 5. Normal Cost per Participant | 2,000 | 2,338 |
| 6. Normal Cost of Plan | 8,000 | 9,352 |

Alternative Solution: Revised number of years may be obtained by adjusting as follows:

$$(34.72 - 4)(1.06) = 32.56$$

**Problem 2 - 3**

Key Concept: The annuity rate must be revised to be a weighted average of those given. The only quantity in our calculations that is affected is the Present Value of Future Benefits.

Annuity Rate:

40% of 12 = 4.80
60% of 14 = 8.40
Total = 13.20
Ratio of revised to old = 13.20 ÷ 12

| | Original Calculations | Difference | Revised Calculation |
|---|---|---|---|
| PVFB - Active | 450,000 | × 13.20 ÷ 12 | 495,000 |
| PVFB - Retirees | 100,000 | | 100,000 |
| PVFB - Total | 550,000 | | 595,000 |
| Plan Assets | 250,000 | | 250,000 |
| PV - Future Normal Costs | 300,000 | | 345,000 |
| PV - Future Years | 12.0 | | 12.0 |
| Normal Cost | 25,000 | | 28,750 |

**Problem 2 - 4**

Key Concepts: (1) When using a salary increase assumption, some attention should be given to the number of years to retirement or the number of "salary increases" that each participant will enjoy. If the compensation used is based upon the current year's salary, then if there are n years to retirement, there will be (n - 1) salary increases. If, however, salary is based upon the prior year's salary (as in this problem), there are n salary increases.

The difference arises because prior year salaries are usually used for a beginning of year valuation. Hence, provision should be made for a salary increase during the year of valuation. Therefore, to compute the present value of future benefits that are expected to be paid in n years, the current salary must be projected for n years.

(2) Assumed earnings of 6% with a salary scale of 4% is the equivalent of an earnings assumption only of 1.923%, i.e. $[(1.06 \div 1.04) - 1]$. This can be seen by listing the terms of $\ddot{a}_{\overline{n|}}$ at 6% earnings and 4% salary scale:

$$= 1 + (1.06)^{-1}(1.04) + (1.06)^{-2}(1.04)^2 + (1.06)^{-3}(1.04)^3 + \ldots + (1.06)^{-(n-1)}(1.04)^{(n-1)}$$

$$= 1 + (1.01923)^{-1} + (1.01923)^{-2} + (1.01923)^{-3} + \ldots + (1.01923)^{-(n-1)}$$

In this problem:

$$\text{PVFB} = 25\%(2{,}400) \times 12\ddot{a}_{65}^{(12)} \times (1.04)^{11}(1.06)^{-11}$$
$$= 600 \times 120 \times (1.01923)^{-11} = 58{,}390$$

Present Value of Future Salaries is determined by multiplying last year's salary ($2,400) by the sum of the following series:

$$(1.04) + (1.04)^2(1.06)^{-1} + (1.04)^3(1.06)^{-2} + \ldots + (1.04)^{11}(1.06)^{-10}$$
$$=(1.04)[1 + (1.04)(1.06)^{-1} + \ldots + (1.04)^{10}(1.06)^{-10}]$$
$$=(1.04)(\ddot{a}_{\overline{11|}\,1.923\%}) = 10.0189(1.04) = 10.4197$$

Present Value of Future Salaries = 2,400 × 12 × 10.4197 = 300,087

| | |
|---|---|
| PV Future Benefits | 58,390 |
| Plan assets | 22,000 |
| PV of Future Normal Cost | 36,390 |
| PV of Future Salary | 300,087 |
| Normal Cost Rate | 12.13% |
| 1986 Salary | 29,952 |
| Normal Cost | 3,633 |

Note: The expected 1986 salary (with one year salary scale) is used above to be consistent with the determination of the present value of future salaries which anticipated that increase.

**Problem 2 - 5**

The solution is as follows:

| | Pension | PVFB |
|---|---|---|
| A | $(2\% \times 50{,}000 \times 25 \text{ years}) \times 12.5 \times (1.04)^6 \times .630$ | 249,110 |
| B | $(2\% \times 30{,}000 \times 19 \text{ years}) \times 12.5 \times (1.04)^{17} \times .270$ | 74,945 |
| | | 324,055 |

| | Salary | PVFS |
|---|---|---|
| A | $50{,}000 \times 5.471$ | 273,550 |
| B | $30{,}000 \times 12.786$ | 383,580 |
| | | 657,130 |

| | |
|---|---|
| 1. Present Value of Future Benefits | 324,055 |
| 2. Less: Plan Assets | 125,000 |
| 3. Present Value of Future Normal Costs | 199,055 |
| 4. Present Value of Future Salaries | 657,130 |
| 5. Normal Cost Rate $3 \div 4$ | 30.29% |
| 6. Current Salaries | 80,000 |
| 7. Normal Cost $5 \times 6$ | 24,232 |

The astute student has probably noticed that the above solution does not adjust the PVFS or current salaries for the increase assumed to occur during 1986, as was done in problem 2 - 4 above. Did we miss something? Not really - the result would be the same as can be seen with a little algebra. The Normal Cost calculation can be written as:

$$NC = PVFNC \div (PVFS \div \text{Current Salaries})$$

If we adjust for the assumed salary increase:

$$\begin{aligned} PVFS' &= PVFS \times 1.04 \\ Salary' &= Salary \times 1.04 \end{aligned}$$

The Normal Cost would be:

$$\begin{aligned} NC &= PVFNC \div [PVFS' \div Salary')] \\ &= PVFNC \div (PVFS \times 1.04) \div (Salary \times 1.04) \\ &= PVFNC \div (PVFS \div Salary) \end{aligned}$$

Since the result is the same, why bother with the extra work of adjusting the PVFS and current salary? In other words, it does not matter if prior year or current year salaries are used in this part of the calculations. However, care must always be taken in projecting salary by the correct number of years to compute the projected benefit as discussed in problem 2 - 4.

**Problem 2 - 6**

The salary freeze impacts the Present Value of Future Benefits and the Temporary Annuity Factor.

(A) To calculate the effect upon the Present Value of Future Benefits:

The freeze will result in the loss of one year's increase in salary, so the revised Present Value of Future Benefits will be:

$$= (75{,}000)(1.03)^{-1} = 72{,}816$$

(B) Effect upon Present Value of Future Salaries:

The original annuity factor may be written as:

$$\ddot{a}_{\overline{10}|} = (1.03) + (1.08)^{-1}(1.03)^2 + (1.08)^{-2}(1.03)^3 + ... + (1.08)^{-9}(1.03)^{10}$$

The revised annuity factor is:

$$\begin{aligned} \text{Revised } \ddot{a}_{\overline{10}|} &= 1 + (1.08)^{-1}(1.03) + \ldots + (1.08)^{-9}(1.03)^9 \\ &= (1.03)^{-1} \times (\text{original } \ddot{a}_{\overline{10}|}) \\ \text{Original factor} &= 407{,}777 \div 50{,}000 = 8.1555 \\ \text{Revised factor} &= 8.1555 \times (1.03)^{-1} = 7.918 \\ \text{Present Value Future Salaries} &= 50{,}000 \times 7.918 = 395{,}900 \end{aligned}$$

**Problem 2 - 7**

Effect upon the Present Value of Future Benefits:

$$\text{PVFB} = {}_{35}p_{30}^{(T)} \times v^{35} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

$$= {}_{1}p_{30}^{(T)} \times {}_{34}p_{31}^{(T)} \times v^{35} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

$$= (1 - q_{30}^{\prime(w)})(1 - q_{30}^{\prime(d)})\,{}_{34}p_{31}^{(T)} \times v^{35} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

Changing the value of $q_{30}^{\prime(w)}$ from .25 to .30 has the effect of decreasing the Present Value of Future Benefits by .70 ÷ .75 or 93.3%.

In a similar manner, the temporary annuity factor can be written as:

$$\ddot{a}_{30:\overline{35|}} = 1 + {}_{1}p_{30}^{(T)} \times v \times \ddot{a}_{31:\overline{34|}}$$

$$= 1 + (1 - q_{30}^{\prime(w)})(1 - q_{30}^{\prime(d)}) \times v \times \ddot{a}_{31:\overline{34|}}$$

Therefore, the revised value of

$$\ddot{a}_{30:\overline{35|}} = 1 + (.70/.75)(\ddot{a}_{30:\overline{35|}} - 1)$$

**Problem 2 - 8**

Key Concepts: (1) The question is really asking what the 1/1/85 normal cost would have been if the salary increase were equal to that assumed, versus actual 1/1/85 normal cost.

(2) If the salary increase had been equal to that assumed, the 1/1/85 projected benefit would be equal to the 1/1/84 projected benefit.

Step I: Calculate the Present Value of projected benefits.

$$\text{Actual} = 59{,}208 \times \ddot{a}_{60}^{(12)} \times D_{60}/D_{36}$$
$$= 59{,}208 \times 12.5 \times .247 = 182{,}805$$

$$\text{Assumed} = 50{,}820 \times 12.5 \times .247 = 156{,}907$$

Step II: Calculate the Normal Cost.

| | Actual | Assumed |
|---|---|---|
| 1) Present Value of Benefits | \$182,805 | \$156,907 |
| 2) Value of Plan Assets | 9,000 | 9,000 |
| 3) Present Value Future Normal Costs (1) - (2) | 173,805 | 147,907 |
| 4) Temporary Annuity Factor | 17.445 | 17.445 |
| 5) Normal Cost (3) ÷ (4) | 9,963 | 8,478 |

Difference (Actual - Assumed) = \$1,485

Answer is D.

**Problem 2 - 9**

The change in the withdrawal rate impacts the Present Value of Future Benefits and the Temporary Annuity Factor.

(A) To revise the Present Value of Future Benefits:

$$\text{PVFB} = {}_{40}p_{25}^{(T)} \times v^{40} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

$$= {}_{1}p_{25}^{(T)} \times {}_{39}p_{26}^{(T)} \times v^{40} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

$$= (1 - q_{25}^{\prime(d)})(1 - q_{25}^{\prime(w)})\, {}_{39}p_{26}^{(T)} \times v^{40} \times \text{Benefit}_{65} \times \ddot{a}_{65}^{(12)}$$

The original PVFB reflects a withdrawal decrement at age 25 of (1 - .20) = .80
The revised PVFB reflects a withdrawal decrement at age 25 of (1 - .30) = .70

Therefore, changing the withdrawal rate from .20 to .30 reduces the PVFB by .70/.80:

$$\text{PVFB} = 20{,}000 \times (.70/.80) = 17{,}500$$

(B) To revise the temporary annuity factor, the temporary annuity factor may be written as:

$$\ddot{a}_{25:\overline{40|}} = 1 + {}_{1}p_{25}^{(T)} \times v \times \ddot{a}_{26:\overline{39|}}$$

$$= 1 + (1 - q_{25}^{\prime(w)})(1 - q_{25}^{\prime(d)}) \times v \times \ddot{a}_{26:\overline{39|}}$$

Therefore, the revised value of $\ddot{a}_{25:\overline{40|}}$ is:

$$= 1 + (.70/.80)(\ddot{a}_{25:\overline{40|}} - 1)$$

The original temporary annuity factor is calculated as:

$$\begin{aligned}\ddot{a}_{25:\overline{40|}} &= (\text{PVFB - Assets}) \div \text{Normal Cost} \\ &= (20{,}000 - 2000) \div 1080 = 16.667\end{aligned}$$

The revised temporary annuity factor is, therefore:

$$\ddot{a}_{25:\overline{40|}} = 1 + (.70/.80)(16.667 - 1) = 14.709$$

Revised Normal Cost = (17,500 - 2000) ÷ 14.709 = 1,054

Answer is C.

**Problem 2 - 10**

Key Concept: Under the aggregate method, a plan amendment or a change in benefits generally changes the present value of future benefits for <u>active</u> participants only.

Because benefits are directly proportional to salaries, a 5% increase in salaries results in a 5% increase in benefits as well as a 5% increase in the present value of future benefits for the active participants.

PVFB for Active Participants = 2,000,000 - 400,000 = 1,600,000

| | <u>Original</u> | | <u>Revised</u> |
|---|---|---|---|
| (1) PVFB (Actives) | \$1,600,000 | × 1.05 | \$1,680,000 |
| (2) PVFB (Inactives) | 400,000 | | 400,000 |
| (3) Plan Assets | 500,000 | | 500,000 |
| (4) Present Value of Future Normal Costs | | | |
| (1) + (2) - (3) | 1,500,000 | | 1,580,000 |
| (5) PV Future Salaries | 24,000,000 | × 1.05 | 25,200,000 |
| (6) Normal Cost Rate (4) ÷ (5) | .06250 | | .062698 |
| (7) Annual Salaries | 4,000,000 | × 1.05 | 4,200,000 |
| (8) Normal Cost (6) × (7) | 250,000 | | 263,332 |

(9) Difference = 263,332 - 250,000 = 13,332

Answer is B.

**Problem 2 - 11**

Key Concept: Under the aggregate method, generally the only valuation quantity affected by a plan amendment is the present value of future benefits. In this instance, only the present value of future benefits of the active participants is affected.

Prior to the amendment, the plan could be valued by using the life annuity factor at retirement because joint-and-survivor benefits were actuarially equivalent at normal retirement age. After the amendment, an adjusted annuity value can be used to value the actives. The adjustment is based on the annuity value for each form of payment weighted by the percentage of pensions assumed to be paid under that form of payment.

Adjustment to annuity factor (weighted average):

Prior to amendment, annuity value = 10

| | |
|---|---|
| Pensions to be paid under life annuity | $10 \times 20\% = 2.00$ |
| Pensions to be paid under J & S | $90\% \times 13 \times 80\% = \underline{9.36}$ |
| Adjusted Annuity Factor | =11.36 |

| | | |
|---|---|---|
| Adjusted PVFB for active Participants | $960{,}000 \times (11.36/10)$ | 1,090,560 |
| Present Value of Future Benefits (retired)* | | 700,000 |
| Total Present Value of Future Benefits | | 1,790,560 |
| Plan Assets* | | 800,000 |
| Present Value Future Normal Costs | | 990,560 |
| Present Value of Future Salaries* | | 12,000,000 |
| Normal Cost Rate | | .082547 |
| Salaries | | 1,500,000 |
| Normal Cost | | 123,820 |

*Same as original valuation.

Answer is B.

Note: If the same amendment applied to retirees, the calculations would be considerably more complex because the adjusted annuity value depends upon the ratio of ($\ddot{a}_{\overline{x:y}} \div \ddot{a}_x$), which varies by age. Therefore, separate calculations would have to be made for each age of retirees.

**Problem 2 - 12**

Key Concept: The relationship between the Present Value of Benefits at any age and at any subsequent age must be understood. This may be seen from the following calculation:

$$(1)\ PV_x = (\text{Values at NRD}) \times (D_{65}/D_x)$$
$$(2)\ PV_{x+n} = (\text{Values at NRD}) \times (D_{65}/D_{x+n})$$

Dividing (1) by (2):

$$(PV_x/PV_{x+n}) = D_{x+n}/D_x$$
$$\text{and} \qquad PV_{x+n} = PV_x(D_x/D_{x+n})$$

In this problem, we must calculate the Present Value of Benefits at age 41 when given the Present Value of Benefits at age 40.

Step I: Determine the Present Value of Benefits at 1/1/85.

$$NC_{85} = (PVFB_{85}) \div [(N_{40} - N_{65}) \div D_{40}] = 1{,}500$$
$$1{,}500 = (PVFB_{85}) \div (787 \div 67)$$
$$PVFB_{85} = (1{,}500 \div 67) \times 787 = 17{,}619$$

Step II: Determine the Present Value of Benefits at 1/1/86.

$$PVFB_{86} = (PVFB_{85})(D_{40}/D_{41})$$
$$= 17{,}619 \times (67 \div 62) = 19{,}040$$

Step III: Determine the Normal Cost at 1/1/86.

$$NC_{86} = (PVFB_{86} - Assets_{86}) \div [(N_{41} - N_{65})/D_{41}]$$
$$= (19{,}040 - 1{,}675) \times 62 \div 720 = 1{,}495$$

Answer is D.

**Problem 2 - 13**

Key Concept: The problem calls for determination of Normal Cost as a <u>level</u> <u>dollar</u> <u>amount</u> as opposed to a level percentage of salary. Hence, the solution involves the division of the Present Value of Future Normal costs by $\ddot{a}_{\overline{30}|.06}$. If the problem called for a level percentage, the divisor would be $\ddot{a}_{\overline{30}|(1.06/1.04 - 1)}$.

| | | |
|---|---|---|
| (1) Current Salary | | \$ 50,000 |
| (2) Projected Annual Salary | $50{,}000 \times (1.04)^{29}$ | 155,933 |
| (3) Projected Annual Benefit | $50\% \times (2)$ | 77,967 |
| (4) Value at Retirement | $(3) \times \ddot{a}_{65}^{(12)}$ | 779,670 |
| (5) Net Value at Retirement | (4) - cash value = 779,670-110,000 | 669,670 |
| (6) Present Value Benefits | $(5) \times v^{30} = 669{,}670 \times .17411$ | 116,596 |
| (7) Present Value Future Normal Costs | (6) - Plan Assets = (6) - 20,000 | 96,596 |
| (8) Normal Cost 1/1/86 | $(7) \div \ddot{a}_{\overline{30}|} = 96{,}596 \div 14.59$ | 6,621 |

Answer is B.

**Problem 2 - 14**

| | <u>Smith</u> | <u>Brown</u> |
|---|---|---|
| (1) Attained age | 30 | 40 |
| (2) Years to Retirement | 35 | 25 |
| (3) Retirement Benefit | \$ 1,000 | \$1,000 |
| (4) Values at NRD $(3) \times 12 \times \ddot{a}_{65}^{(12)}$ | 120,000 | 120,000 |
| (5) Insurance CV at NRD | 40,000 | 30,000 |
| (6) Value of Side Fund at NRD | 80,000 | 90,000 |
| (7) Present Value of Benefits $(6) \times v^n$ | 7,493 | 16,582 |
| (8) Annuity Rate $\ddot{a}_{\overline{n}|}$ | 13.854 | 12.469 |

Present Value of Benefits = 7,493 + 16,582 = 24,075
Plan Annuity Rate = 13.854 + 12.469 = 26.323
Normal Cost = (24,075 - Plan Assets) ÷ 26.323
= (24,075 - 5,000) ÷ 26.323 = 724.65 per participant.
Side Fund Normal Cost = 724.65 × 2 participants = 1,449

Answer is A.

**Problem 2 - 15**

Key Concept: Since Brown retires at an age other than 65, attention must be given to its impact upon elements of the valuation.

| | Age at Hire | Age at Participation | Attained Age | Retirement Age |
|---|---|---|---|---|
| Smith | 33 | 42 | 50 | 65 |
| Brown | 40 | 57 | 65 | 67 |

| | --Years of Service-- | | | ---Participation--- | | | Projected |
|---|---|---|---|---|---|---|---|
| | Past | Future | Total | Past | Future | Total | Pension |
| Smith | 17 | 15 | 32 | 8 | 15 | 23 | $320 |
| Brown | 25 | 2 | 27 | 8 | 2 | 10 | 270 |

Determine Present Value of Benefits:

$$PVFB_{Smith} = (320 \times 12)(\ddot{a}_{65}^{(12)})(1.08)^{-15}$$
$$= (320)(12 \times 8.142)(.31524) = 9{,}856$$

$$PVFB_{Brown} = (270)(12 \times \ddot{a}_{67}^{(12)})(1.08)^{-2}$$
$$= (270)(12 \times 7.702)(.85734) = 21{,}394$$

Determine Normal Cost:

Total PVFB = 9,856 + 21,394 = 31,250
Asset Value = 10,000
Difference = 21,250

Present Value of Future Years = $\ddot{a}_{\overline{15|}} + \ddot{a}_{\overline{2|}}$ = 11.1702
Normal Cost per Participant = 21,250 ÷ 11.1702 = 1,902.38
Normal Cost = 1902.38 × 2 = 3,805

Answer is E.

**Problem 2 -16**

Key Concept: The items affected in making the revised valuation are (1) Present Value of Future Benefits and (2) Average Temporary Annuity. Actuarial Asset Value will, of course, remain the same.

Step I: Determine the Present Value of Future Benefits with the employee added. Since all are the same age, the Present Value of Future Benefits will increase in exactly the same proportion as salaries.

$$PVFB_{new} = PVFB_{old} \times (Salaries_{new} \div Salaries_{old})$$
$$= 500{,}000 \times (225{,}000 \div 200{,}000) = 562{,}500$$

Step II: Determine the Average Temporary Annuity under original valuation.

| | |
|---|---|
| PVFB | 500,000 |
| Less Assets | -100,000 |
| Present Value Future Normal Costs | 400,000 |

ATA (per participant) = (PVFNC ÷ NC) = 400,000 ÷ 40,000 = 10

Since all participants are the same age, the Average Temporary Annuity is still 10 under the revised valuation.

Step III: Determine the revised Normal Cost.

$$NC = [(PVFNC - Assets) \div 50] \times 5$$
$$= [(562{,}500 - 100{,}000) \div 50] \times 5 = 46{,}250$$

Answer is E.

**Problem 2 - 17**

Key Concept: To determine the final average salary in problems involving salary increase assumptions, use either of the following formulae:

$$Comp_{avg} = C_{ra-n}(s_{\overline{n}|} \div n)$$

or

$$Comp_{avg} = C_{ra}(a_{\overline{n}|} \div n) \text{ where } ra = \text{retirement age.}$$

n = number of years to be averaged.

C = Salary.

For example, in this problem, the participant is 55, normal retirement age is 65 and the number of years to be averaged is 5. Hence, we can project the salary to age 60 and multiply by ($s_{\overline{5}|} \div 5$).

Note: There are 6 salary changes to age 60 since we are given the 1986 salary.

$$\begin{aligned}\text{Average Salary} &= (\text{Salary}_{86} \times 1.06^6)(s_{\overline{5}|} \div 5)\\ &= (32{,}000 \times 1.41852)(5.6371 \div 5) = 51{,}177\end{aligned}$$

We could also project salary to age 65 and multiply by $a_{\overline{5}|} \div 5$.

$$\begin{aligned}\text{Average salary} &= (\text{Salary}_{86} \times 1.06^{11})(a_{\overline{5}|} \div 5)\\ &= (32{,}000 \times 1.8983)(4.2124 \div 5) = 51{,}177\end{aligned}$$

$$\text{Benefit} = 51{,}177 \times 40\% = 20{,}471$$

$$\text{PVFB} = (\text{Ben} \times \ddot{a}_{65}^{(12)})(1.08)^{-10} = 20{,}471 \times 8.65 \times .4632 = 82{,}021$$

$$\begin{aligned}\text{PVFS} &= (\text{Salary})[1 + (1.06/1.08) + (1.06/1.08)^2 + ... + (1.06/1.08)^9]\\ &= (32{,}000)(a_{\overline{10}|j}) \quad (\text{where } j = 1.08/1.06 - 1 = .018868)\\ &= (32{,}000)(9.2065) = 294{,}608\end{aligned}$$

$$\begin{aligned}\text{NC} &= (\text{PVFB - Assets}) \times (\text{Salary} \div \text{PVFS})\\ &= (82{,}020 - 30{,}000)(32{,}000 \div 294{,}608) = 5{,}650\end{aligned}$$

Answer is C.

**Problem 2 - 18**

Key Concepts: The Normal Cost rate is Present Value of Future Normal Costs divided by the Present Value of Future Salaries. We can determine each quantity from the 1/1/86 data. The Present Value of Future Salaries at 1/1/86 is not given. However, it can be calculated from the Normal Cost and Current Salaries which are given.

Step I: Determine Present Value of Benefits at 1/1/87 from data given from 1/1/86 valuation. If benefits were to remain the same, the Present Value of Benefits at 1/1/87 would be equal to the Present Value of Benefits at 1/1/86 multiplied by (1 + i). Since salary increases did not equal the assumed increase, benefits need to be reduced to (1.05/1.06) of benefits projected at 1/1/86.

$$\begin{aligned}\text{PVFB}_{87} &= \text{PVFB}_{86} \times (1.05/1.06) \times 1.08\\ &= 950{,}000 \times (1.05/1.06) \times 1.08 = 1{,}016{,}321\end{aligned}$$

Step II: Determine Actuarial Value of Assets at 1/1/87.

$$\begin{aligned} \text{Assets}_{87} &= (\text{Assets}_{86} + \text{NC}_{86}) \times \text{Investment return} \\ &= (500{,}000 + 60{,}000) \times 1.10 = 616{,}000 \end{aligned}$$

Step III: Determine the Present Value of Future Salaries at 1/1/86.

$$\begin{aligned} \text{NC}_{86} &= (\text{PVFNC}_{86} \times \text{Salaries}_{86}) \div \text{PVFS}_{86} \\ &= (\text{PVFB}_{86} - \text{Assets}_{86}) \times \text{Salaries} \div \text{PVFS}_{86} \end{aligned}$$

or

$$\begin{aligned} \text{PVFS}_{86} &= (\text{PVFNC}_{86} \times \text{Salaries}_{86}) \div \text{NC}_{86} \\ &= (950{,}000 - 500{,}000) \times 1{,}000{,}000 \div 60{,}000 = 7{,}500{,}000 \end{aligned}$$

Step IV: Determine the PVFS at 1/1/87. Again, if Salaries were to increase by exactly the assumed rate, the PVFS at 1/1/87 would equal the PVFS at 1/1/86 decreased by Annual Salaries and multiplied by $(1 + i)$. However, salaries did not increase by 6% and an adjustment equal to (1.05/1.06) must be made.

$$\begin{aligned} \text{PVFS}_{87} &= (\text{PVFS}_{86} - \text{Salaries}_{86})(1.05/1.06)(1 + i) \\ &= (7{,}500{,}000 - 1{,}000{,}000)(1.05/1.06)(1.08) = 6{,}953{,}774 \end{aligned}$$

Step V: Determine Normal Cost Rate at 1/1/87.

$$\begin{aligned} \text{NC}_{87} \text{ percentage rate} &= (\text{PVFB}_{87} - \text{Assets}_{87}) \div \text{PVFS}_{87} \\ &= (1{,}016{,}321 + 616{,}000) \div 6{,}953{,}774 = 5.76\% \end{aligned}$$

Answer is D.

**Problem 2 - 19**

Key Concept: The Present Value of Benefits is the missing item. It can be calculated from information given for the Unit Credit Method used for 1987.

Step I: Determine a value for $12\ddot{a}_{65}^{(12)}$.

$$\begin{aligned} \text{Accrued Liability} &= (25 \times 9 \text{ years})(12\ddot{a}_{65}^{(12)})(1.06)^{-18} \\ &= (225)(12\ddot{a}_{65}^{(12)})(.35034) \\ &= (12\ddot{a}_{65}^{(12)})(78.83) \end{aligned}$$

$$\begin{aligned} \text{Accrued Liability} &= (\text{Actuarial Value of Assets}) + \text{UAL}_{87} \\ &= 6{,}300 + 2{,}500 = 8{,}800 \end{aligned}$$

Therefore $(12\ddot{a}_{65}^{(12)})(78.83) = 8{,}800$

and $12\ddot{a}_{65}^{(12)} = 111.63$

Step II: Determine the Present Value of Future Benefits as of 1/1/88.

Value of Benefits @ NRA = (111.63)(27 years × 25) = 75,350

$$PVFB_{88} = (1.06)^{-17}(75{,}350) = 27{,}982$$

Step III: Determine Actuarial Value of Assets at 1/1/88.

$$\begin{aligned} Assets_{88} &= Assets_{87} + \text{Contribution} + \text{Interest} \\ &= 6{,}300 + 1{,}250 + (10\% \text{ of } 6{,}300) = 8{,}180 \end{aligned}$$

Step IV: Determine Normal Cost at 1/1/88.

$$\begin{aligned} NC_{88} &= (PVFB_{88} - Assets_{88})/\ddot{a}_{\overline{17}|} \\ &= (27{,}982 - 8{,}180) \div 11.11 = 1{,}782 \end{aligned}$$

Answer is B.

**Problem 2 - 20**

Key Concept: Normal Cost must be determined assuming Brown retires at age 65 and also assuming he retires at age 62.

| | | Brown | |
|---|---|---|---|
| | Smith | NRA at 65 | NRA at 62 |
| (1) Attained Age | 55 | 62 | 62 |
| (2) Past Service | 30 | 37 | 37 |
| (3) Future Service | 10 | 3 | 0 |
| (4) Projected Benefit | \$400 | \$400 | \$370 × .82 = 303.40 |
| (5) PV of Benefits $(4) \times 12\ddot{a}_{ra}^{(12)} \times (D_{ra}/D_{aa})$ | 21,767 | 35,664 | 36,772 |
| (6) PV of Future Years $(N_{aa} - N_{ra})/D_{aa}$ | 7.44 | 2.78 | 0 |

| | |
|---|---|
| If Brown retires at age 65, then Present Value of Benefits | \$57,431 |
| Less: Actuarial Value of Assets | - 15,000 |
| Equals Present Value Future Normal Costs | 42,431 |
| Divided by Present Value of Future Years | ÷ 10.22 |
| Equals Normal Cost per Participant | 4,151.76 |
| Total Normal Cost (2 participants) | 8,303.52 |
| If Brown retires at Age 62, then Present Value of Benefits | \$58,539 |
| Less: Actuarial Value of Assets | - 15,000 |
| Equals PV Future Normal Costs | 43,539 |
| Divided by PV of Future Years | ÷ 7.44 |
| Equals Normal Cost per Participant (1 participant) | 5,852.02 |

Change in Normal Cost = 5,852.02 - 8,303.52 = (2,451.50)

Answer is A.

**Problem 2 - 21**

Key Concept: The normal cost under the Aggregate method remains a level percentage of salary each year if there are no gains or losses. Gains (losses) will decrease (increase) the normal cost.

Step I: Determine the Present Value of Future Salary as of 1/1/89.

$$NC_{89} = \frac{PVFB_{89} - Assets_{89}}{PVFS_{89}} \times Salary_{89}$$

$$31{,}250 = \frac{1{,}000{,}000 - 500{,}000}{PVFS_{89}} \times 1{,}000{,}000$$

$$PVFS_{89} = 16{,}000{,}000$$

Step II: Determine the Present Value of Future Salary as of 1/1/90.

The relationship for PVFS at time 0 to PVFS at time 1 is:

$$\text{PVFS}_1 = (\text{PVFS}_0 - \text{Salary}_0) \times (1+i)$$

$$\text{In this case, } \text{PVFS}_{90} = (\text{PVFS}_{89} - \text{Salary}_{89}) \times 1.06$$

$$= (16{,}000{,}000 - 1{,}000{,}000) \times 1.06 = 15{,}900{,}000$$

Step III: Determine the Salary as of 1/1/90.

$$\text{Salary}_{90} = \text{Salary}_{89} \times 1.05 = 1{,}000{,}000 \times 1.05 = 1{,}050{,}000$$

Step IV: Calculate the Normal Cost as of 1/1/90.

If there had been no gains or losses during 1989,

$$NC_{90} = NC_{89} \times 1.05 = 31{,}250 \times 1.05 = 32{,}812$$

The experience gain must be amortized (weighted by Salaries) as follows:

$$\begin{aligned}\text{Decrease in } NC_{90} &= 30{,}000/(\text{PVFS}/\text{Salary}) \\ &= 30{,}000/(15{,}900{,}000/1{,}050{,}000) = 1{,}981\end{aligned}$$

$$\text{Total } NC_{90} = 32{,}812 - 1{,}981 = 30{,}831$$

Answer is B.

**Problem 2 - 22**

Step I: Calculate the Normal Cost using the retirement age of 65.

$$\text{Benefit} = 20 \times 35 \text{ Years of Service} = 700$$

$$\text{PVFB} = 700 \times 12\ddot{a}_{65}^{(12)} \times v^{10} = 31{,}671$$

$$NC = (\text{PVFB} - \text{Assets}) \div \ddot{a}_{\overline{10}|} = (31{,}671 - 10{,}000) \div \ddot{a}_{\overline{10}|} = 2{,}990$$

Step II: Calculate the Normal Cost using the retirement age of 64.

$$\text{Benefit} = 20 \times 34 \text{ Years of Service} = 680$$

$$PVFB = 680 \times 12\ddot{a}_{64}^{(12)} \times v^9 = 34{,}085$$

$$NC = (PVFB - Assets) \div \ddot{a}_{\overline{9}|} = (34{,}085 - 10{,}000) \div \ddot{a}_{\overline{9}|} = 3{,}570$$

Step III: Calculate the increase in the Normal Cost.

$$Increase = 3{,}570 - 2{,}990 = 580$$

Answer is A.

**Problem 2 - 23**

Step I: Calculate the Present Value of Future Benefits for active participants before the change in benefit form.

$$NC = (PVFB - Assets)/(PVFS/Salary)$$
$$85{,}000 = (Active\ PVFB + 400{,}000 - 600{,}000)/(4{,}200{,}000/350{,}000)$$

$$Active\ PVFB = 1{,}220{,}000$$

Step II: Calculate the new PVFB for active participants.

$$Annuity\ Value\ for\ 50\%\ J\ \&\ S = \ddot{a}_{65}^{(12)} + .5(\ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)})$$

$$= 9.9 + .5(9.9 - 7.82) = 10.94$$

Note that spouses are assumed to be the same age as participants. Since 85% of participants are assumed to be married,

$$Unmarried\ active\ PVFB = (1{,}220{,}000)(.15) = 183{,}000$$

$$and\ Married\ active\ PVFB = (1{,}220{,}000)(.85)(10.94/9.9) = 1{,}145{,}937$$

$$Total\ Active\ PVFB = 183{,}000 + 1{,}145{,}937 = 1{,}328{,}937$$

Step III: Calculate the Normal Cost.

$$NC = (1{,}328{,}937 + 400{,}000 - 600{,}000)/(4{,}200{,}000/350{,}000) = 94{,}078$$

Answer is B.

**Problem 2 - 24**

Key concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' contributions.

| | |
|---|---|
| Present Value of Future Retirement Benefits | \$1,500,000 |
| Plus Present Value of Future Death Benefits | + 30,000 |
| Equals Total Present Value of Future Benefits | 1,530,000 |
| Less Value of Assets | - 400,000 |
| Equals Present Value of Future Normal Costs | 1,130,000 |
| Divided by PV of Future Compensation | ÷8,000,000 |
| Equals Normal Cost Percentage | 14.125% |
| Multiplied by Current Compensation | ×1,000,000 |
| Equals Total Normal Cost at 1/1/90 | 141,250 |
| Less Employee Contributions | 15,000 |
| Equals Employer's Normal Cost at 1/1/90 | \$ 126,250 |

Answer is A.

**Problem 2 - 24** (Alternative Solution):

Present Value of Benefits = Present Value of Future Employer costs plus Present Value of Future Employee Costs plus Assets.

$$PVFB = PVFNC_{ER} + PVFNC_{EE} + \text{Assets}$$

$$\text{where } PVFNC_{ER} = NC\%_{ER} \times PVFS$$

$$\text{and } PVFNC_{EE} = 1.5\% \times PVFS$$

We will use the above relationship to solve for Employer NC rate:

$$1{,}530{,}000 = NC\%_{ER} \times 8{,}000{,}000 + (.015)(8{,}000{,}000) + 400{,}000$$
$$1{,}010{,}000 = NC\%_{ER} \times 8{,}000{,}000$$

$$NC\%_{ER} = .12625$$

$$NC_{ER} = (.12625) \times (1{,}000{,}000) = 126{,}250$$

Answer is A.

**Problem 2 - 25**

Key concept: The Normal Cost rate is the Present Value of Future Normal Costs divided by the Present Value of Future Salaries. We can determine each quantity from the 1/1/90 data.

Step I: Determine Present Value of Benefits at 1/1/91 from the 1/1/90 valuation. If benefits were to remain the same, the Present Value of Benefits at 1/1/91 would be equal to the Present Value of Benefits at 1/1/90 multiplied by $(1 + i)$. Since salary increases exceeded the assumed increase, benefits need to be increased by $(1.08 \div 1.05)$ of benefits projected at 1/1/90. Since the benefit formula increased benefits, benefits need to be increased by $(50\% \div 40\%)$ of benefits projected at 1/1/90.

$$\begin{aligned} PVFB_{91} &= [PVFB_{90} \times (1.08/1.05) \times (.5/.4) \times 1.07] \\ &= 800{,}000 \times (1.08/1.05) \times (.5/.4) \times 1.07 = 1{,}100{,}571 \end{aligned}$$

Step II: Determine Present Value of Future Salaries at 1/1/91.

$$\begin{aligned} PVFS_{91} &= [PVFS_{90} - Salaries_{90}] \times (1.07) \times (1.08/1.05) \\ &= (11{,}250{,}000 - 900{,}000) \times (1.07) \times (1.08/1.05) = 11{,}390{,}914 \end{aligned}$$

Step III: Determine Salaries at 1/1/91.

$$\begin{aligned} Salaries_{91} &= Salaries_{90} \text{ x } 1.08 \\ &= 900{,}000 \times 1.08 = 972{,}000 \end{aligned}$$

Step IV: Determine Normal Cost rate at 1/1/91.

$$\begin{aligned} NC_{91} \text{ percentage rate} &= NC_{91}/Salaries_{91} \\ &= 60{,}000/972{,}000 = 6.1728\% \end{aligned}$$

Step V: Determine Assets at 1/1/91.

$$\begin{aligned} NC_{91} \text{ percentage rate} &= (PVFB_{91} - Assets_{91})/PVFS_{91} \\ 6.1728\% &= (1{,}100{,}571 - Assets_{91})/11{,}390{,}914 \end{aligned}$$

Therefore, $Assets_{91} = 397{,}433$, which are the value of assets as of 12/31/90.

Answer is E.

**Problem 2 - 26**

Key Concept: The benefits of retired participants are included in the Present Value of Future Benefits, but the Present Value of Future Normal Costs are amortized over the future working lifetimes of only the active participants

Step I: Calculate Present Value of Future Benefits.

$$\text{Smith PVFB} = 1{,}000 \times 12 \times \ddot{a}_{65}^{(12)} = 1{,}000 \times 12 \times 8.74 = 104{,}880$$

$$\text{Brown PVFB} = [30{,}000 \times .5] \times \ddot{a}_{65}^{(12)} \times v^{19} = 15{,}000 \times 8.74 \times .27651 = 36{,}250$$

$$\text{Total PVFB} = 104{,}880 + 36{,}250 = 141{,}130$$

Step II: Calculate the Normal Cost.

Note that although Brown is age 46 as of the 12/31/90 valuation date, there will be 20 future normal costs since the last cost for Brown will be the day before normal retirement

| | |
|---|---|
| Present Value of Future Benefits | \$141,130 |
| Less Value of Assets | - 94,650 |
| Equals Present Value of Future Normal Costs | 46,480 |
| Divided by Present Value of Future Years ($\ddot{a}_{\overline{20}|}$) | ÷11.3356 |
| Equals Normal Cost as of 12/31/90 | 4,100 |

Answer is D.

**Problem 2 - 27**

Each participant will have a total of 40 years of service at retirement and a benefit as follows:

Benefit = 10 x 40 years of service = 400

Since we are given $N_x$ and we need $N_x^{(12)}$, and since $N_x^{(12)} = N_x - (11/24)D_x$,

$$N_{65}^{(12)} = 8{,}872 - (11/24)(965) = 8{,}430$$

The Present Value of Future Benefits is equal to the sum of the PVFB's for each age group.

$$\text{PVFB} = 15 \text{ participants} \times \left[400 \times 12\ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{45}}\right]$$

$$+ 10 \text{ participants} \times \left[400 \times 12\ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{55}}\right]$$

$$+ 5 \text{ retirees} \times \left[400 \times 12\ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{65}}\right]$$

$$= 400 \times 12N_{65}^{(12)} \times \left[\frac{15}{D_{45}} + \frac{10}{D_{55}} + \frac{5}{D_{65}}\right] = 528{,}724$$

$$\text{Average Temporary Annuity} = \frac{\left(15 \times \dfrac{N_{45} - N_{65}}{D_{45}} + 10 \times \dfrac{N_{55} - N_{65}}{D_{55}}\right)}{25} = 9.4047$$

NC = (PVFB - Assets) ÷ Average Temporary annuity
= (528,724 - 300,000) ÷ 9.4047 = 24,320

Answer is C.

**Problem 2 - 28**

Key Concept: Present Value of Future Benefits is equal to half the PVFB if the participant retires at 62 plus half the PVFB if the participant retires at 65. Similarly, the PVFS equals half the PVFS if the participant retires at 62 plus half the PVFS if the participant retires at 65.

Final salary at 62 = 100,000 × $(1.07)^6$ = 150,073
Final salary at 65 = 100,000 × $(1.07)^9$ = 183,846

Benefit at 62 = 150,073 × .01 × 7 years of service = 10,505
Benefit at 65 = 183,846 × .01 × 10 years of service = 18,385

$$\text{PVFB} = (.5)(10{,}505)(\ddot{a}_{62}^{(12)})v^6 + (.5)(18{,}385)(\ddot{a}_{65}^{(12)})v^9$$

$$= (.5)(10{,}505)(9.394) \times v^6 + (.5)(18{,}385)(8.736) \times v^9$$

$$= 32{,}879 + 43{,}681 = 76{,}560$$

Note that $\ddot{a}_{\overline{6|}} = 6$, and $\ddot{a}_{\overline{9|}} = 9$,

$$\text{Where } (1+j) = \frac{1+i}{1+s} = \frac{1.07}{1.07} = 1.00$$

$$\text{PVFS} = 100{,}000 \times \ddot{a}_{\overline{6|}} \times q_{62}^{(r)} + 100{,}000 \times \ddot{a}_{\overline{9|}} \times {}_3p_{62}^{(r)} \times q_{65}^{(r)}$$

$$= 100{,}000 \times 6 \times (.5) + 100{,}000 \times 9 \times (.5) \times (1)$$

$$= 300{,}000 + 450{,}000 = 750{,}000$$

$$\text{NC} = \frac{\text{PVFB} - \text{Assets}}{(\text{PVFS/Salary})}$$

$$\frac{76{,}560 - 0}{(750{,}000/100{,}000)} = 10{,}208$$

Answer is D.

**Problem 2 - 29**

Key Concept: The normal form has been amended from a subsidized 100% J&S benefit for married participants to a life annuity. This change applies to active participants only. Since only 85% of the participants are assumed to be married, it is necessary to determine the portion of the PVFB attributable to these participants.

Step I: Determine PVFB for active participants as of 1/1/90, before the amendment.

$$NC_{90} = (PVFB_{90} - Assets_{90})/(PVFS_{90} \div Salary_{90})$$
$$94{,}000 = (PVFB_{90} - 730{,}000)/(4{,}850{,}000/375{,}000)$$

$$PVFB_{90} = 1{,}945{,}733$$

$$\text{Active } PVFB_{90} = PVFB_{90} - \text{Inactive } PVFB_{90}$$
$$= 1{,}945{,}733 - 500{,}000 = 1{,}445{,}733$$

Step II: Determine portion of active $PVFB_{90}$ due to assumed married participants.

$$\text{Value of \$1 per year on life only basis} = \ddot{a}_{65}^{(12)} = 10$$

$$\text{Value of \$1 per year on 100\% J and S basis} = \ddot{a}_{65}^{(12)} + \ddot{a}_{65}^{(12)} - \ddot{a}_{65:65}^{(12)} = 11.8$$

Each participant's PVFB consists of 15% of a life only benefit and 85% of a 100% J&S benefit.

Life benefit = (.15)(10) = 1.5
J&S benefit = (.85)(11.8) = 10.03
Total benefit = 1.5 + 10.03 = 11.53

$$\text{Married Portion of PVFB} = PVFB_{90} \times 10.03/11.53$$
$$= 1{,}445{,}733 \times 10.03/11.53 = 1{,}257{,}650$$

Step III: Determine revised active $PVFB_{90}$.

Convert married portion of PVFB to life only:

$$= 1{,}257{,}650 \times 10/11.8 = 1{,}065{,}805$$

PVFB on life only basis = 1,445,733 - 1,257,650 + 1,065,805 = 1,253,888

Increased active PVFB due to amendment:

$$= 1{,}253{,}888 \times 100\%/50\% = 2{,}507{,}776$$

Step IV: Calculate revised Normal Cost.

$$\text{Total PVFB} = 2{,}507{,}776 + 500{,}000 = 3{,}007{,}776$$

$$NC = \frac{(3{,}007{,}776 - 730{,}000)}{4{,}850{,}000} \times 375{,}000 = 176{,}117$$

Answer is B.

**Problem 2 - 30**

Calculate the retirement benefit with and without salary scale. The salary used is the 1993 salary (the year preceding the 1/1/94 retirement date).

$$\text{Benefit without salary scale} = 40{,}000 \times .5 = 20{,}000$$

$$\text{Benefit with salary scale} = 40{,}000 \times (1.05)^2\,(.5) = 22{,}050$$

Under the general conditions of the exam, benefits are paid monthly. Thus, we need to determine $N_{65}^{(12)}$:

$$N_{65}^{(12)} = N_{65} - \frac{11}{24} \times D_{65}$$

$$= 2{,}508 - \frac{11}{24} \times 279 = 2{,}380$$

The previous Normal Cost was 23,615. We need this information to determine the valuation assets.

$$NC_{\text{Old Assumption}} = \frac{\sum PVFB - Assets}{\ddot{a}_{62:\overline{3}|}}$$

$$23{,}615 = \frac{20{,}000 \times \dfrac{D_{65}}{D_{62}} \ddot{a}_{65}^{(12)} - Assets}{2.75616}$$

$$= \frac{20{,}000 \times \left(\dfrac{2380}{365}\right) - Assets}{2.75616}$$

$$Assets = 65{,}324$$

The temporary annuity for the new salary scale assumption must incorporate the salary scale.

$$^{s}D_{62} = D_{62} \times (1+s)^{62} = D_{62} \times (1.05)^{62} = 7{,}516.74$$

$$^{s}D_{63} = D_{63} \times (1+s)^{63} = D_{63} \times (1.05)^{63} = 7{,}243.87$$

$$^{s}D_{64} = D_{64} \times (1+s)^{64} = D_{64} \times (1.05)^{64} = 6{,}947.63$$

Temporary Annuity = $(^{s}D_{62} + {}^{s}D_{63} + {}^{s}D_{64})/{}^{s}D_{62} = 2.88799$

$$NC(\text{new assumptions}) = \frac{(22{,}050)\left(\dfrac{2380}{365}\right) - 65{,}324}{2.88799} = 27{,}166$$

Answer is B.

**Problem 2 - 31**

Key Concept: If employee contributions were discontinued, then the normal cost is to be calculated using only Present Value of Future Benefits which exclude benefits due to employee contributions and assets which exclude accumulated employee contributions.

Step I: Calculate Normal Cost Rate including employee contributions:

$$\text{Normal Cost Rate} = \frac{\text{PVFB - Assets - PV of Future EE Contributions}}{\text{PVFS}}$$

$$= \frac{10{,}850{,}000 - 4{,}800{,}000 - 2{,}000{,}000}{50{,}600{,}000} = .08004$$

Step II: Calculate Normal Cost Rate excluding employee contributions:

$$PVFB_{ER} = 10{,}850{,}000 - 850{,}000 = 10{,}000{,}000$$

$$\text{Assets} = 4{,}800{,}000 - 1{,}200{,}000 = 3{,}600{,}000$$

New Normal Cost Rate:

$$\frac{PVFB_{ER} - \text{Assets}}{PVFS} = \frac{10{,}000{,}000 - 3{,}600{,}000}{50{,}600{,}000} = .12648$$

Step III: Calculate increase in normal cost.

$$\Delta NC = (.12648 - .08004) \times 3{,}500{,}000 = 162{,}540$$

Answer is E.

**Problem 2 - 32**

Key Concepts:

(1) Normal cost is determined based on <u>expected</u> retirement age and <u>expected</u> benefit at that age.

(2) Since benefit is <u>not</u> salary based, normal cost is determined as dollars per participant, <u>not</u> rate per dollars of covered payroll.

Step I: Original Valuation at 1/1/92

$$\text{Benefit} = 25 \times 20 \text{ years of service} = 500$$

$$NC = \frac{PVFB - \text{Assets}}{\ddot{a}_{\overline{8}|}} = \frac{500 \times 12\ddot{a}_{60}^{(12)} \times v^8 - 5{,}000}{\ddot{a}_{\overline{8}|}}$$

$$= \frac{500 \times 12 \times 9.82 \times .582 - 5{,}000}{6.3893} = 4{,}584$$

Step II: Revised Valuation at 1/1/92

Benefit = 25 × 25 years of service = 625

$$NC = \frac{\text{PVFB} - \text{Assets}}{\ddot{a}_{\overline{13|}}} = \frac{625 \times 12\ddot{a}_{65}^{(12)} \times v^{13} - 5{,}000}{\ddot{a}_{\overline{13|}}}$$

$$= \frac{625 \times 12 \times 8.74 \times .41496 - 5{,}000}{8.94268} = 2{,}483$$

$$\text{Decrease in NC} = 4{,}584 - 2{,}483 = 2{,}101$$

Answer is D.

**Problem 2 - 33**

The assets earned exactly 7% during the year (7,750 × 1.07 = 8,292). If salaries had increased 5%, all assumptions would have been met and

$$\text{Expected } NC_{92} = NC_{91} \times 1.05 = 7{,}750 \times 1.05 = 8{,}137.50$$

Note: The Normal Cost increases 5% if all assumptions are met in order for it to remain a level percentage of salary.

$$\text{Expected } PVFNC_{92} = 8{,}137.50 \times \ddot{a}_{\overline{24|}j} = 158{,}549 \quad (\text{where } j = 1.07/1.05 - 1)$$

$$\begin{aligned}\text{Expected } PVFB_{92} &= \text{Expected } PVFNC_{92} + \text{Accumulation of prior NC} \\ &= 158{,}549 + (7{,}750 \times 1.07) = 166{,}841\end{aligned}$$

Since the salary actually increased 8%,

$$\begin{aligned}PVFB_{92} &= \text{Expected } PVFB_{92} \times 1.08/1.05 \\ &= 166{,}841 \times 1.08/1.05 = 171{,}608\end{aligned}$$

$$\begin{aligned}NC_{92} &= (PVFB_{92} - Assets_{92})/\ddot{a}_{\overline{24|}j} \\ &= (171{,}608 - 8{,}292)/\ddot{a}_{\overline{24|}j} = 8{,}382.16\end{aligned}$$

Increase in NC = 8,382.16 - 8,137.50 = 244.66

Answer is D.

**Problem 2 - 34**

Key Concept: The side fund normal cost is developed using the lump sum value of the normal retirement benefit at retirement age less the cash value of the insurance at retirement age.

Projected benefit = $40 \times 30$ years of service = 1,200

Insurance = $50 \times$ Projected Benefit = $50 \times 1{,}200 = 60{,}000$

Cash Value at 65 = $60 \times 200 = 12{,}000$

Net Lump Sum = $(12 \times 1{,}200 \times \ddot{a}_{65}^{(12)}) - 12{,}000 = 113{,}856$

$$NC = \frac{PVFB - Assets}{\ddot{a}_{\overline{25}|}}$$

$$= \frac{113{,}856 \times v^{25} - 5{,}000}{\ddot{a}_{\overline{25}|}} = \frac{15{,}977.87}{12.4693} = 1{,}281$$

Answer is B.

**Problem 2 - 35**

Key Concept: An additional 7% increase in compensation serves to increase PVFB for active participants and PVFS by 7%.

$PVFB_{Actives}$ = 1,600,000 - 320,000 = 1,280,000

Recalculated $PVFB_{Actives}$ = 1,280,000 × 1.07 = 1,369,600

Recalculated $PVFB_{Total}$ = 1,369,600 + 320,000 = 1,689,600

Since the Normal Cost is based on the factor PVFS ÷ S, there is no need to increase each term by 1.07 due to the cancellation that will occur.

Thus,

$$NC = \frac{PVFB - Assets}{\left(\frac{PVFS}{Salary}\right)} = \frac{1{,}689{,}600 - 400{,}000}{\left(\frac{19{,}200{,}000}{3{,}200{,}000}\right)} = 214{,}933$$

Answer is C.

**Problem 2 - 36**

Key Concept: The employer's Normal Cost is equal to the total Normal Cost needed to fund all benefits less the contributions for the year to be paid by the employees.

$$PVFB = 2{,}000{,}000 + 70{,}000 + 200{,}000 = 2{,}270{,}000$$

$$\begin{aligned} NC &= (PVFB\text{-}Assets)/(PVFS/Salary) \\ &= (2{,}270{,}000 - 500{,}000) \div (7{,}000{,}000/1{,}000{,}000) = 252{,}857 \end{aligned}$$

Note that the amount of the accumulated employee contributions is irrelevant since it is included in the total assets.

$$\text{Employee contribution} = (.015)(1{,}000{,}000) = 15{,}000$$

$$\text{Employer NC} = 252{,}857 - 15{,}000 = 237{,}857$$

Answer is C.

**Problem 2 - 37**

Key Concept: As in Problem 6 - 28, we must determine the value of $\ddot{a}_{65}^{(12)}$ in order to solve this problem.

Step I: Calculate the Final Average Salary at each salary scale rate.

Using 5% salary scale,

$$\text{Final Average Salary} = (100{,}000)\left[(1.05)^4 + (1.05)^3 + (1.05)^2 + (1.05) + 1\right]/5$$
$$= 110{,}513$$

Using 6% salary scale,

$$\text{Final Average Salary} = (100{,}000)\left[(1.06)^4 + (1.06)^3 + (1.06)^2 + (1.06) + 1\right]/5$$
$$= 112{,}742$$

See the discussion in Problem 4 - 53 concerning the calculation of Final Average Salary.

Step II: To determine the single sum value of the retirement benefit at age 65, calculate the annuity purchase rate using the valuation results before the salary scale was increased to 6%.

$$\text{Retirement Benefit} = (.5)(110{,}513) = 55{,}256$$

$$\text{PVFNC} = \text{NC} \times \ddot{a}_{\overline{5}|j} \qquad \text{where } j = 1.07/1.05 - 1 = .0190476$$
$$= 38{,}120 \times 4.8165 = 183{,}605$$

$$\text{PVFB} = \text{PVFNC} + \text{Assets}$$
$$= 183{,}605 + 200{,}000 = 383{,}605$$

$$\text{Also, PVFB} = (\text{Benefit})(\ddot{a}_{65}^{(12)})v^5$$
$$383{,}605 = (55{,}256)(\ddot{a}_{65}^{(12)})v^5$$

$$\text{Therefore, } \ddot{a}_{65}^{(12)} = 9.737$$

Step III: Determine the new normal cost using the 6% salary scale.

$$\text{Retirement Benefit} = (.5)(112{,}742) = 56{,}371$$

$$\text{PVFB} = (\text{Benefit})(\ddot{a}_{65}^{(12)})v^5$$
$$= (56{,}371)(9.737)(.712986) = 391{,}347$$

$$\text{PVFNC} = \text{PVFB - assets}$$
$$= 391{,}347 - 200{,}000 = 191{,}347$$

$$\text{NC} = \text{PVFNC}/\ddot{a}_{\overline{5}|j}, \qquad \text{where } j = 1.07/1.06 - 1 = .009434$$
$$= 191{,}347/4.9074 = 38{,}992$$

Answer is B.

**Problem 2 - 38**

Step I: Calculate the Present Value of Future Benefits for the side fund.

$$PVFB_{65} = (\text{Benefit})(12\ddot{a}_{65}^{(12)})$$
$$= (2{,}000)(12)(8.78) = 210{,}720$$

$$\text{Side Fund } PVFB_{45} = (PVFB_{65} - \text{Cash Value})(D_{65}/D_{45})$$
$$=(210{,}720 - 50{,}000)(94/445) = 33{,}950$$

Step II: Calculate the side fund Normal Cost.

$$\text{Side Fund NC} = (PVFB_{45} - \text{assets})/\left[(N_{45} - N_{65})/D_{45}\right]$$
$$= (33{,}950 - 10{,}000)/\left[(5{,}691 - 868)/445\right] = 2{,}210$$

Step III: Calculate the total Normal Cost.

$$\text{Total NC} = \text{Side Fund NC} + \text{Premium}$$
$$= 2{,}210 + 2{,}000 = 4{,}210$$

Answer is B.

**Problem 2 - 39**

Step I: Calculate the retirement benefit. Each participant will have a total of 44 years of service at retirement.

$$\text{Benefit} = \$10 \times 44 \text{ years of service} = 440$$

Step II: Calculate the Present Value of Future Benefits. Note that the calculation is identical for Smith and Brown, since they each have the same date of birth.

$$\text{Smith/Brown PVFB} = \$440 \times 12\ddot{a}_{65}^{(12)} \times D_{65}/D_{35}$$
$$= \$440 \times 12 \times 8.74 \times 94{,}414/894{,}190 = 4{,}873$$

$$\text{Green PVFB} = \$440 \times 12\ddot{a}_{65}^{(12)} \times D_{65}/D_{45}$$
$$= \$440 \times 12 \times 8.74 \times 94{,}414/445{,}008 = 9{,}791$$

$$\text{Total PVFB} = 4{,}873 + 4{,}873 + 9{,}791 = 19{,}537$$

Step III: Calculate the Average Temporary Annuity. Again, the calculation is identical for Smith and Brown.

$$\text{Smith/Brown TA} = (N_{35} - N_{65})/D_{35}$$
$$= (12{,}364{,}650 - 868{,}052)/894{,}190 = 12.857$$

$$\text{Green TA} = (N_{45} - N_{65})/D_{45}$$
$$= (5{,}690{,}850 - 868{,}052)/445{,}008 = 10.8376$$

$$\text{Average TA} = (12.857 + 12.857 + 10.8376)/3 = 12.1839$$

Step IV: Calculate the Normal Cost.

$$NC = \text{Total PVFB/ATA}$$
$$= 19{,}537/12.1839 = 1{,}604$$

Answer is B.

**Problem 2 - 40**

Key Concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' mandatory contributions. The actual amount of employees' voluntary contributions and the assets associated with the employees' voluntary contributions are irrelevant as they are kept as a separate account.

$$PVFB = 1{,}000{,}000 + 20{,}000 + 80{,}000 = 1{,}100{,}000$$

$$\text{Assets} = 30{,}000 + 150{,}000 = 180{,}000$$

$$PVFNC = PVFB - \text{Assets}$$
$$= 1{,}100{,}000 - 180{,}000 = 920{,}000$$

$$NC\ \% = PVFNC \div PVFS$$
$$= 920{,}000 \div 4{,}800{,}000 = 19.1667\%$$

$$NC = NC\% \times \text{Salary}$$
$$= 19.1667\% \times 600{,}000 = 115{,}000$$

$$\text{Employer NC} = \text{NC} - \text{Mandatory employee contributions}$$
$$= 115{,}000 - 3{,}000 = 112{,}000$$

Answer is C.

**Problem 2 - 40 (Alternative Solution)**

Key Concept: Present Value of Future Benefits equals Present Value of Future Employer Costs plus Present Value of Future Employee Costs plus Assets.

$$PVFB = PVFNC_{ER} + PVFNC_{EE} + \text{Assets}$$

$$\text{Where } PVFNC_{ER} = NC\%_{ER} \times PVFS$$

$$\text{And } PVFNC_{EE} = .5\% \times PVFS$$

We will use the above relationship to solve for $NC\%_{ER}$:

$$1{,}100{,}000 = (NC\%_{ER})(4{,}800{,}000) + (.005)(4{,}800{,}000) + 180{,}000$$
$$896{,}000 = NC\%_{ER} \times 4{,}800{,}000$$

$$NC\%_{ER} = 18.6667\%$$
$$NC_{ER} = (.186667)(600{,}000) = 112{,}000$$

Answer is C.

**Problem 2 - 40 (Alternative Solution)**

The following equation of value can be written:

$$\text{Total } PVFB = PVFNC_{ER} + PVFNC_{EE} + \text{Assets}$$

The total PVFB includes retirement benefits, return of mandatory employee contributions, and employer-provided termination benefits.

The Assets include both employer and mandatory employee contributions with interest.

Also, $$PVFNC_{EE} = (.005)(PVFS)$$
$$= (.005)(4{,}800{,}000) = 24{,}000$$

So, $1{,}100{,}000 = PVFNC_{ER} + 24{,}000 + 180{,}000$

and $PVFNC_{ER} = 896{,}000$

$$\begin{aligned} \text{NC Rate}_{ER} &= PVFNC_{ER}/PVFS \\ &= 896{,}000/4{,}800{,}000 = .186667 \end{aligned}$$

$$\begin{aligned} NC_{ER} &= (\text{NC Rate}_{ER})(\text{Salary}) \\ &= (.186667)(600{,}000) = 112{,}000 \end{aligned}$$

Answer is C.

**Problem 2 - 41**

Key Concept: The Normal Cost under the Aggregate method remains the same each year (since the benefit formula is a flat dollar amount), unless there are gains or losses. Since the benefit formula increased in 1995, the 1995 Normal Cost is equal to the 1994 Normal Cost plus an amount to fund the increased benefit.

Step I: Calculate the increase in the PVFB due to the plan amendment. Note that the benefit increased by a factor of 5/30.

$$\begin{aligned} \text{Expected } PVFB_{95} &= PVFB_{94} \times (1 + i) \\ &= 900{,}000 \times 1.07 = 963{,}000 \end{aligned}$$

$$\Delta PVFB_{95} = 963{,}000 \times 5/30 = 160{,}500$$

Step II: Calculate the Temporary Annuity as of 1/1/95.

$$\begin{aligned} TA_{95} &= (TA_{94} - 1)(1 + i) \\ &= (11 - 1)(1.07) = 10.7 \end{aligned}$$

Step III: Calculate the increase in Normal Cost for 1995.

$$\begin{aligned} \Delta NC_{95} &= \Delta PVFB_{95} / TA_{95} \\ &= 160{,}500/10.7 = 15{,}000 \end{aligned}$$

Step IV: Calculate the Normal Cost for 1994.

$$\begin{aligned} NC_{94} &= (PVFB_{94} - \text{Assets}_{94})/TA_{94} \\ &= (900{,}000 - 500{,}000)/11 = 36{,}364 \end{aligned}$$

Step V: Calculate the total Normal Cost for 1995.

$$\begin{aligned} NC_{95} &= NC_{94} + \Delta NC_{95} \\ &= 36{,}364 + 15{,}000 = 51{,}364 \end{aligned}$$

Answer is D.

**Problem 2 - 41 (Alternative Solution)**

The total Normal Cost for 1995 could have been calculated directly, as follows.

The total PVFB for 1995 is:

$$\begin{aligned} PVFB_{95} &= \text{Expected } PVFB_{95} \times 35/30 \quad \text{(due to amendment)} \\ &= 963{,}000 \times 35/30 = 1{,}123{,}500 \end{aligned}$$

The Assets as of 1/1/95 are:

$$\begin{aligned} Assets_{95} &= (Assets_{94} + Contribution_{94})(1 + i) \\ &= (500{,}000 + 36{,}364)(1.07) = 573{,}909 \end{aligned}$$

And,

$$\begin{aligned} NC_{95} &= (PVFB_{95} - Assets_{95})/TA_{95} \\ &= (1{,}123{,}500 - 573{,}909)/10.7 = 51{,}364 \end{aligned}$$

Answer is D.

**Problem 2 - 42**

Step I: Calculate the 1995 Normal Cost.

| | |
|---|---|
| Present Value of Future Benefits | \$149,000 |
| Less Value of Assets | (20,000) |
| Equals Present Value of Future Normal Costs | 129,000 |
| Divided by Present Value of Future Salary | ÷1,200,000 |
| Equals Normal Cost Rate | 10.75% |
| Multiplied by Annual Salary | × 150,000 |
| Equals Normal Cost | 16,125 |

Step II: Calculate the expected 1995 Normal Cost. Since compensation increased by 10% instead of the expected 3%, the Present Value of Future Benefits increased by a ratio of 1.1/1.03.

$$\begin{aligned} PVFB &= e PVFB \times (1.1/1.03) \\ 149{,}000 &= e PVFB \times (1.1/1.03) \\ e PVFB &= 139{,}518 \end{aligned}$$

$$e\text{Assets} = 20{,}000 - 1{,}600 = 18{,}400$$

$$\begin{aligned} \text{Expected NC} &= (e PVFB - e\text{Assets})/(PVFS/\text{Salary}) \\ &= (139{,}518 - 18{,}400)/(1{,}200{,}000/150{,}000) = 15{,}140 \end{aligned}$$

$$\text{Increase in NC} = 16{,}125 - 15{,}140 = 985$$

Answer is A.

Note: The ratio of PVFS/Salary is unchanged due to the 10% salary increase even though the individual amounts (PVFS and Salary) have changed since both have increased by the ratio 1.1/1.03.

**Problem 2 - 42 (Alternative Solution)**

The solution can be found more quickly by direct calculation of the experience gain/loss.

$$\text{Expected Compensation} = (150{,}000/1.1) \times 1.03 = 140{,}455$$

$$\begin{aligned} e PVFB &= (PVFB) \times (\text{Expected Compensation}/\text{Actual Compensation}) \\ &= (149{,}000) \times (140{,}455/150{,}000) = 139{,}519 \end{aligned}$$

The loss due to the 10% compensation increase can be calculated as:

$$\begin{aligned} \text{Loss} &= \text{Actual } PVFB - e PVFB \\ &= 149{,}000 - 139{,}519 = 9{,}481 \end{aligned}$$

The total experience loss is

$$\begin{aligned} \text{Total Loss} &= \text{Compensation loss} - \text{Investment gain} \\ &= 9{,}481 - 1{,}600 = 7{,}881 \end{aligned}$$

The difference in the Normal Cost is

$$\Delta NC = \text{(Experience loss)/(PVFS/Salary)}$$
$$= (7{,}881)/(1{,}200{,}000/150{,}000) = 985$$

Answer is A.

**Problem 2 - 43**

The increased Normal Cost also increased the Present Value of Future Normal Costs by

$$100 \times \text{(PVFS/Salary)} = 100 \times (9{,}500{,}000/750{,}000) = 1{,}267$$

This amount also represents the asset loss. Since the assets were expected to earn 7% and only earned 4%, the loss amounted to 3% of the 1/1/94 assets.

$$1{,}267 = (.03)(\text{Assets}_{1/1/94})$$
$$\text{Assets}_{1/1/94} = 42{,}233$$

Therefore,

$$\text{Assets}_{1/1/95} = 42{,}233 \times 1.04 = 43{,}922$$

Answer is E.

**Problem 2 - 44**

Step I: Determine the Present Value of Future Benefits as of 1/1/96

Since compensation increased by 6% instead of 4%, the Present Value of Future Benefits increased by a ratio of 1.06/1.04. Since the benefit formula increased from 60% of average compensation to 70% of average compensation, the Present Value of Future Benefits also increased by a ratio of 70%/60%.

$$\begin{aligned} PVFB_{96} &= PVFB_{95} \times (1.06/1.04) \times (70\%/60\%) \times 1.07 \\ &= 1{,}500{,}000 \times (1.06/1.04) \times (70\%/60\%) \times 1.07 \\ &= 1{,}908{,}510 \end{aligned}$$

Step II: Determine the Average Temporary Annuity (ATA) as of 1/1/96

The average temporary annuity as of 1/1/95 can be derived from the valuation results as of 1/1/95 as follows.

$$ATA_{95} = (PVFB_{95} - \text{Assets}_{95})/NC_{95}$$

$$= (1{,}500{,}000 - 300{,}000)/96{,}000 = 12.5$$

The average temporary annuity as of 1/1/96 can be derived from the average temporary annuity as of 1/1/95 as follows.

$$\begin{aligned} ATA_{96} &= (ATA_{95} - 1)/[(1+i)/(1+s)] \\ &= (12.5 - 1)/(1.07/1.04) = 11.8317 \end{aligned}$$

Note that the actual 6% compensation increase is irrelevant because the ATA is actually just a ration of the present value of future salary to current salary. Since each of these increase by the same 6%, the actual salary increase becomes a cancellation in the ratio.

Step III: Determine the value of assets as of 1/1/96.

$$\begin{aligned} NC_{96} &= (PVFB_{96} - Assets_{96})/ATA_{96} \\ &= (1{,}908{,}510 - Assets_{96})/11.8317 \end{aligned}$$

Therefore, $Assets_{96} = 457{,}352$

Answer is D.

**Problem 2 - 45**

Key Concept: The gain due to Smith's death will be the value of the life portion of the benefit. Under the Aggregate method, the gain will be amortized over the 20 year future working lifetime of Brown, the sole active participant.

Step I: Calculate the present value of the decrease in Smith's benefit.

$$\text{Smith Monthly Benefit} = (50)(20 \text{ years of service}) = 1{,}000$$

$$\text{Decrease in Present Value} = (1{,}000)(12)(N_{75}^{(12)}/\ D_{67}) = 33{,}165$$

Step II: Calculate the decrease in the Normal Cost

$$\text{Decrease in Normal Cost} = 33{,}165/\ \ddot{a}_{\overline{20}|} = 2{,}926$$

Answer is E.

**Problem 2 - 46**

Key Concept: Since Smith is the only active participants, the Present Value of Future Benefits will be amortized over Smith's future working lifetime.

Step I: Calculate the final compensation for Smith.

$$\text{Final Compensation} = 48{,}000 \times (1.05)^{29} = 197{,}575$$

Step II: Calculate the Present Value of Future Benefits for each participant.

$$\begin{aligned} PVFB_{Smith} &= (197{,}575)(.01)(40 \text{ years of service})({}_{30|}\ddot{a}_{35}^{(12)}) \\ &= (197{,}575)(.01)(40)(1) \\ &= 79{,}030 \end{aligned}$$

$$\begin{aligned} PVFB_{Brown} &= (200)(12)({}_{15|}\ddot{a}_{50}^{(12)}) \\ &= (200)(12)(4) \\ &= 9{,}600 \end{aligned}$$

$$\begin{aligned} PVFB_{Green} &= (300)(12)(\ddot{a}_{65}^{(12)}) \\ &= (300)(12)(10) \\ &= 36{,}000 \end{aligned}$$

$$\begin{aligned} PVFB_{Total} &= 79{,}030 + 9{,}600 + 36{,}000 \\ &= 124{,}630 \end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned} \text{Normal Cost} &= (\text{PVFB - Assets})/[({}^sN_{35} - {}^sN_{65})/{}^sD_{35}] \\ &= (124{,}630 - 60{,}000)/12.5 \\ &= 5{,}170 \end{aligned}$$

Answer is B.

**Problem 2 - 47**

Key Concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' mandatory contributions. Note that it is assumed that the employee contribution does not change from 1995 to 1996 since the employee base does not change.

Step I: Calculate the employee contribution.

The equation representing successive years Present Value of Future Employee Contributions (PVFEC) is:

$$PVFEC_{96} = (PVFEC_{95} - \text{Employee Contribution})(1 + i)$$
$$24{,}500 = (25{,}000 - \text{Employee Contribution})(1.07)$$

Therefore,

$$\text{Employee Contribution} = 2{,}103$$

Step II: Calculate the average temporary annuity for the 1995 valuation.

The equation representing the 1/1/95 Normal Cost is:

$$NC_{95} = (PVFB_{95} - Assets_{95}) / \ddot{a}_{\overline{n}|}$$
$$7{,}200 + 2{,}103 = (100{,}000 - 10{,}000) / \ddot{a}_{\overline{n}|}$$

Therefore,

$$\ddot{a}_{\overline{n}|} = 9.673$$

Step III: Calculate the average temporary annuity for the 1996 valuation.

$$\ddot{a}_{\overline{n-1}|} = (\ddot{a}_{\overline{n}|} - 1)(1.07) = (9.673 - 1)(1.07) = 9.2815$$

Step IV: Calculate the Normal Cost.

$$PVFB_{96} = (PVFB_{95})(1 + i)$$
$$= (100{,}000)(1.07)$$
$$= 107{,}000$$

$$\text{Total } NC_{96} = (PVFB_{96} - Assets_{96}) / \ddot{a}_{\overline{n-1}|}$$
$$= (107{,}000 - 22{,}000)/9.2815$$
$$= 9{,}158$$

$$\text{Employer } NC_{96} = 9{,}158 - 2{,}103 = 7{,}055$$

Answer is B.

**Problem 2 - 47 (Alternative Solution)**

The Present Value of Future Benefits are funded through the existing assets, the future employer contributions (normal cost), and the future employee contributions.

The average temporary annuity for 1995 can be calculated from the equation representing the 1/1/95 employer Normal Cost:

$$\text{Employer } NC_{95} = (PVFB_{95} - PVFEC_{95} - Assets_{95})/\ddot{a}_{\overline{n}|}$$
$$7{,}200 = (100{,}000 - 25{,}000 - 10{,}000)/\ddot{a}_{\overline{n}|}$$

Therefore,

$$\ddot{a}_{\overline{n}|} = 9.0278$$

Note: This annuity value is slightly different from the annuity value arrived at in the previous solution. This appears to be due to an inconsistency in the PVFEC from 1995-1996.

The average temporary annuity for the 1996 valuation is:

$$\ddot{a}_{\overline{n-1}|} = (\ddot{a}_{\overline{n}|} - 1)(1.07) = (9.0278 - 1)(1.07) = 8.5897$$

Now calculate the employer Normal Cost for 1996:

$$\text{Employer } NC_{96} = (PVFB_{96} - PVFEC_{96} - Assets_{96})/\ddot{a}_{\overline{n-1}|}$$
$$= (107{,}000 - 24{,}500 - 22{,}000)/8.8597$$
$$= 7{,}043$$

Answer is B.

**Problem 2 - 48**

Key Concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' mandatory contributions. The actual amount of the employees' voluntary contributions and the assets associated with the employees' voluntary contributions are irrelevant as they are kept as a separate account.

PVFB = 2,500,000 + 84,000 + 25,000 = 2,609,000

Assets = 725,000 - 35,000 = 690,000

$$\begin{aligned} PVFNC &= PVFB - Assets \\ &= 2{,}609{,}000 - 690{,}000 = 1{,}919{,}000 \end{aligned}$$

$$\begin{aligned} NC\% &= PVFNC/PVFS \\ &= 1{,}919{,}000/8{,}400{,}000 = 22.8452\% \end{aligned}$$

$$\begin{aligned} NC &= NC\% \times Salary \\ &= 22.8452\% \times 1{,}200{,}000 = 274{,}142 \end{aligned}$$

$$\begin{aligned} \text{Employer } NC &= NC - \text{Mandatory employee contributions} \\ &= 274{,}142 - (.015)(1{,}200{,}000) = 256{,}142 \end{aligned}$$

Answer is C.

**Problem 2 - 48 (Alternative Solution)**

Key Concept: Present Value of Future Benefits equals Present Value of Future Employer Costs plus Present Value of Future Employee Costs plus Assets.

$$PVFB = PVFNC_{ER} + PVFNC_{EE} + Assets$$

where $PVFNC_{ER} = NC\%_{ER} \times PVFS$

and $PVFNC_{EE} = NC\%_{EE} \times PVFS = 1.5\% \times PVFS$

We will use the above relationships to solve for $NC\%_{ER}$:

$$\begin{aligned} PVFB &= PVFNC_{ER} + PVFNC_{EE} + Assets \\ 2{,}609{,}000 &= (NC\%_{ER})(8{,}400{,}000) + (.015)(8{,}400{,}000) + 690{,}000 \\ 1{,}793{,}000 &= NC\%_{ER} \times 8{,}400{,}000 \\ NC\%_{ER} &= 21.3452\% \\ NC_{ER} &= (.213452)(1{,}200{,}000) = 256{,}142 \end{aligned}$$

Answer is C.

**Problem 2 - 49**

Key Concept: The normal form of payment has been amended from a life annuity or actuarial equivalent for all participants to a life annuity for single participants and a fully subsidized joint and 100% survivor annuity for married participants. The increase in the Normal Cost will be equal to an amortization of the increase in the Present Value of Future Benefits attributable to the normal form change.

Step I: Determine the PVFB under the plan amendment.

Note that it is assumed that 20% of the participants are single and 80% of the participants are married.

It is given that $PVFB_{LIFE} = 2,000,000$.

Determine $PVFB_{J\&S}$:

$$\begin{aligned} PVFB_{J\&S} &= PVFB_{LIFE} \times (12\ddot{a}^{(12)}_{\overline{65:65}}/12\ddot{a}^{(12)}_{65}) \\ &= (2,000,000) \times (126.9/104.83) \\ &= 2,421,063 \end{aligned}$$

Determine the total PVFB:

$$\begin{aligned} PVFB &= (.2)(PVFB_{LIFE}) + (.8)(PVFB_{J\&S}) \\ &= (.2)(2,000,000) + (.8)(2,421,063) \\ &= 2,336,850 \end{aligned}$$

Step II: Determine the increase in the PVFB.

$$PVFB_{INCREASE} = 2,336,850 - 2,000,000 = 336,850$$

Step III: Determine the Normal Cost increase.

$$\begin{aligned} NC_{INCREASE} &= PVFB_{INCREASE} \,/\, (PVFS/Salary) \\ &= 336,850/(27,000,000/1,600,000) \\ &= 19,961 \end{aligned}$$

Answer is B.

# Chapter 3

# Individual Aggregate Cost Method

## 3.1 Normal Cost

Suppose that a plan is installed with two participants, ages 25 and 60 on the effective date. Each participant has a projected benefit of $1,000 per month which is to be provided by an insurance annuity purchased at retirement age of 65. The plan assets increase at exactly the same rate as the assumed rate of 6% and annuities are purchased at the same rate as our assumed annuity rate ($120 per $1 of monthly income). The actuary did not know any better and used the Aggregate Method. What is the Normal Cost?

The first valuation would be as follows:

| | |
|---|---|
| Present value of benefits | $101,338 |
| Present value of future years | ÷ 20.42 |
| Normal cost per participant | $4,963 |
| Number of participants | × 2 |
| Plan normal cost | $ 9,926 |

Extending the process until the normal retirement date of the older participant, we find that there is only $59,311 in plan assets for which to purchase an annuity which costs $120,000. This could be a real problem!

The above is meant to illustrate that the Aggregate Method is not a suitable method for use in a small plan where there is a wide disparity of ages and salaries, particularly when the first to retire has significantly greater benefits. Its advantage lies in its simplicity since gains and losses are automatically spread over the remaining working years of the participants.

To offset the glaring disadvantage of the use of the Aggregate Cost Method in small plans while retaining its simplicity, the Individual Aggregate Cost Method was developed. Under this method, plan Assets are allocated to individual participants on the basis of the present value of past normal costs. These amounts are subtracted from each participant's present value of benefits, with the difference to be paid by level payments over the remaining years of each. Thus, the Normal Cost as of the beginning of the year for each Participant is:

$$NC(x) = (\text{PVFB - share of assets}) \div \ddot{a}_{\overline{65-n|}}$$

The Normal Cost of the Plan is the sum of the normal costs of the participants.

For example, suppose the Flea Flicker Pension Plan were to be valued by this method, the following would be the result:

| | Homer | Jake |
|---|---|---|
| Current Age | 45 | 52 |
| Value of Benefits at NRD | \$93,600 | \$36,000 |
| Present Value of Benefits | 29,185 | 16,878 |
| Temporary Annuity Factor | 12.16 | 9.38 |
| Normal Cost | 2,400 | 1,799 |

Total Normal Cost (Homer + Jake) = 2,400 + 1,799 = 4,199

On the next valuation date, the Plan Assets would be allocated to Homer and Jake on the basis of the Present Value of the Accumulated Normal Cost deposits, which would be 2,400 × 1.06 for Homer and 1,799 × 1.06 for Jake. If there were plan assets of \$5,000, the following would be the result:

| | Homer | Jake |
|---|---|---|
| Current Age | 46 | 53 |
| Value of Benefits at NRD | \$93,600 | \$36,000 |
| Present Value of Benefits | 30,936 | 17,891 |
| Allocation basis | 2,544 | 1,907 |
| Plan Assets | 2,858 | 2,142 |
| PV of Future Normal Costs | 28,078 | 15,749 |
| Temporary Annuity Factor | 11.83 | 8.89 |
| Normal Cost | 2,373 | 1,772 |

Total Normal Cost (Homer + Jake) = 2,373 + 1,772 = 4,145

In the case of a takeover or if a change in funding method is made, the Present Value of Accumulated Normal Cost Deposits is generally not readily known. In this situation, it is permissible to allocate assets on the basis of the Present Value of Accrued Benefits, or on the Entry Age Normal Accrued Liability, or a different measure of the Accrued Liability in order to get the method started.

In subsequent years, the assets are allocated based on the prior year asset allocation plus prior Normal Cost. This amount can be thought of as the expected asset value for each participant.

If there are inactive participants (for example, vested terminees), they are allocated assets equal to the present value of their future benefits, and the remaining assets are allocated to the active participants in the manner described above.

### 3.2 Characteristics of the Individual Aggregate Cost Method

1. It is simple to administer since actuarial gains or losses need not be calculated and amortized separately. Any deviations from the expected values are automatically spread over the remaining working careers of participants.

2. It is reasonably certain that funds to provide retirement benefits will be present at normal retirement date. Neither will there be any great excess over the amount needed.

3. It is useful in small plans, particularly where there is a wide diversity in ages and salaries.

4. Accrued Liability is not defined in this method but is assumed to be equal to the actuarial value of plan assets.

## 3.3 Problems

### Problem 3 - 1

Refer to the illustration at the start of this section and compute the normal cost for the first two plan years using the Individual Aggregate Method. Actuarial value of plan assets at the second valuation date is $20,000

### Problem 3 - 2

Data: Assumed interest rate: 7%
Assumed retirement age: 65
No mortality nor withdrawals are assumed.

There were 3 participants as of the 1/1/85 valuation date. Data from that valuation is:

| | Jackson | Johnson | Jameson |
|---|---|---|---|
| Age | 35 | 55 | 47 |
| Present Value of Benefits | $ 9,931 | $115,294 | $55,918 |
| Allocation Basis | 3,144 | 51,903 | 9,544 |
| Normal Cost | 511 | 8,435 | 4,309 |

On 12/31/85, Jameson terminated employment with a vested accrued benefit of $125, which has a present value of $4,700. There were no changes in salaries since 1/1/85.

Actuarial Value of Plan Assets on 1/1/86: $64,000

Compute the plan Normal Cost as of 1/1/86.

### Problem 3 - 3

You, as consultant and actuary, have taken over a plan and decide to use the Individual Aggregate Method. The plan has the following specifications:

Retirement Benefit: 50% of compensation.
Normal Retirement Age: 65
Accrued benefit is determined by the fractional method on the basis of service.

Census data for the three participants are as follows:

| | A | B | C |
|---|---|---|---|
| Age | 35 | 47 | 62 |
| Years of Service | 5 | 12 | 20 |
| Compensation | \$18,000 | \$36,000 | \$60,000 |

Actuarial Value of Plan Assets on valuation date: \$280,000
Assumed interest rate: 6%

$\ddot{a}_{65}^{(12)} = 10.5$

Determine the normal cost of the plan.

**Problem 3 - 4**

A plan has two participants with census data as shown below. The retirement benefit is 50% of final year salary.

| | Smith | Brown |
|---|---|---|
| Age | 48 | 52 |
| Monthly Salary | \$2,400 | \$3,000 |
| Allocated Assets | 9,000 | 21,000 |

$\ddot{a}_{65}^{(12)} = 10$ Salary Scale is 4% Retirement Age - 65

There are no death benefits.

| Age x | $D_x$ | $N_x$ | ${}^sD_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 48 | 4,497 | 58,300 | 29,547 | 604,414 |
| 52 | 3,421 | 41,980 | 26,294 | 491,150 |
| 65 | 1,280 | 12,148 | 16,384 | 209,526 |

Using commutation functions as shown above:

(1) Determine the normal cost assuming a level dollar amount of payment, and;
(2) Determine the normal cost assuming a level percentage of salary.

**Problem 3 - 5**

Valuation date: 1/1/86
Retirement benefit: 40% of average compensation of the five years preceding retirement.
Normal retirement date: 65

There were two participants on 1/1/85 and 1/1/86. Both received salary increases as of 1/1/86.

Census data as of 1/1/86 is as follows:

| | Brown | Green |
|---|---|---|
| Current Age | 50 | 55 |
| 1986 compensation | \$16,500 | \$25,000 |
| Allocated Plan Assets | 6,735 | 19,265 |

The 1/1/85 valuation report showed plan normal cost of \$5,398 and actuarial value of plan assets of \$17,500. The contribution of \$5,398 was made on 1/1/85.

If the Normal Cost is computed on the basis of a level percentage of compensation, determine the amount of increase of Normal Cost in 1986 over 1985 that is attributable to actual salary increases over those assumed.

Assumptions: Retirement age: 65 $\quad \ddot{a}_{65}^{(12)} = 9$
Interest: 7% $\quad D_{65}/D_{50} = .362446$
Salary increases: 3% $\quad D_{65}/D_{55} = .508349$

$({}^sN_{50} - {}^sN_{65})/{}^sD_{50} = 11.647 \qquad ({}^sN_{55} - {}^sN_{65}) / {}^sD_{55} = 8.476$

**Problem 3 - 6**

Effective Date: 1/1/85
Retirement Benefit: 50% of final year's salary.
Retirement age: 60

Employee data as of 1/1/85

| | Hurdy | Gurdy |
|---|---|---|
| Current Age | 42 | 50 |
| Normal Cost | \$4,579 | \$22,367 |

Total Normal Cost of \$26,946 was contributed on 1/1/85. Actuarial Value of Plan Assets on 1/1/86 was \$25,000.

There were no salary changes on 1/1/86; however, a new employee became eligible. His normal cost, based on a level payment from 1/1/86 to his retirement date, was \$2,678. Determine the Normal Cost for 1986 as of 1/1/86.

Assumptions:

Interest: 6%
Retirement age: 60
No pre-retirement deaths or terminations.

**Problem 3 - 7**

Actuarial cost method: Individual Aggregate, with assets allocated to each active participant in proportion to the sum, as of the prior valuation date, of normal cost and allocated assets.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Selected valuation results as of 1/1/84:

| | Normal Cost as of 1/1 | Allocated Assets |
|---|---|---|
| Smith | \$1,705 | \$30,000 |
| Brown | 858 | 4,000 |

Participant data as of 1/1/85:

| | Age | Projected Monthly Benefit |
|---|---|---|
| Smith | 55 | $700 |
| Brown | 40 | 600 |
| Green (new entrant) | 55 | 200 |

Actuarial value of assets as of 1/1/85: $40,000

$\ddot{a}_{65}^{(12)} = 10$

In what range is the total normal cost as of 1/1/85?

(A) Less than $4,100
(B) $4,100 but less than $4,150
(C) $4,150 but less than $4,200
(D) $4,200 but less than $4,250
(E) $4,250 or more.

**Problem 3 - 8**

Plan effective date: 1/1/83
Normal retirement benefit: $10 per month for each year of service.
Pre-retirement death benefit: none

Actuarial cost method: Individual aggregate method. Assets are allocated to individuals in proportion to their respective accrued liabilities under the individual level premium method.

Assumed interest rate: 7%
Assumed retirement age: 65
It is assumed there are no deaths or other terminations of employment prior to age 65.

Participant data and valuation results at 1/1/83

| | Smith | Brown | Green |
|---|---|---|---|
| Current Age | 29 | 49 | 59 |
| Normal Cost as of 1/1/83 | $301 | $1,609 | $6,271 |

Green died on 12/1/83 and there are no new entrants on 1/1/84.
Actuarial value of assets as of 1/1/84: $10,000

In what range is the normal cost for 1984 as of 1/1/84?

(A) Less than $1,100
(B) $1,100 but less than $1,300
(C) $1,300 but less than $1,500
(D) $1,500 but less than $1,700
(E) $1,700 or more.

**Problem 3-9**

Actuarial cost method: Individual aggregate. Assets are allocated to each active participant in proportion to the sum, as of the prior valuation date, of normal cost and allocated assets.

Actuarial assumptions: Interest: 6%.
Pre-retirement deaths and terminations: None.
Retirement age: 65.

Valuation results as of 1/1/85:

| | Normal Cost as of 1/1 | Allocated Assets |
|---|---|---|
| Smith | $2,000 | $10,000 |
| Brown | 1,000 | 3,000 |
| Green | 1,000 | 2,000 |
| Total | $ 4,000 | $ 15,000 |

During 1985 Green terminated with no vested benefit.

Participant data as of 1/1/86:

| | Age | Projected Monthly Benefit |
|---|---|---|
| Smith | 50 | $800 |
| Brown | 45 | $600 |

Actuarial value of assets as of 1/1/86: $22,000.

$\ddot{a}_{65}^{(12)} = 10$

In what range is Smith's normal cost for 1986 as of 1/1/86?

(A) Less than $1,800
(B) $1,800 but less than $2,000
(C) $2,000 but less than $2,200
(D) $2,200 but less than $2,400
(E) $2,400 or more

**Problem 3 - 10**

| | |
|---|---|
| Normal retirement benefit: | $600 per month. |
| Actuarial cost method: | Individual aggregate. Assets are allocated to each active participant in proportion to the sum, as of the prior valuation date, of the participant's normal cost and allocated assets. |

Actuarial assumptions:

Interest: 8%
Preretirement deaths and terminations: None.
Retirement age: 65

Selected valuation results as of 1/1/86:

| | Age | Normal Cost as of 1/1 | Allocated Assets |
|---|---|---|---|
| Smith | 40 | $510 | $ 2,705 |
| Brown | 50 | 274 | 16,000 |

Actuarial value of assets as of 1/1/87: $20,000.
There were no deaths, terminations, retirements or new entrants during 1986.

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than \$800
(B) \$800 but less than \$830
(C) \$830 but less than \$860
(D) \$860 but less than \$890
(E) \$890 or more

**Problem 3 - 11**

Plan effective date: 1/1/83
Normal retirement benefit: \$10 per month for each year of service.

Actuarial cost method:

Before 1988: Aggregate.
After 1987: Individual aggregate with assets allocated in proportion to the present value of accrued benefits as of 1/1/88.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Actuarial value of assets as of 1/1/88: \$10,000

Participant data as of 1/1/88:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/38 | 1/1/28 |
| Date of hire | 1/1/83 | 1/1/73 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is Smith's normal cost for 1988 as of 1/1/88?

(A) Less than \$300
(B) \$300 but less than \$500
(C) \$500 but less than \$700
(D) \$700 but less than \$900
(E) \$900 or more

**Problem 3 - 12**

Plan effective date: 1/1/89
Actuarial cost method: Individual aggregate with side fund.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: None.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Pre-retirement death benefit: 100 times the monthly projected retirement benefit; provided by a whole life insurance policy.

Data and valuation results for only participants as of 1/1/90:

| | Smith | Brown |
|---|---|---|
| Age | 60 | 57 |
| Monthly projected retirement benefit | \$500 | \$200 |
| Present value of retirement benefits | 41,900 | 14,072 |
| Allocated assets | 7,500 | 0 |
| Annual premium as of 1/1 | 3,000 | 1,000 |
| Cash value at age 65 per \$1,000 of insurance | 200 | 250 |

In what range is the normal cost plus premiums for 1990 as of 1/1/90?

(A) Less than $11,000
(B) $11,000 but less than $11,600
(C) $11,600 but less than $12,200
(D) $12,200 but less than $12,800
(E) $12,800 or more

**Problem 3 - 13**

Plan effective date: 1/1/91
Normal retirement benefit: 2% of final year's compensation for each year of service.

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for all participants:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/31 | 1/1/61 |
| Date of hire | 1/1/66 | 1/1/91 |
| 1991 compensation | $90,000 | $12,000 |
| Status as of 1/1/91 | Active | Active |

$\ddot{a}_{65}^{(12)} = 9.345$

In what range is the excess (shortfall) of the normal cost for 1991 as of 1/1/91 determined under the level dollar aggregate method over that determined under the level dollar individual aggregate method?

(A) Less than $(50,000)
(B) $(50,000) but less than $(25,000)
(C) $(25,000) but less than $0
(D) $0 but less than $25,000
(E) $25,000 or more

**Problem 3 - 14**

Valuation date: 12/31/92.
Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Individual aggregate.

Actuarial assumptions:

> Interest rate: 7% per year.
> Compensation increases: 3% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/40 |
| 1992 compensation | $50,000 |
| Allocated assets as of 12/31/92 | 10,000 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is Smith's normal cost for 1992 as of 12/31/92?

(A) Less than $12,500
(B) $12,500 but less than $13,000
(C) $13,000 but less than $13,500
(D) $13,500 but less than $14,000
(E) $14,000 or more

**Problem 3 - 15**

Plan effective date: 1/1/94
Normal retirement benefit: $900 per month.
Early retirement eligibility: Age 60.

Early retirement benefit: Normal retirement benefit minus $50 per month for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

| Age x | |
|---|---|
| 63 | 33.33% |
| 64 | 66.67% |
| 65 | 100.00% |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/49 |
| Date of hire | 1/1/94 |

Selected annuity values:

$\ddot{a}_{63}^{(12)} = 8.96$ $\ddot{a}_{64}^{(12)} = 8.74$ $\ddot{a}_{65}^{(12)} = 8.51$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $2,200
(B) $2,200 but less than $2,300
(C) $2,300 but less than $2,400
(D) $2,400 but less than $2,500
(E) $2,500 or more

**Problem 3 - 16**

Lump sum death benefit: $100,000, payable at end of year of death.

Actuarial cost method for all benefits: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Probability of mortality at each age from 50 through 64: .005
Preretirement terminations other than deaths: None.
Retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/45 |
| Projected monthly retirement benefit | \$5,000 |

Value of assets as of 1/1/95: \$71,500.

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than \$11,500
(B) \$11,500 but less than \$11,550
(C) \$11,550 but less than \$11,600
(D) \$11,600 but less than \$11,650
(E) \$11,650 or more

**Problem 3 - 17**

Normal retirement benefit: 60% of final year's compensation.

Actuarial cost method: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 3% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Valuation data for participant Smith (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/40 |
| 1994 compensation | \$90,000 |
| Allocated assets as of 1/1/95 | 18,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost attributable to Smith for 1995 as of 1/1/95?

(A) Less than $35,000
(B) $35,000 but less than $36,500
(C) $36,500 but less than $38,000
(D) $38,000 but less than $39,500
(E) $39,500 or more

**Problem 3 - 18**

Benefit: $20 per month for each year of service.

Actuarial cost method:

Before 1996: Aggregate.
After 1995: Individual aggregate with assets allocated in proportion to the entry age normal accrued liability.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Value of assets as of 1/1/96: $20,000.

Valuation data for all participants (both active as of 1/1/96):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/56 | 1/1/41 |
| Date of hire | 1/1/91 | 1/1/81 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 10$

In what range is the normal cost for 1996 for Brown as of 1/1/96?

(A) Less than $1,500
(B) $1,500 but less than $1,700
(C) $1,700 but less than $1,900
(D) $1,900 but less than $2,100
(E) $2,100 or more

**Problem 3 - 19**

Normal retirement benefit: 50% of final 3-year average compensation.

Actuarial cost method: Individual aggregate (level percentage of compensation).

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for participant Smith (active as of 1/1/97):

| | |
|---|---|
| Date of birth | 1/1/55 |
| 1997 valuation compensation | $20,800 |
| Allocated value of assets as of 1/1/97 | 6,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost attributable to Smith for 1997 as of 1/1/97?

(A) Less than \$2,030
(B) \$2,030 but less than \$2,130
(C) \$2,130 but less than \$2,230
(D) \$2,230 but less than \$2,330
(E) \$2,330 or more

**Problem 3 - 20**

Actuarial cost method: Individual aggregate (level dollar). Assets are allocated first to inactive participants; the remaining assets are allocated to each active participant in proportion to the sum of the prior year's normal cost and allocated assets.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for all participants and selected valuation results:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/62 | 1/1/57 | 1/1/60 |
| Normal cost as of 1/1/96 | \$ 12,000 | \$ 7,500 | \$ 6,800 |
| Allocated value of assets as of 1/1/96 | 35,000 | 41,000 | 27,000 |
| Present value of future benefits as of 1/1/97 | 212,000 | 136,000 | 11,000 |
| Status as of 1/1/97 | Active | Active | Terminated 12/31/96 |

Value of assets as of 1/1/97: \$115,000.

In what range is the normal cost for 1997 as of 1/1/97?

(A) Less than \$16,000
(B) \$16,000 but less than \$17,000
(C) \$17,000 but less than \$18,000
(D) \$18,000 but less than \$19,000
(E) \$19,000 or more

## 3.4 Solutions to Problems

**Problem 3 - 1**

Key Concept: Assets are allocated on second valuation date on basis of first year's normal cost plus interest at assumed rate.

First Valuation:

| | A | B | Total |
|---|---|---|---|
| (1) Age (aa) | 25 | 60 | |
| (2) Pension | \$1,000 | \$1,000 | |
| (3) Values at NRD (2) × 120 | 120,000 | 120,000 | |
| (4) Present Value of Benefits (3) × $v^{(ra - aa)}$ | 11,667 | 89,671 | |
| (5) Normal Cost (4) ÷ $\ddot{a}_{\overline{ra - aa}\rceil}$ | 731 | 20,083 | \$20,814 |

Second Valuation

| | A | B | Total |
|---|---|---|---|
| (1) Age | 26 | 61 | |
| (2) Pension | \$1,000 | \$1,000 | |
| (3) Present Value of Benefits | 12,367 | 95,051 | |
| (4) Allocation Basis (prior NC × 1.06) | 775 | 21,288 | 22,063 |
| (5) Allocated Assets ((4) × 20,000)/22,063 | 703 | 19,297 | 20,000 |
| (6) PV Future Normal Costs (3) - (5) | 11,664 | 75,754 | |
| (7) Normal Cost (6) ÷ $\ddot{a}_{\overline{ra - aa}\rceil}$ | 736 | 20,624 | 21,360 |

The answer is \$20,814 and \$21,360.

**Problem 3 - 2**

Key concepts: (1) If benefits remain unchanged from the preceding valuation date, the present value of benefits will increase by $(1 + i)$.

(2) Before allocating assets under the Individual Aggregate Cost Method, liabilities for inactive participants (retirees, terminees, etc.) must be subtracted.

Assets to be allocated = Total Assets less Jameson's Benefit
= 64,000 - 4,700 = 59,300

As of 1/1/86:

| | Jackson | Johnson |
|---|---|---|
| (1) Present Value of Benefits | \$10,626 | \$123,365 |
| 1.07 × prior year Present Value | | |
| (2) Allocation Basis | 3,911 | 64,562 |
| Prior year allocation basis + NC | | |
| (3) Allocated assets | 3,387 | 55,913 |
| (4) PV of Future Normal Costs (1) - (3) | 7,239 | 67,452 |
| (5) $\ddot{a}_{\overline{ra-aa}\rceil}$ | 13.14 | 6.97 |
| (6) Normal Cost (4) ÷ (5) | 551 | 9,677 |

(7) Total Normal Cost 551 + 9,677 = 10,228

**Problem 3 - 3**

Key Concept: On a takeover, using the Individual Aggregate Method, the value of past Normal Cost deposits is generally not known. In that case, assets are allocated in the year of the takeover on the basis of Present Value of Accrued Benefits. In ensuing years, normal costs and interest are added to the allocated assets in the usual manner.

Determination of Present Value of Accrued Benefits:

| | A | B | C |
|---|---|---|---|
| (1) Current Age (aa) | 35 | 47 | 62 |
| (2) Completed years of service | 5 | 12 | 20 |
| (3) Total service to NRD | 35 | 30 | 23 |
| (4) Projected Annual Pension | \$9,000 | \$18,000 | \$30,000 |
| (5) Accrued Benefit (4) × (2)/(3) | 1,286 | 7,200 | 26,087 |
| (6) Value of Accrued Benefit at NRD 10.5 × (5) | 13,503 | 75,600 | 273,914 |
| (7) Present Value of Accrued Benefits | 2,351 | 26,486 | 229,983 |
| (6) × $v^{(ra - aa)}$ | | | |

Determination of Normal Cost

| | | | |
|---|---|---|---|
| (8) Present Value of Benefits<br>(4) × 10.50 × $v^{(ra - aa)}$ | 16,453 | 66,215 | 264,480 |
| (9) Allocated Assets ((7) × 280,000)/258,820 | 2,543 | 28,654 | 248,803 |
| (10) Present Value future Normal Cost (8) - (9) | 13,910 | 37,561 | 15,677 |
| (11) Normal Cost (10) / $\ddot{a}_{\overline{ra - aa}\rceil}$ | 953 | 3,273 | 5,533 |

Plan Normal Cost = 953 + 3,273 + 5,533 = 9,759

Note: The total Present Value of Accrued Benefits is $258,820.

**Problem 3 - 4**

Key concepts: When using a salary scale, it is important to determine the number of assumed salary increases that will occur from valuation date until retirement date. If salary is based upon compensation used in the year of valuation (as in this problem), there will be (n - 1) salary changes. Also, as there are no death benefits, the Present Value of Future Benefits will be discounted for mortality.

| | | Smith | Brown |
|---|---|---|---|
| (1) Current Age (aa) | | 48 | 52 |
| (2) Monthly Compensation | | $2,400 | $3,000 |
| (3) Projected Compensation | $(1.04)^{ra - aa - 1}$ × (2) | 4,495 | 4,803 |
| (4) Projected Pension | 50% of (3) | 2,248 | 2,402 |
| (5) Values at NRD | 10 × 12 × (4) | 269,760 | 288,240 |
| (6) Present Value of Benefits | (5) × $(D_{65}/D_x)$ | 76,783 | 107,848 |
| (7) Allocated Assets | | 9,000 | 21,000 |
| (8) PV of Future Normal Costs | (6) - (7) | 67,783 | 86,848 |

Calculation assuming a level dollar normal cost:

| | | | |
|---|---|---|---|
| (9) Present Value Factor | $(N_x - N_{65})/D_x$ | 10.263 | 8.720 |
| (10) Normal Cost | (8) ÷ (9) | 6,605 | 9,960 |

(11) Plan Normal Cost = 6,605 + 9,960 = 16,565

Calculation assuming a level percent of salary:

| | | | |
|---|---|---|---|
| (12) Present Value Factor | $({}^sN_x - {}^sN_{65}) / {}^sD_x$ | 13.365 | 10.711 |
| (13) Normal Cost | (8) ÷ (12) | 5,072 | 8,108 |

(14) Plan Normal Cost = 5,072 + 8,108 = 13,180

Note: It should be noticed that the normal cost under the level percentage of compensation method is smaller than that under the level dollar method. However, in later years the former will exceed the latter which is a result which you probably expect.

**Problem 3 - 5**

Key Concepts: The formula to determine the average compensation of the n years preceding retirement is:

Salary $\times\ \ddot{a}_{\overline{n}|} \div n$ (where Salary is the final year's compensation).

Another method is to multiply the salary in the nth year preceding retirement by $\ddot{s}_{\overline{n}|}/n$. In the solution below, we shall use the former method.

In the problem, there are only two factors involved that can change the normal cost. These are the changes in salaries and the change in plan assets. The solution, then, involves the determination of (1) the total change in normal cost and (2) the change attributable to the change of plan assets over what is expected.

The expected value of actuarial plan assets is:

$$\begin{aligned} e\text{Assets} &= [\text{Assets}_{85} + NC_{85}](1+i) \\ &= (17{,}500 + 5{,}398)(1.07) = 24{,}501 \end{aligned}$$

Determination of total change in Normal Cost:

| | | Brown | Green | Total |
|---|---|---|---|---|
| (1) Current Age (aa) | | 50 | 55 | |
| (2) Current Year Salary | | \$16,500 | \$25,000 | |
| (3) Projected Final Year Salary | (2) × $(1.03)^{ra-aa-1}$ | 24,958 | 32,619 | |
| (4) $\ddot{a}_{\overline{5}\|.03}$ | | 4.717 | 4.717 | |
| (5) Average Compensation | (3) × (4) ÷ 5 | 23,546 | 30,773 | |
| (6) Projected Pension | (5) × 40% | 9,418 | 12,309 | |
| (7) Values at Retirement | (6) × 9 | 84,762 | 110,781 | |
| (8) $D_{65}/D_x$ | | .362446 | .508349 | |
| (9) Present Value of Benefits | (7) × (8) | 30,722 | 56,315 | |
| (10) Plan Assets | | 6,735 | 19,265 | 26,000 |
| (11) PV of Future Normal Cost | (9) - (10) | 23,987 | 37,050 | |
| (12) Present Value Factor | $({}^sN_x - {}^sN_{65})/{}^sD_x$ | 11.647 | 8.476 | |
| (13) Normal Cost | (11) ÷ (12) | 2,060 | 4,371 | 6,431 |

The total change in Normal Cost 6,431 - 5,398 = 1,033

Determination of change in Normal Cost due to change in plan assets over the amount expected (use the expected value of plan assets rather then actual):

| | | Brown | Green | Total |
|---|---|---|---|---|
| (14) Plan Assets allocated to (10) | | 6,347 | 18,154 | 24,501 |
| (15) PV of Future Normal Costs | (9) - (14) | 24,375 | 38,161 | |
| (16) Normal Cost | (15) ÷ (12) | 2,093 | 4,502 | 6,595 |

The impact of the increase in plan assets over that expected

= 6,595 - 6,431 = 164

Therefore, if the total change is $1,033 and the change in plan assets causes a reduction of $164 to the normal cost, the change in salaries must increase the normal cost by 1,033 + 164, or $1,197.

**Problem 3 - 6**

Key Concept: If benefits remain the same, the Present Value of Benefits will increase by (1+*i*) in the following year.

| | | Hurdy | Gurdy |
|---|---|---|---|
| (1) Years to Retirement at 1/1/85 | | 18 | 10 |
| (2) Normal Cost on 1/1/85 | | $4,579 | $22,367 |
| (3) Present Value of Benefits (1/1/85) | | 52,554 | 174,500 |
| (4) Present Value of Benefits (1/1/86) | (3) × 1.06 | 55,707 | 184,970 |
| (5) Asset Allocation Basis | (2) × 1.06 | 4,854 | 23,709 |
| (6) Allocated Assets | $25,000 allocated using (5) | 4,249 | 20,751 |
| (7) Pres. Value of Future Normal Cost | (4) - (6) | 51,458 | 164,219 |
| (8) Normal Cost | (7)/$\ddot{a}_{\overline{ra-aa}\rceil}$ | 4,633 | 22,777 |

Since the new participant is not allocated any assets, his normal cost will be 2,678.

Plan Normal Cost = 4,633 + 22,777 + 2,678 = 30,088

**Problem 3 - 7**

The first step is to allocate the 1/1/85 Assets:

| | (1) Assets 1/1/84 | (2) Normal Cost 1/1/84 | (3) Allocation Base | (4) Actual Assets (1/1/85) |
|---|---|---|---|---|
| Smith | 30,000 | 1,705 | 31,705 | 34,686 |
| Brown | 4,000 | 858 | 4,858 | 5,314 |

Note: (3) = (1) + (2)
(4) = (3) × [40,000 ÷ ∑ (3)]

Next, calculate the normal cost for each participant:

| | Smith | Brown | Green |
|---|---|---|---|
| Years to Retirement ra - aa | 10 | 25 | 10 |
| Benefit | 700 | 600 | 200 |
| PV of Benefit (1) × 12 × $\ddot{a}_{ra}^{(12)} \times v^{ra-aa}$ | 46,906 | 16,776 | 13,402 |
| Asset Allocation | 34,686 | 5,314 | 0 |
| PV of Future Normal Cost | 12,220 | 11,462 | 13,402 |
| $\ddot{a}_{\overline{ra-aa}\rceil}$ | 7.8017 | 13.5504 | 7.8017 |
| Normal Cost | 1,566 | 846 | 1,718 |

Total Normal Cost = 1,566 + 846 + 1,718 = 4,130

Answer is B.

**Problem 3 - 8**

Key Concept: The method of allocating assets as stated in the actuarial cost method of the problem is a little different than the usual approach of allocating based upon prior assets plus normal costs. However, the results as of 1/1/84 will be the same under either approach because the plan was effective on 1/1/83 and there were no assets on that date. Also note, in the first year of the plan (1/1/83 valuation results), the results under Individual Aggregate are identical to Individual Level Premium.

To perform the 1/1/84 valuation, we need to determine the present value of future benefits and allocate assets to each participant. These amounts for Green are both zero.

For Smith and Brown:

$$PVFB_{84} = PVFB_{83} \times 1.07$$
$$PVFB_{83} = PVFNC_{83} \text{ (assets = 0)}$$
$$PVFNC_{83} = NC_{83} \times \ddot{a}_{\overline{65-aa}|}$$

| | | Smith | Brown |
|---|---|---|---|
| (1) Years to Retirement on 1/1/83 | | 36 | 16 |
| (2) Normal Cost on 1/1/83 | | 301 | 1,609 |
| (3) $PVFB_{83}$ | (2) × $\ddot{a}_{\overline{65-aa}|}$ | 4,198 | 16,264 |
| (4) $PVFB_{84}$ | (3) × 1.07 | 4,492 | 17,402 |
| (5) Asset Allocation Basis | (2) × 1.07 | 322 | 1,722 |
| (6) Allocated Assets | (Allocate on basis of (5)) | 1,575 | 8,425 |
| (7) PV Future Normal Costs (4) - (6) | | 2,917 | 8,977 |
| (8) Normal Cost 1/1/84 | (7) / $\ddot{a}_{\overline{65-aa}|}$ | 211 | 921 |

Normal Cost = 211 + 921 = 1,132

Answer is B.

Note: The individual normal costs decreased significantly due to the reallocation of Green's assets to the surviving participants.

**Problem 3 - 9**

Considering Smith alone:

Years to retirement: 15
Monthly Benefit: $800
Value at NRD: $800 \times 12 \times \ddot{a}_{65}^{(12)} = 96,000$

Present Value of Benefits: $96,000 \times v^{15} = 40,057$

Basis of Asset allocation: 75% (From prior year valuation, Normal Cost plus allocated assets was $12,000 of the total $16,000 for Smith and Brown)

$$\text{Normal Cost (Smith)} = (9{,}272 - 3{,}299) \div \ddot{a}_{\overline{24}|} = (9{,}272 - 3{,}299) \div 11.3711 = 525$$

$$\text{Normal Cost (Brown)} = (20{,}016 - 16{,}701) \div \ddot{a}_{\overline{14}|} = (20{,}016 - 16{,}701) \div 8.9038 = 372$$

$$\text{Total Normal Cost} = 525 + 372 = 897$$

Answer is E.

**Problem 3 - 11**

Key Concept: This problem calls for determining the present value of Smith's projected benefit, allocating assets based upon the present value of the accrued benefit, subtracting the allocated assets from the present value of future benefits, then computing the normal cost over Smith's remaining years.

| As of 1/1/88: | Smith | Brown |
|---|---|---|
| Attained Age | 50 | 60 |
| Entry Age | 45 | 45 |
| Past years of service | 5 | 15 |
| Future years of service | 15 | 5 |
| Total years of service | 20 | 20 |
| Projected benefit | \$200 | \$200 |
| Accrued Benefit | (200)(5/20) = 50 | (200)(15/20) = 150 |
| PV of Accrued Benefit | $(50)(120)(1.06)^{-15}$ =2,504 | $(150)(120)(1.06)^{-5}$ = 13,451 |

$$\text{Asset Allocation (Smith)} = 2{,}504 \,/\, (2{,}504 + 13{,}451) \times 10{,}000 = 1{,}569$$

$$\begin{aligned}\text{PVFB (Smith)} &= (\text{Benefit})(120)(1.06)^{-15} \\ &= (200)(120)(.4173) = 10{,}015\end{aligned}$$

$$\text{PVFNC} = 10{,}015 - 1{,}569 = 8{,}446$$

$$\begin{aligned}\text{NC (Smith)} &= 8{,}446/\ddot{a}_{\overline{15}|} \\ &= (8{,}446)/10.2950 = 820\end{aligned}$$

Answer is D.

Assets allocated: 75% of assets $= .75 \times 22{,}000 = 16{,}500$
Present Value of Future Normal Costs: PVFB - Assets $= 40{,}057 - 16{,}500 = 23{,}557$
Normal Cost: $23{,}557/\ddot{a}_{\overline{15|}} = 23{,}557 \div 10.295 = 2{,}288$

Answer is D.

**Problem 3 - 10**

Key Concept: The Present Value of Future Benefits is not provided. Therefore, it must be calculated from the 1/1/86 valuation for each participant. Since there were no changes in salary, participant data, etc., the Present value of Future Benefits at 1/1/87 will equal the Present Value of Future Benefits at 1/1/86 multiplied by (1 + i).

Step I: Determine the Present Value of Future Benefits at 1/1/86.

For Smith

$$\text{PVFB} = (\ddot{a}_{\overline{25|}})(510) + 2{,}705$$

$$= (11.5288)(510) + 2{,}705 = 8{,}585$$

For Brown

$$\text{PVFB} = (\ddot{a}_{\overline{15|}})(274) + 16{,}000$$

$$= (9.2442)(274) + 16{,}000 = 18{,}533$$

Step II: Determine Normal Cost at 1/1/87.

For Smith: $\text{PVFB}_{87} = 8{,}585 \times 1.08 = 9{,}272$

For Brown: $\text{PVFB}_{87} = 18{,}533 \times 1.08 = 20{,}016$
Asset Allocation:

| | Asset Allocation Basis | Allocation |
|---|---|---|
| Smith | 510 + 2,705 = 3,215 | 3,299 |
| Brown | 274 + 16,000 = 16,274 | 16,701 |
| Total | 19,489 | 20,000 |

**Problem 3 - 12**

Key Concept: The cash value at retirement of the life insurance policy must be subtracted from the value of the benefit at retirement in order to determine the normal cost for the side fund. Alternatively, the present value of the cash value at retirement can be subtracted from the present value of future benefits.

Step I: Calculate side fund Normal Cost for Smith.

$$\text{Face Amount of Insurance} = 100 \times \text{Monthly retirement benefit}$$
$$= 100 \times 500 = 50{,}000$$

$$\text{Cash Value at 65} = 50 \times 200 \text{ (per thousand of face amount)} = 10{,}000$$

$$\text{Present Value of Cash Value} = 10{,}000 \times v^5 = 7{,}473$$

$$\text{Normal Cost} = (\text{PVFB - CV - Assets}) \div \ddot{a}_{\overline{5|}}$$
$$= (41{,}900 - 7{,}473 - 7{,}500) \div \ddot{a}_{\overline{5|}} = 6{,}031$$

Step II: Calculate side fund Normal Cost for Brown.

$$\text{Face Amount of Insurance} = 100 \times \text{Monthly retirement benefit}$$
$$= 100 \times 200 = 20{,}000$$

$$\text{Cash Value at 65} = 20 \times 250 \text{ (per thousand of face amount)} = 5{,}000$$

$$\text{Present Value of Cash Value} = 5{,}000 \times v^8 = 3{,}137$$

$$\text{Normal Cost} = (\text{PVFB - CV - Assets}) \div \ddot{a}_{\overline{8|}}$$
$$= (14{,}072 - 3{,}137 - 0) \div \ddot{a}_{\overline{8|}} = 1{,}661$$

Step III: Calculate Total Normal Cost.

$$\text{Total Premiums} = 3{,}000 + 1{,}000 = 4{,}000$$
$$\text{Normal Cost} = 6{,}031 + 1{,}661 + 4{,}000 = 11{,}692$$

Answer is C.

**Problem 3 - 13**

Calculate the Present Value of Future Benefits:

$$\text{Smith:} \quad 54{,}000 \times v^5 \times \ddot{a}_{65}^{(12)} = 343{,}441$$

$$\text{Brown:} \quad 8{,}400 \times v^{35} \times \ddot{a}_{65}^{(12)} = 5{,}306$$

Calculate the Normal Cost under the Aggregate Cost Method:

$$NC_{agg} = \frac{\sum \text{PVFB}}{\sum \text{TemporaryAnnuity}} \times \text{number of participants}$$

$$= \frac{343{,}441 + 5{,}306}{4.312 + 12.587} \times 2 = 41{,}274$$

Calculate Normal Cost under the Individual Aggregate Cost Method:

$$NC_{\text{Ind. Agg.}} = \text{PVFB}/\ddot{a}_{\overline{n}|}$$

$$NC_{\text{Smith}} = 343{,}441/4.312 = 79{,}648$$

$$NC_{\text{Brown}} = 5{,}306/12.587 = 422$$

Total Normal Cost = 79,648 + 422 = 80,070

$$\Delta NC = 41{,}274 - 80{,}070 = (38{,}796)$$

Answer is B.

**Problem 3 - 14**

Key Concept: In a last day valuation, there is a normal cost to be made the day before retirement age is attained (unlike a first day valuation in which the final normal cost is contributed one year before retirement). As a result, there will be a total of 13 future normal costs for Smith even though Smith will attain retirement in 12 years.

Step I: Calculate Final Salary.

$$\text{Final Salary} = (50{,}000)(1.03)^{12} = 71{,}288$$

Step II: Calculate the Present Value of Future Benefits.

$$\text{PVFB} = (71{,}288)(.5)(\ddot{a}_{65}^{(12)})(v^{12}) = 138{,}322$$

Step III: Calculate the Normal Cost.

$$\text{NC} = (\text{PVFB - Assets})/\ddot{a}_{\overline{13}|j} \quad (\text{where } j = 1.07/1.03 - 1)$$

$$= (138{,}322 - 10{,}000)/10.448813 = 12{,}281$$

Answer is A.

**Problem 3 - 15**

Step I: Calculate the Present Value of Future Benefits for each retirement age.

For RA = 63, Retirement Benefit = 900 - (50)(2 years of service) = 800
For RA = 64, Retirement Benefit = 900 - 50 = 850
For RA = 65, Retirement Benefit = 900

For RA = 63, $\text{PVFB}_{63} = (\text{Benefit})(12\ddot{a}_{63}^{(12)})(v^{18})$
$= (800)(12)(8.96)(.29586) = 25{,}449$

For RA = 64, $\text{PVFB}_{64} = (\text{Benefit})(12\ddot{a}_{64}^{(12)})(v^{19})$
$= (850)(12)(8.74)(.27651) = 24{,}650$

For RA = 65, $\text{PVFB}_{65} = (\text{Benefit})(12\ddot{a}_{65}^{(12)})(v^{20})$
$= (900)(12)(8.51)(.25842) = 23{,}751$

Step II: Calculate the total Present Value of Future Benefits. The total Present Value of Future Benefits is equal to the sum of the individual Present Value of Future Benefits at each retirement age, each multiplied by the probability of retirement at that age.

$$\begin{aligned}\text{PVFB} &= (\text{PVFB}_{63})(q_{63}^{(r)}) + (\text{PVFB}_{64})(p_{63}^{(r)})(q_{64}^{(r)}) + (\text{PVFB}_{65})(p_{64}^{(r)})(p_{63}^{(r)})(q_{65}^{(r)}) \\ &= (25{,}449)(1/3) + (24{,}650)(2/3)(2/3) + (23{,}751)(2/3)(1/3)(1) \\ &= 24{,}717\end{aligned}$$

Step III: Calculate the Normal Cost. The temporary annuity used to calculate the Normal Cost is equal to the sum of the individual temporary annuities at each retirement age, each multiplied by the probability of retirement at that age.

$$\begin{aligned}\text{Temporary Annuity} &= \ddot{a}_{\overline{18}|}(q_{63}^{(r)}) + \ddot{a}_{\overline{19}|}(p_{63}^{(r)})(q_{64}^{(r)}) + \ddot{a}_{\overline{20}|}(p_{64}^{(r)})(p_{63}^{(r)})(q_{65}^{(r)}) \\ &= \ddot{a}_{\overline{18}|}(1/3) + \ddot{a}_{\overline{19}|}(2/3)(2/3) + \ddot{a}_{\overline{20}|}(2/3)(1/3)(1) \\ &= 3.58774 + 4.91515 + 2.51902 = 11.02191\end{aligned}$$

$$\begin{aligned}\text{NC} &= \text{PVFB}/(\text{Temporary Annuity}) \\ &= 24{,}717/11.02191 = 2{,}243\end{aligned}$$

Answer is B.

**Problem 3-15 (Alternative Solution):**

The temporary annuity factor determined in Step III above could also be calculated as the present value of future annual payments of $1, with each payment multiplied by the probability that the participant is still active.

$$\begin{aligned}\text{Temporary Annuity} &= \ddot{a}_{\overline{18}|} + v^{18}(p_{63}^{(r)}) + v^{19}(p_{63}^{(r)})(p_{64}^{(r)}) + v^{20}(p_{63}^{(r)})(p_{64}^{(r)})(p_{65}^{(r)}) \\ &= \ddot{a}_{\overline{18}|} + v^{18}(.6667) + v^{19}(.6667)(.3333) + v^{20}(.6667)(.3333)(0) \\ &= 10.7632 + .1973 + .0614 = 11.0219\end{aligned}$$

$$\begin{aligned}\text{NC} &= \text{PVFB}/(\text{Temporary Annuity}) \\ &= 24{,}717/11.0219 = 2{,}243\end{aligned}$$

Answer is B.

**Problem 3 - 16**

Key Concept: The Present Value of Future Benefits is equal to the Present Value of the retirement benefits plus the Present Value of the death benefits.

Step I: Calculate the Present Value of the Retirement Benefit:

$$\text{PV of Retirement Benefit} = (5{,}000)(12\ddot{a}_{65}^{(12)})(v^{15})({}_{15}p_{50})$$
$$= (5{,}000)(12)(8.74)(.3624)(.9276) = 176{,}283$$

Note that since $q_x = .005$ at each age from 50 through 64,
$p_x = .995$ at each age from 50 through 64.

Step II: Calculate the Present Value of the death benefit.

$$\text{PV of death benefit} = (100{,}000)\left[(q_{50})(v) + ({}_{1|}q_{50})(v^2) + \ldots + ({}_{14|}q_{50})(v^{15})\right]$$
$$= (100{,}000)\left[(.005)(v) + (.995)(.005)(v^2) + \ldots + (.995)^{14}(.005)(v^{15})\right]$$
$$= (100{,}000)(.005)/(.995)\left[.995v + (.995v)^2 + \ldots + (.995v)^{15}\right]$$
$$= 502.51(a_{\overline{15}|j}) \qquad \text{where } j = 1.07/.995 - 1 = .07538$$
$$= (502.51)(8.8063) = 4{,}425$$

Step III: Calculate the Normal Cost.

$$\text{PVFNC} = \text{PVFB} - \text{Assets}$$

$$\text{PVFB} - \text{Assets} = \text{NC}(1 + vp_{50} + v^2{}_2p_{50} + \ldots + v^{14}{}_{14}p_{50})$$

$$176{,}283 + 4{,}425 - 71{,}500 = \text{NC}(1 + .995v + (.995v)^2 + \ldots + (.995v)^{14})$$

$$(\text{NC})(\ddot{a}_{\overline{15}|j}) = 109{,}208$$
$$\text{NC} = 109{,}208/9.4701 = 11{,}532$$

Answer is B.

**Problem 3 - 16 (additional information)**

The correct numerical solution is also obtained if the death benefit is funded using a one-year term cost.

The Normal Cost associated with the death benefit would be

$$\begin{aligned} NC_{\text{Death Benefit}} &= 100{,}000vq_{50} \\ &= (100{,}000)(.9346)(.005) = 467 \end{aligned}$$

The Normal Cost associated with the retirement benefit would be

$$\begin{aligned} NC_{RB} &= (PVFB_{RB} - \text{Assets})/\ddot{a}_{\overline{15}|j} \\ &= (176{,}283 - 71{,}500)/9.4701 = 11{,}065 \end{aligned}$$

The total Normal Cost would be

$$NC = 11{,}065 + 467 = 11{,}532$$

Note that while coincidentally this produces the same result as the previous solution, the methodology is incorrect since the problem states that Individual Aggregate is the cost method for <u>all</u> benefits.

**Problem 3 - 17**

Step I: Calculate the final Salary.

$$\text{Salary} = 90{,}000 \times (1.03)^{10} = 120{,}952$$

Step II: Calculate the Present Value of Future Benefits.

$$\begin{aligned} PVFB &= (120{,}952)(.6)(\ddot{a}_{65}^{(12)})(v^{10}) \\ &= (120{,}952)(.6)(8.74)(.5083) = 322{,}401 \end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned} NC &= (PVFB - \text{Assets})/\ \ddot{a}_{\overline{10}|j} \qquad \text{where } j = 1.07/1.03 - 1 = .038835 \\ &= (322{,}401 - 18{,}000)/8.475 = 35{,}918 \end{aligned}$$

Answer is B.

**Problem 3 - 18**

Key Concept: Since the Individual Aggregate funding method is to be used for the first time in 1996, a reasonable method of allocating assets must be used. The description of the funding method indicates that the method to be used in this problem is to allocate the assets in proportion to Entry Age Normal Accrued Liability.

Step I: Calculate the Present Value of Future Benefits.

$$\text{Projected Retirement Benefit} = (20)(25 \text{ years of service}) = 500.00$$

$$\text{Value at Retirement} = (500.00)(12)(\ddot{a}_{65}^{(12)}) = 60{,}000$$

$$\text{PVFB} = (60{,}000)(v^{10}) = 30{,}501$$

Step II: Calculate the Entry Age Normal Accrued Liability for each participant.

$$\begin{aligned}\text{Smith Accrued Liability} &= (20)(30 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{5|}} / \ddot{s}_{\overline{30|}}) \\ &= 4{,}383\end{aligned}$$

$$\begin{aligned}\text{Brown Accrued Liability} &= (20)(25 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{15|}} / \ddot{s}_{\overline{25|}}) \\ &= 23{,}838\end{aligned}$$

$$\text{Total Accrued Liability} = 4{,}383 + 23{,}838 = 28{,}221$$

Step III: Calculate the allocation of assets for Brown.

$$\begin{aligned}\text{Asset Allocation}_{\text{Brown}} &= (\text{Total Assets})(\text{EANAL}_{\text{Brown}} / \text{EANAL}_{\text{Total}}) \\ &= (20{,}000)(23{,}838/28{,}221) = 16{,}894\end{aligned}$$

Step IV: Calculate the Normal Cost for Brown.

$$\begin{aligned}\text{Normal Cost}_{\text{Brown}} &= (\text{PVFB - Assets}) / \ddot{a}_{\overline{10|}} \\ &= (30{,}501 - 16{,}894) / 7.515232 = 1{,}811\end{aligned}$$

Answer is C.

**Problem 3 - 19**

Step I: Calculate the Final Average Salary.

$$\begin{aligned}\text{Final Average Salary} &= (20{,}800)[(1.04)^{22} + (1.04)^{21} + (1.04)^{20}]/3 \\ &= 47{,}423\end{aligned}$$

Note that since retirement is assumed to occur on 1/1/2020 (age 65), the final salary used will be in the 1/1/2019 valuation, 22 years from now.

Step II: Calculate the Present Value of Future Benefits.

$$\begin{aligned} PVFB &= (47{,}423)(.5)(\ddot{a}_{65}^{(12)})(v^{23}) \\ &= (47{,}423)(.5)(8.74)(.2109) \\ &= 43{,}707 \end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned} NC &= (PVFB - Assets)/\ddot{a}_{\overline{23}|j} \qquad \text{(where } j = 1.07/1.04 - 1 = .028846) \\ &= (43{,}707 - 6{,}000)/17.1227 \\ &= 2{,}202 \end{aligned}$$

Answer is C.

**Problem 3 - 20**

Step I: Allocate Assets.

The funding method provides that assets are first allocated to inactive participants. Green has terminated as of the 1/1/97 valuation date, with a Present Value of Future Benefits equal to $11,000. The assets to be allocated to the active participants are:

$$Assets_{ACTIVE} = 115{,}000 - 11{,}000 = 104{,}000.$$

The asset allocation is as follows:

| | Asset Allocation Basis | Allocation |
|---|---|---|
| Smith | 12,000 + 35,000 = 47,000 | 51,183 |
| Brown | 7,500 + 41,000 = 48,500 | 52,817 |
| Total | 95,500 | 104,000 |

Step II : Determine Normal Cost.

$$\begin{aligned} \text{Normal Cost (Smith)} &= (212{,}000 - 51{,}183)/\ddot{a}_{\overline{30}|} \\ &= (212{,}000 - 51{,}183)/13.2777 \\ &= 12{,}112 \end{aligned}$$

$$\begin{aligned} \text{Normal Cost (Brown)} &= (136{,}000 - 52{,}817)/\ddot{a}_{\overline{25}|} \\ &= (136{,}000 - 52{,}817)/12.4693 \\ &= 6{,}671 \end{aligned}$$

$$\text{Total Normal Cost} = 12{,}112 + 6{,}671$$
$$= 18{,}783$$

Answer is D.

# Chapter 4

# Entry Age Normal Cost Method

### 4.1 Assuming Pre-retirement Interest Rates Only

Let us review the Unit Credit Method of funding. You will recall that we began by defining the Accrued Liability as the Present Value of Accrued Benefits. From this definition, it was a natural development to define the Normal Cost as that amount needed to fund that year's increase in Accrued Liability. For example, using an interest assumption of 6% and $12\ddot{a}_{65}^{(12)} = 120$, we used the definition to calculate the Accrued Liability of Homer as:

$$AL(\text{Homer}) = 19 \text{ years} \times 20 \text{ per month} \times 120 \times v^{20} = 14{,}218$$

and the Normal Cost as:

$$NC(\text{Homer}) = 20 \times 120 \times v^{20} = 748$$

One of the important characteristics of the Unit Credit Method is that the costs tend to increase more rapidly than salary. In fact, even if benefits remain the same, the Normal Cost will still increase from year to year. If a plan has a high rate of turnover and terminating employees are replaced with younger employees, the annual contribution rate may tend to level off. However, as the employee group begins to age, contributions will again begin their ascent. For this reason, the Unit Credit Method is unsuitable for many plans.

It is also for this reason that the Entry Age Normal Method came into being. Under this method, we define the Normal Cost as a level dollar amount (or percentage) of cost beginning at Entry Age and continuing until Normal Retirement Age. Hence by the very definition, if benefits neither increase or decrease, the Normal Cost will neither increase or decrease. (Entry Age is defined as the earliest age at which an employee would have become a participant had the plan always been in existence. If a plan has no age or service eligibility requirement, Entry Age would be the Age at Hire.) Let us begin by stating the definition:

Definition: The Entry Age Normal Cost is the amount of level contribution such that the Present Value of Future Normal Costs at Entry Age (ea) is exactly equal to the Present Value of Future Benefits at Entry Age. Thus, for a particular employee, the Entry Age Normal Cost would be:

$$NC = [B(ra) \times 12\ddot{a}_{ra}^{(12)} \times v^{ra-ea}] \div \ddot{a}_{\overline{ra-ea}|}$$

For example, under this definition, the Present Value of Future Benefits at Entry Age for Homer is:

$$= \text{Monthly Benefit} \times 12\ddot{a}_{65}^{(12)} \times v^{39}$$
$$= 780 \times 120 \times .10306 = 9{,}646$$

And Homer's Normal Cost would be:

$$NC(\text{Homer}) = \text{PV of Benefits} \div \ddot{a}_{\overline{39}|}$$
$$= 9{,}646 \div 15.846 = 609$$

And, of course, the Normal Cost for the Plan would be:

$$NC = \sum (B(ra) \times 12\ddot{a}_{ra}^{(12)} \times v^{ra-ea}) \div \ddot{a}_{\overline{ra-ea}|}$$

Later on, when we introduce a salary scale, pre-retirement mortality and other decrements into our assumptions, the above definition will need be to altered somewhat. But, for the present, let's go with what we have.

From the definition of Normal Cost, it is a natural development to then define Accrued Liability as the Present Value of prior Normal Cost deposits. For example, Homer's Accrued Liability at his attained age of 45 is:

$$AL(\text{Homer}) = NC \times \ddot{a}_{\overline{45-ea}|} \times (1 + i)^{45-ea}$$

$$\text{or,} \quad AL(\text{Homer}) = NC \times \ddot{s}_{\overline{45-ea}|}$$

$$= 609 \times 35.786 = 21{,}794$$

This is the equivalent of subtracting the Present Value of Future Normal Costs from the Present Value of Future Benefits, which we prefer to use for our formal definition.

Definition: Accrued Liability is the Present Value of Future Benefits less the Present Value of Future Normal Costs. Thus, for a particular employee, the Accrued Liability may be expressed as:

$$AL = (B(ra) \times 12\ddot{a}_{ra}^{(12)} \times v^{ra-aa}) - (NC \times \ddot{a}_{\overline{ra-aa|}})$$

Using this definition, we can compute Homer's Accrued Liability at his attained age of 45 to be:

$$\begin{aligned} AL(\text{Homer}) &= (780 \times 120 \times v^{20}) - (609 \times \ddot{a}_{\overline{20|}}) \\ &= (93{,}600 \times .3118) - (609 \times 12.158) \\ &= 29{,}184 - 7{,}404 = 21{,}780 \end{aligned}$$

which compares to the $21,794 which we calculated first, the small difference being due to rounding. Both definitions need to be remembered, but in practice, the second has more utility value.

Notice that so long as the benefits remain the same, the Normal Cost will remain the same throughout an employee's working career. If benefits increase as they normally are expected to do, the normal cost will also increase but only in proportion to the increase in benefits.

Furthermore, the Accrued Liability increases each year by the Normal Cost plus interest. Thus, the relationship between the Accrued Liability at time $(t + 1)$ and time $t$ can be expressed as follows:

$$AL_{t+1} = (AL_t + NC_t)(1 + i)$$

As with the Unit Credit Method, gains or losses occur because results seldom follow the assumptions used. These gains and/or losses are amortized over future years and become a part of the required contributions. To determine these gains or losses, we introduce the term Unfunded Accrued Liability.

Definition: Unfunded Accrued Liability is the amount by which the Accrued Liability for the Plan exceeds the actuarial value of Plan Assets.

$$UAL_t = AL_t - \text{Assets}_t$$

Of course, the value of plan assets could exceed the Accrued Liability in which case there would be a funding excess and the Unfunded Accrued Liability would be a negative number. Nevertheless, we shall use the term Unfunded Accrued Liability to denote either situation of a positive or negative result.

To determine the Actuarial Gain or Loss, we introduce the term of Expected Unfunded Accrued Liability which, believe it or not, is the expected Unfunded at year $(t + 1)$ when we know the results at year $t$. The variables that must be considered to determine the Expected Unfunded are:

UAL at year $t$ plus assumed interest;
Normal Cost for year $t$ plus assumed interest;
Contributions made plus interest assumed earned.

The formula for determining the Expected Unfunded Liability at year $(t + 1)$ is:

$$eUAL_{t+1} = (UAL_t + NC_t)(1 + i) - (C_t + I_c)$$

where $C_t$ is the contribution made;
and $I_c$ is interest earned on the contribution at the expected interest rate.

Finally, the difference between the Expected and Actual Unfunded Accrued Liability at $(t + 1)$ is the Actuarial Gain or Loss.

$$\text{Gain/Loss} = (eUAL_{t+1} - UAL_{t+1})$$

An Actuarial Gain results if the answer is positive and an Actuarial Loss results if the answer is negative.

### 4.2 Assuming Pre-retirement Interest and Decrements

In the above section, we defined the terms Normal Cost, Accrued Liability and other functions using only an actuarial assumption of interest prior to retirement. If we consider mortality and other withdrawals along with interest, the definitions hold but we need to use Commutation Functions rather than only interest. Normal Cost for the Plan may be determined by the following formula:

$$NC = \sum B(ra)(12\ddot{a}_{ra}^{(12)})(D_{ra}/(N_{ea} - N_{ra}))$$

where, for each participant, B(ra) represents the monthly benefit at retirement age, ra represents Retirement Age, ea represents Entry Age, and D and N represent the usual commutation functions. Notice that the Normal Cost depends only upon the ages of entry and retirement and not upon the attained age.

In a similar manner, we can develop the <u>retrospective</u> definition of Accrued Liability to be:

$$AL = \sum NC\,(N_{ea} - N_{aa}) \div D_{aa} \text{ (where aa represents attained age.)}$$

Or, we can show that the <u>prospective</u> definition of Accrued Liability is:

$$AL = \sum B(ra)(12\ddot{a}^{(12)}_{ra})(D_{ra}/D_{aa}) - \sum NC\,(N_{aa} - N_{ra}) / D_{aa}$$

The determination of Gain or Loss follows the same procedure as that described in Section 4.1.

Let us try a sample problem.

Data: Effective Date of Plan: 1/1/83
Valuation Date: 1/1/85
Retirement Benefit: \$500 per month.
Normal retirement age: 65
Census data for sole participant:

| | |
|---|---|
| Date of Birth | 1/1/35 |
| Date of Hire | 1/1/75 |

| | <u>1/1/85</u> | <u>1/1/86</u> |
|---|---|---|
| Actuarial Value of Assets | \$3,000 | \$4,850 |

Contribution of \$1,528 was made on 1/1/85.

$\ddot{a}^{(12)}_{65} = 9$ $\quad i = 6\%$

Selected Commutation Functions

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 40 | 763 | 10,732 |
| 47 | 481 | 6,308 |
| 50 | 393 | 4,957 |
| 51 | 367 | 4,564 |
| 65 | 128 | 1,215 |

Part A: Determine the Normal Cost and Accrued Liability by both Retrospective and Prospective methods as of the valuation date.

$$\begin{aligned}\text{Normal Cost at Entry age} &= [B(ra)](12\ddot{a}_{ra}^{(12)})[D_{ra}/(N_{ea} - N_{ra})]\\ &= (500 \times 12 \times 9) \times [128 / (10{,}732 - 1{,}215)]\\ &= 54{,}000 \times (128 \div 9{,}517) = 726\end{aligned}$$

Accrued Liability by the Retrospective Method:

$$\begin{aligned}AL &= NC_{ea} \times (N_{ea} - N_{aa})/D_{aa}\\ &= 726 \times (10{,}732 - 4{,}957) \div 393\\ &= 726 \times (5{,}775 \div 393) = 10{,}668\end{aligned}$$

Accrued Liability by the Prospective Method:

$$AL = [B(ra)](12\ddot{a}_{ra}^{(12)})(D_{ra} / D_{aa}) - NC[(N_{aa} - N_{ra})/D_{aa}]$$

$$\begin{aligned}&= (500 \times 12 \times 9)(128/393) - (726)(4{,}957 - 1{,}215)/393\\ &= 17{,}588 - 6{,}913 = \underline{10{,}675} \quad \text{(Difference due to rounding.)}\end{aligned}$$

Part B: Determine (1) the Unfunded Accrued Liability as of 1/1/85 and 1/1/86 and (2) the Actuarial Gain or Loss for 1985.

$$\begin{aligned}UAL_{85} &= AL_{85} - Assets_{85}\\ &= 10{,}668 - 3{,}000 = 7{,}668\end{aligned}$$

To determine the Unfunded Accrued Liability as of 1/1/86, we must first compute the Accrued Liability as of 1/1/86.

$$AL_{86} = NC \times (N_{ea} - N_{aa})/D_{aa}$$

$$= (726)(10{,}732 - 4{,}564) \div 367 = 12{,}202$$

$$UAL_{86} = 12{,}202 - 4{,}850 = 7{,}352$$

$$\text{Expected } UAL_{86} = (UAL_{85} + NC_{85})(1 + i) - (\text{Contribution} + I_c)$$

$$\begin{aligned}&= (7{,}668 + 726)(1.06) - (1{,}528 + 92)\\ &= (8{,}394)(1.06) - 1{,}620\\ &= 8{,}898 - 1{,}620 = 7{,}278\end{aligned}$$

$$\text{Gain(Loss)} = e\text{UAL}_{86} - \text{UAL}_{86}$$

$$= 7{,}278 - 7{,}352 = -74$$

Since the Actual Unfunded Liability is greater than the Expected Unfunded Liability, the $74 represents an Actuarial Loss.

### 4.3 Entry Age Normal with Salary Increases

One of the important characteristics of the Entry Age Normal Method is that it tends to generate level normal costs for each employee from date of hire until date of retirement. However, when we anticipate salary increases in our valuations, we change this concept somewhat by seeking a normal cost that is a level percentage of salary rather than a level dollar amount. This percentage of salary is calculated by dividing the present value of benefits at entry age by the present value of future salaries, also computed at entry age, *ea*.

Normal Cost % = (PV Benefits at *ea*) ÷ (PV Future Salaries at *ea*) = % of Salary

This formula would also hold when salary increases are not considered but, generally, its use is cumbersome.

Having the Normal Cost Percentage at Entry Age, it is a simple calculation to obtain the dollar amount of Normal Cost on any valuation date by applying the percentage to the then current salary. Another problem arises on the first valuation date, however, since the employee data provides salary information for Attained Ages and we need to know salaries at Entry Ages. In this case, each Entry Age salary can be calculated by discounting the Attained Age salary back to Entry Age by the assumed salary increase percentage.

Let us walk through an example.

Effective Date: 1/1/86
Valuation Date: 1/1/86

Retirement Benefit: 30% of final year salary.
Normal Retirement Age: 65

Census Data for sole participant:

| | Age | Years of Service | 1985 Salary |
|---|---|---|---|
| Jack Sprat | 48 | 8 | \$20,000 |

Actuarial Assumptions:

Plan Earnings 7%
Salary Increases 4%

$\ddot{a}_{65}^{(12)} = 9.50$

Step I: Determine salary at Entry Age.

$$\text{Salary } (ea) = 20{,}000 \times (1.04)^{-8} = 14{,}614$$

Step II: Determine Projected Retirement Benefit.

$$\text{Salary } (ra) = 20{,}000 \times (1.04)^{17} = 38{,}958$$
$$\text{Benefit at NRD} = 30\% \times 38{,}958 = 11{,}687$$

Step III: Determine Present Value of Future Benefits at Entry Age.

$$\begin{aligned} PVFB_{ea} &= \text{Benefit} \times \ddot{a}_{ra}^{(12)} \times v^{ra-ea} \\ &= 11{,}687 \times 9.50 \times (1.07)^{-25} = 20{,}457 \end{aligned}$$

Step IV: Determine Present Value of Future Salaries at Entry Age.

Key Concept: Salaries are assumed to increase at 4% per year and then discounted at 7% per year. This is equivalent to discounting at a rate equal to an interest only assumption of:

$$j = [(1.07/1.04) - 1] = 2.8846\%$$

$$\begin{aligned} PVFS_{ea} &= 14{,}614 \times \ddot{a}_{\overline{25|}j} \\ &= 14{,}614 \times 18.148 = 265{,}215 \end{aligned}$$

Step V: Determine Normal Cost percentage at Entry Age.

$$NC\ \% = PVFB\ (\text{Step III})\ \text{divided by}\ PVFS\ (\text{Step IV})$$
$$= 20{,}457 \div 265{,}215 = 7.71\%$$

Step VI: Determine Normal Cost at Attained Age.

$$= \text{Salary at attained age} \times \text{Normal Cost}\ \%$$
$$= 20{,}000 \times 7.71\% = 1{,}542$$

Accrued Liability follows the same pattern, defined as the Present Value of Future Benefits less the Present Value of Future Normal Costs, both computed at attained age. To extend our example to computation of Accrued Liability:

Step I: Compute Present Value of Future Benefits at attained age.

$$PVFB_{aa} = B_{ra} \times \ddot{a}_{ra}^{(12)} \times v^{ra-aa}$$
$$= 11{,}687 \times 9.50 \times (1.07)^{-17} = 35{,}148$$

Step II: Compute Present Value of Future Normal Costs.

$$PVFNC_{aa} = \text{Normal Cost (Attained age)} \times \ddot{a}_{\overline{ra-aa}|j} \quad \text{where } j = 2.8846\%$$
$$= 1{,}542 \times 13.6726 = 21{,}083$$

Step III: Compute Accrued Liability.

$$AL = 35{,}148 - 21{,}083 = 14{,}065$$

When using Commutation Functions, the procedures are consistent with those described above under the Entry Age Normal Cost Method:

PV of Future Benefits at attained age = Benefit $\times\ \ddot{a}_{ra}^{(12)} \times (D_{ra} / D_{aa})$
PV of Future Salaries at Entry Age = Salary(ea) $\times\ [({}^{s}N_{ea} - {}^{s}N_{ra}) / {}^{s}D_{ea}]$
Normal $\text{Cost}_{ea}$ % = $PVFB_{ea} \div PVFS_{ea}$

### 4.4 Characteristics of Entry Age Normal Method

1. Contributions consist of Normal Cost plus amortization payment of initial Unfunded Accrued Liability.

2. Normal Costs are fairly level as either dollar amounts or percentages of salaries.

3. It usually generates a higher Accrued Liability than any other method and, hence, has the greatest range of acceptable contributions for the employer.

4. Gains or losses occur each year which must be identified and amortized.

5. Terminations and deaths greater than assumed decrements generate actuarial gains.

6. For salary related plans, actuarial gains or losses result when salaries change at a rate other than the assumed rate.

7. Because amortization of the annual gain or loss is required, the method is not desirable for most small plans where greater simplicity is desired.

## 4.5 Problems

### Problem 4 - 1

Effective Date: 1/1/85
Valuation Date: 1/1/86
Retirement Benefit: 30% of final year's salary.
Normal Retirement Age: 60

Assumptions:

> Interest: 7%
> Pre-retirement Deaths or Terminations: None.
> Salary Increases: None.

Actuarial Cost Method: Entry Age Normal.

Census data for plan participants:

| | Age | Years of Service | Salary |
|---|---|---|---|
| Jack | 50 | 6 | $30,000 |
| Jill | 35 | 2 | 20,000 |

$\ddot{a}_{60}^{(12)} = 12.50$

Determine entry age normal cost and accrued liability under both the retrospective and prospective methods.

### Problem 4 - 2

Assumptions:

> Interest: 6.5%
> Pre-retirement deaths or terminations: None.
> Salary increases: None.

Actuarial Cost Method: Entry Age Normal.

Selected Valuation Results:

| | 1/1/85 | 1/1/86 |
|---|---|---|
| Present Value of Future Benefits | \$800,000 | \$1,125,000 |
| Actuarial Value of Plan Assets | 200,000 | 290,000 |
| Accrued Liability | 450,000 | 530,000 |
| Normal Cost | 50,000 | |
| Contribution paid on 12/31 | 75,000 | |

Determine actuarial gain or loss as of 1/1/86.

**Problem 4 - 3**

Effective date: 1/1/85
Valuation date: 1/1/86
Normal Retirement Benefit: \$25 per month for each year of service.
Normal Retirement Age: 65

Employee data for plan participants:.

| | Date of Birth | Date of Hire | $N_{ea} - N_{ra}$ | $N_{ea} - N_{aa}$ | $N_{ea} - N_{aa-1}$ |
|---|---|---|---|---|---|
| Winken | 1/1/31 | 1/1/71 | 9,517 | 7,492 | 7,195 |
| Blinken | 1/1/46 | 1/1/81 | 14,166 | 4,650 | 3,835 |
| Nod | 1/1/56 | 1/1/81 | 29,413 | 8,834 | 7,286 |

where *ea* = entry age, *aa* = attained age on 1/1/86 and *ra* = normal retirement age.

$i = 6\%$ $\quad \ddot{a}_{65}^{(12)} = 10.50$

The contribution of \$7,200 was made on 12/31/85.
Nod terminated on 12/31/85 with Present Value of Vested Accrued Benefit of \$833.

Selected Commutation Functions

$D_{25} = 2{,}000 \quad D_{40} = 763$
$D_{29} = 1{,}548 \quad D_{54} = 297$
$D_{30} = 1{,}452 \quad D_{55} = 277$
$D_{39} = 814 \quad D_{65} = 128$

Determine Normal Cost under the Entry Age Normal Cost Method as of 1/1/86.

**Problem 4 - 4**

Refer to the data provided in Problem 3. Determine the Actuarial Gain or Loss for 1985.

**Problem 4 - 5**

| | 1/ 1/85 | 1/ 1/ 86 |
|---|---|---|
| Present Value of Future Benefits | $580,000 | $640,000 |
| Accrued Liability | 435,000 | 495,000 |
| Actuarial Value of Plan Assets | 300,000 | 365,000 |
| Normal Cost | 34,000 | |

There were no distributions paid in 1985. Contribution for 1985 of $42,000 was made on 12/31/85.

Plan Assumptions:

Interest: 8%
Mortality: 1971 GAM
Turnover: Actuary's Pension Handbook T - 3

Determine amount of Actuarial Gain or Loss attributable to factors other than investment earnings.

**Problem 4 - 6**

Normal retirement benefit: $10 per month per year of service.
Early retirement benefit: None.
Actuarial cost method: Entry age normal.

Two valuations (X and Y) are performed using identical assumptions, except that valuation X uses a withdrawal assumption and valuation Y does not. There are active participants at ages at which withdrawals are assumed.

Consider the following statements:

I. The present value of the accrued benefits which are vested on the valuation date is the same in valuation X as in valuation Y.

II. The accrued liability in valuation X may be greater than that in valuation Y.

III. The accrued liability in valuation X may be less than in valuation Y.

Which, if any of these statements is (are) true?

(A) III only
(B) I and II only
(C) I and III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D) above.

**Problem 4 - 7**

Valuation Date: 1/1/86
Normal Retirement Benefit: 2% of final year's salary for each year of service.
Funding Method: Entry Age Normal
Normal Retirement Age: 65

Actuarial Assumptions:

Investment Earnings: 6%
Salary Increases: 4%
Terminations: T-2 Scale
$\ddot{a}_{65}^{(12)} = 9.50$

| Census Data: | Date of Birth | Date of Hire | 1985 Salary |
|---|---|---|---|
| Tinker | 1/1/36 | 1/1/76 | $40,000 |
| Bell | 1/1/56 | 1/1/85 | 25,000 |

Selected Commutation Functions:

| x | $D_x$ | $N_x$ | ${}^sD_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 29 | 1,548 | 23,341 | 4,828 | 134,216 |
| 30 | 1,452 | 21,793 | 4,709 | 129,388 |
| 40 | 763 | 10,732 | 3,663 | 87,199 |
| 50 | 393 | 4,957 | 2,793 | 54,615 |
| 65 | 128 | 1,215 | 1,638 | 20,953 |

(A) Determine Normal Cost as of 1/1/86.
(B) Determine Accrued Liability as of 1/1/86

**Problem 4 - 8**

Valuation Date: 1/1/86
Normal Retirement Age: 65
Normal Retirement Benefit: 10% of Final Year's Salary.
Salary Scale: 4%
Interest: 7%

$$\ddot{a}_{65}^{(12)} = 10.50$$

Data for Sole Participant:

Age: 55
Years of Past Service: 8

Determine the Normal Cost Percentage.

**Problem 4 - 9**

Valuation Date: 1/1/86
Normal Retirement Benefit: \$200 per month
Normal Retirement Age: 65

$$12\ddot{a}_{65}^{(12)} = 120 \qquad 12\ddot{a}_{62}^{(12)} = 126$$

Assumptions used:

No provision for withdrawals or salary increases.

Valuation results:

Normal Cost: $2,084

The sole participant has not attained age 62 as of 1/1/86.

After the initial valuation, it was decided to amend the plan to provide for retirement age of 62 with same benefit. Determine the increase in Normal Cost as a result of this amendment.

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 62 | 328 | 3,585 |
| 63 | 302 | 3,257 |
| 64 | 278 | 2,955 |
| 65 | 256 | 2,677 |

**Problem 4 - 10**

Valuation Date: 1/1/86
Assumed Interest Rate: 8%

Selected Valuation Results:

| | 1/1/85 | 1/1/86 |
|---|---|---|
| Accrued Liability | $600,000 | $625,000 |
| Actuarial Asset Value | 370,000 | 415,000 |

The contribution for 1985 equal to Normal Cost and interest plus $40,000 was made on 12/31/85.

Determine the Actuarial Gain or Loss for 1985.

**Problem 4 - 11**

Normal retirement age: 60
Normal retirement benefit: 50% of final year's salary.
Actuarial cost method: Entry age normal method (applied on an individual basis).
Assumed retirement age: 60

It is assumed that there are no terminations prior to age 60 other than by death.

Participant data as of 1/1/84 and selected commutation functions:

| | Hire Age | Att. Age | Monthly Salary |
|---|---|---|---|
| Smith | 35 | 40 | \$3,000 |
| Brown | 25 | 35 | \$1,000 |

| | $s_{59}/s_{ea}$ | $s_{59}/s_{aa}$ | ${}^sD_{ea}$ | ${}^sN_{ea}$-${}^sN_{60}$ | ${}^sN_{aa}$-${}^sN_{60}$ | $D_{60}/D_{ea} \times \ddot{a}_{60}^{(12)}$ |
|---|---|---|---|---|---|---|
| Smith | 2.5 | 2.0 | 125 | 3,000 | 2,500 | 1.0 |
| Brown | 3.5 | 2.5 | 100 | 4,500 | 3,000 | 0.5 |

$s_{59} = s_{60}$

In what range is the normal cost as of 1/1/84?

(A) Less than \$1,500
(B) \$1,500 but less than \$1,750
(C) \$1,750 but less than \$2,000
(D) \$2,000 but less than \$2,250
(E) \$2,250 or more.

**Problem 4 - 12**

Plan effective date: 1/1/84
Normal retirement benefit: 40% of salary in the year preceding retirement.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest: 7%
Salary increases: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole plan participant:

| | |
|---|---|
| Date of birth: | 1/1/34 |
| Date of hire: | 1/1/79 |
| Salary for 1983: | $24,000 |

Contribution for 1984 paid at 12/31/84: $7,000
Unfunded accrued liability as of 1/1/85: $23,000
Selected factors:

$({}^sN_{45} - {}^sN_{65}) \div {}^sD_{45} = 18.320$
$({}^sN_{45} - {}^sN_{50}) \div {}^sD_{45} = 4.907$
$({}^sN_{50} - {}^sN_{65}) \div {}^sD_{50} = 14.057$

$12\ddot{a}_{65}^{(12)} = 115$

In what range is the amount of experience loss during 1984?

(A) Less than $2,000
(B) $2,000 but less than $2,500
(C) $2,500 but less than $3,000
(D) $3,000 but less than $3,500
(E) $3,500 or more

**Problem 4 - 13**

Actuarial cost method: Entry age normal method (Individual basis)
Assumed interest: 7%
Selected valuation results:

| | 1/1/83 | 1/1/84 |
|---|---|---|
| Accrued liability | $304,300 | --- |
| Actuarial value of assets | 235,000 | 275,000 |

Contribution for 1983: Normal cost plus interest to 12/31/83 plus $21,000, paid at 12/31/83.

Participant data as of 1/1/84:

| Attained Age | Number of Participants | Annual Salary | Normal Cost as of 1/1/84 | Present Value of Future Salary | Present Value of Future Benefits |
|---|---|---|---|---|---|
| 50 | 1 | $15,500 | $ 800 | $250,000 | $ 45,000 |
| 55 | 1 | 18,000 | 1,025 | 180,000 | 105,000 |
| 65 | 1 | 35,000 | --- | ---- | 200,000 |
| | | | | | $350,000 |

In what range is the experience gain or loss for 1983?

(A) Loss of $600 or more
(B) Loss of less than $600, or no gain or loss
(C) Gain of less than $600
(D) Gain of $600 but less than $1,200
(E) Gain of $1,200 or more

**Problem 4 - 14**

Normal retirement benefit: $100 per month.
Postponed retirement benefit: $100 per month.
Actuarial cost method: Entry age normal.
Assumed retirement age: 65

It is assumed that there are no pre-retirement terminations other than by death.
All participants have the same entry age and are younger than 65.

Normal cost: $800 per active participant.

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 100 | 1,200 |
| 66 | 95 | 1,103 |

The assumed retirement age is changed to age 66.

In what range is the new normal cost per active participant?

(A) Less than \$700
(B) \$700 but less than \$710
(C) \$710 but less than \$720
(D) \$720 but less than \$730
(E) \$730 or more.

**Problem 4 - 15**

Actuarial cost method: Entry age normal with normal cost determined as a percentage of pay.

Actuarial assumptions:

Interest: 7%
Salary increases: 6%
Pre-retirement deaths: 1971 Group Annuity Mortality Table
Pre-retirement terminations other than death: None
Retirement age: 65

Data for the sole participant as of 1/1/85:

| Hire Age | Attained Age | Projected Monthly Benefit |
|---|---|---|
| 61 | 63 | \$700 |

Selected commutation functions based on the above interest and mortality assumptions:

| Age x | $D_x$ |
|---|---|
| 61 | 385 |
| 62 | 354 |
| 63 | 326 |
| 64 | 299 |
| 65 | 274 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the normal cost for 1985 as of 1/1/85?

(A) Less than \$15,000
(B) \$15,000 but less than \$16,000
(C) \$16,000 but less than \$17,000
(D) \$17,000 but less than \$18,000
(E) \$18,000 or more

**Problem 4 -16**

Plan effective date: 1/1/84
Normal retirement benefit: 60% of salary in the year immediately preceding normal retirement date.

Actuarial cost method: Entry age normal method.

Actuarial assumptions:

Interest: 7%
Pre-retirement mortality and withdrawal: None.
Salary increases: 5%
Retirement age: 65

$12\ddot{a}_{65}^{(12)} = 100$

Data for sole participant:

| | |
|---|---|
| Age at hire: | 55 |
| Age on 1/1/84: | 56 |
| 1/1/84 salary: | \$100,000 |

Selected interest-function values:

| Interest Rate | $\ddot{a}_{\overline{10}\rvert}$ | $v^{10}$ | $\ddot{a}_{\overline{9}\rvert}$ | $v^9$ |
|---|---|---|---|---|
| 1.90% | 9.2013 | .8284 | 8.3572 | .8442 |
| 2.00% | 9.1622 | .8203 | 8.3255 | .8368 |

In what range is the normal cost as of 1/1/84?

(A) Less than $41,000
(B) $41,000 but less than $42,000
(C) $42,000 but less than $43,000
(D) $43,000 but less than $44,000
(E) $44,000 or more

**Problem 4 - 17**

Normal retirement benefit: 50% of final salary.
Actuarial cost method: Entry age normal.
Assumed interest rate: 7.5%
Assumed salary increases: 6.0%

There are no retired participants, and no terminated participants with deferred vested benefits.

Selected valuation results:

| | 1/1/84 | 1/1/85 |
|---|---|---|
| Present value of future benefits | $1,000,000 | $1,115,566 |
| Actuarial value of assets | 300,000 | 400,000 |
| Unfunded accrued liability | 300,000 | 330,696 |
| Normal cost as of 1/1 | 55,000 | --- |
| Contribution paid at 12/31 | 90,000 | --- |

Each participant received a salary increase of 10% during 1984. Experience with regard to actuarial assumptions other than salary increases and interest earnings was exactly equal to that assumed. There were no new entrants and no terminations with benefits during 1984.

In what range is the actuarial loss due to salary increases as of 1/1/85?

(A) Less than $15,000
(B) $15,000 but less than $20,000
(C) $20,000 but less than $25,000
(D) $25,000 but less than $30,000
(E) $30,000 or more.

**Problem 4 - 18**

Vesting schedule: 100% after 10 years.
Amended vesting schedule: 100% after 8 years, effective 1/1/84
Normal retirement benefit: $10 per month per year of service.
Actuarial cost method: Entry age normal method.
Assumed retirement age: 65

It is assumed that all terminations of employment occur at the beginning of the year.

Data for the sole plan participant as of 1/1/84:

| Age | Hire Age | $D_{50}$ | $M_{58}^{(w)}$ | $M_{59}^{(w)}$ | $M_{60}^{(w)}$ |
|---|---|---|---|---|---|
| 50 | 50 | 250 | 45 | 40 | 38 |

$$M_x^{(w)} = \sum_{t=x}^{64} l_t v^t q_t^{(w)}{}_{65-t|}\ddot{a}_t^{(12)} \qquad \ddot{a}_{65}^{(12)} = 10$$

where commutation functions, $l_x$ and $q_x^{(w)}$ are from the active service table, and annuity values are based upon mortality and interest only.

In what range is the increase in the present value of future benefits as of 1/1/84 resulting from the change in the vesting schedule?

(A) Less than $15
(B) $15 but less than $30
(C) $30 but less than $45
(D) $45 but less than $60
(E) $60 or more.

**Problem 4 - 19**

Plan effective date: 1/1/86
Normal retirement benefit: $20 per month for each year of service.
Asset valuation: Market value.

Actuarial assumptions:

Interest: 7%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Participant data as of 1/1/86:

| | Attained Age | Age at Hire |
|---|---|---|
| Smith | 30 | 30 |
| Brown | 50 | 50 |
| Green | 55 | 55 |

Contributions for 1986 and 1987: Normal cost, paid at 12/31 of each year.

Experience:

For 1986: In accordance with assumptions.
For 1987: In accordance with assumptions, except investment return is 14%.

There are no new entrants after 1/1/86.

Consider the following statements regarding the aggregate cost method and the entry age normal cost method (applied on an individual basis).

I. The normal cost for 1986 under the aggregate method is less than the normal cost for 1986 under the entry age normal method.

II. The normal cost for 1988 under the aggregate method is less than the normal cost for 1987 under the aggregate method.

III. The normal cost for 1988 under the entry age normal method is less than the normal cost for 1987 under the entry age normal method.

Which, if any, of these statements is (are) true?

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) The correct answer is not given by (A), (B), (C), or (D) above

**Problem 4 - 20**

Plan effective date: 1/1/86
Normal retirement benefit: \$100 per month.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest: 7%
Pre-retirement deaths: 1971 Group Annuity Mortality Table.
Post-retirement deaths: 1971 Group Annuity Table with ages set back one year.
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant as of 1/1/86:

| | | | |
|---|---|---|---|
| Date of birth: | 1/1/45 | Date of hire: | 1/1/81 |

Selected commutation functions from the 1971 Group Annuity Mortality Table.

| Age x | $D_x$ | $N_x$ | $N_x^{(12)}$ |
|---|---|---|---|
| 35 | 9,207 | 127,908 | 123,688 |
| 36 | 8,595 | 118,701 | 114,761 |
| 40 | 6,522 | 87,606 | 84,617 |
| 41 | 6,085 | 81,084 | 78,295 |
| 64 | 1,080 | 10,121 | 9,626 |
| 65 | 990 | 9,041 | 8,587 |
| 66 | 906 | 8,051 | 7,635 |

In what range is the accrued liability as of 1/1/86?

(A) Less than \$570
(B) \$570 but less than \$580
(C) \$580 but less than \$590
(D) \$590 but less than \$600
(E) \$600 or more

**Problem 4 - 21**

Normal retirement benefit: \$10 per month per year of service.
Actuarial cost method: Entry age normal (applied on an aggregate basis)
Assumed retirement age: 65

It is assumed that there are no terminations prior to age 65 other than by death.

Participant data at 1/1/86 and selected commutation functions.

| Attained Age | Age at Hire | Number of Participants | $D_x$ | $N_x - N_{65}$ |
|---|---|---|---|---|
| 45 | - | 0 | 1,000 | 9,012 |
| 50 | 45 | 1 | 585 | 4,945 |
| 55 | 50 | 1 | 362 | 2,522 |
| 65 | - | 0 | 147 | 0 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the normal cost for 1986 as of 1/1/86?

(A) Less than \$915
(B) \$915 but less than \$920
(C) \$920 but less than \$925
(D) \$925 but less than \$930
(E) \$930 or more

**Problem 4-22**

Normal retirement benefit: \$20 per month for each year of service.
Actuarial cost method: Entry age normal.
Assumed retirement age: 65

It is assumed that there are no deaths or other terminations prior to retirement.

Data for sole participant:

Date of birth: 1/1/31 Date of hire: 1/1/76

Valuation results as of 1/1/86:

| | |
|---|---|
| Actuarial value of assets: | \$8,000 |
| Unfunded accrued liability: | 9,700 |
| Normal cost as of 1/1: | 1,300 |

Present value of \$1.00 per year of future employment: 7.90

In what range is the unfunded accrued liability as of 1/1/86 under the accrued benefit (unit credit) cost method?

(A) Less than \$6,000
(B) \$6,000 but less than \$7,000
(C) \$7,000 but less than \$8,000
(D) \$8,000 but less than \$9,000
(E) \$9,000 or more

**Problem 4 - 23**

Plan effective date: 1/1/86.
Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Entry age normal (applied on an individual basis).

Actuarial assumptions:

Interest: 7%
Preretirement deaths and terminations: None.
Retirement age: 65

Participant data as of 1/1/86:

| | Date of Birth | Date of Hire |
|---|---|---|
| Smith | 1/1/51 | 1/1/81 |
| Brown | 1/1/52 | 1/1/77 |

1986 normal cost for Smith as of 1/1/86: \$320

In what range is the 1986 normal cost for Brown as of 1/1/86?

(A) Less than \$225
(B) \$225 but less than \$275
(C) \$275 but less than \$325
(D) \$325 but less than \$375
(E) \$375 or more

**Problem 4 - 24**

Plan effective date: 1/1/86
Normal retirement benefit: 50% of final year's salary.
Actuarial cost method: Entry age normal.
Assumed salary increases: 6% per year.
Assumed retirement age: 65

It is assumed that there are no terminations prior to retirement, other than by death.

Data for sole participant as of 1/1/86:

| | |
|---|---|
| Date of birth: | 1/1/46 |
| Date of hire: | 1/1/81 |
| Annual salary for 1985: | \$30,000 |

Selected commutation functions:

| Age x | $D_x$ | ${}^sD_x$ | ${}^sN_x$ |
|---|---|---|---|
| 35 | 921 | 7,077 | 239,735 |
| 40 | 652 | 6,708 | 205,091 |
| 65 | 99 | 4,371 | 62,569 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the normal cost for 1986 as of 1/1/86?

(A) Less than \$3,000
(B) \$3,000 but less than \$3,500
(C) \$3,500 but less than \$4,000
(D) \$4,000 but less than \$4,500
(E) \$4,500 or more

**Problem 4 - 25**

Normal retirement benefit: \$400 per month.
Normal form of retirement benefit:

| | |
|---|---|
| Retirements before 1986: | Life annuity. |
| Retirements after 1985: | Life annuity with 120 monthly payments certain. |

Actuarial cost method: Entry age normal.

It is assumed that there are no deaths or other terminations before age 65.

Valuation results as of 1/1/86 based on life annuity normal form:

| | |
|---|---|
| Present value of future benefits - Active participants | \$1,500,000 |
| Present value of future benefits - Retired participants | 500,000 |
| Actuarial value of assets | 600,000 |
| Unfunded accrued liability | 800,000 |
| Normal cost as of 1/1 | 75,000 |

Selected commutation functions and values:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 990 | 9,000 |
| 75 | 350 | 2,400 |

$\ddot{a}_{\overline{10|}}^{(12)} = 7.287$

In what range is the increase in the unfunded accrued liability as of 1/1/86 due to the change in the normal form?

(A) Less than \$45,000
(B) \$45,000 but less than \$60,000
(C) \$60,000 but less than \$75,000
(D) \$75,000 but less than \$90,000
(E) \$90,000 or more

**Problem 4 - 26**

Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest: 8%
Retirement age: 65
Preretirement terminations other than deaths: None

Participant data as of 1/1/87:

| Name | Date of Birth | Date of Hire |
|---|---|---|
| Smith | 1/1/57 | 1/1/87 |
| Brown | 1/1/47 | 1/1/77 |
| Green | 1/1/37 | 1/1/67 |

| Age x | $N_x$ | $D_x$ |
|---|---|---|
| 30 | 12,570 | 980 |
| 40 | 5,485 | 450 |
| 50 | 2,255 | 200 |
| 65 | 465 | 55 |

$\ddot{a}_{65}^{(12)} = 8$

In what range is the accrued liability as of 1/1/87?

(A) Less than $10,000
(B) $10,000 but less than $20,000
(C) $20,000 but less than $30,000
(D) $30,000 but less than $40,000
(E) $40,000 or more

**Problem 4 - 27**

| | |
|---|---|
| Normal retirement benefit: | $10 per month per year of service. |
| Early retirement benefit: | Accrued benefit reduced by 6% for each year by which the early retirement age precedes 65. |

Actuarial cost method: Entry age normal.
Assumed retirement age: 60

It is assumed that there are no terminations prior to retirement other than by death.

Data for sole participant:

Date of birth: 1/1/37 Date of hire: 1/1/77

| Age x | $D_x$ | $N_x$ | $\ddot{a}_x^{(12)}$ |
|---|---|---|---|
| 40 | 652 | 8,761 | 12.98 |
| 50 | 322 | 3,902 | 11.66 |
| 60 | 151 | 1,547 | 9.79 |
| 65 | 99 | 904 | 8.67 |

In what range is the accrued liability as of 1/1/87?

(A) Less than $5,000
(B) $5,000 but less than $7,000
(C) $7,000 but less than $9,000
(D) $9,000 but less than $11,000
(E) $11,000 or more

**Problem 4 - 28**

Normal retirement benefit: 50% of final 3-year average compensation.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest: 6%
Compensation increases: 5% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant as of 1/1/88:

| | |
|---|---|
| Date of birth: | 1/1/38 |
| Date of hire: | 1/1/73 |
| Compensation for 1988: | $50,000 |

$\ddot{a}_{65}^{(12)} = 9$

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than $5,750
(B) $5,750 but less than $6,000
(C) $6,000 but less than $6,250
(D) $6,250 but less than $6,500
(E) $6,500 or more

**Problem 4 - 29**

Actuarial cost method: Entry age normal.
Assumed interest rate: 6%

Data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/18 |
| Date of retirement | 1/1/83 |
| Date of death | 12/31/87 |
| Retirement benefit: | $1,000 per month, payable as a life annuity with ten years certain. |

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 1,886 | 17,624 |
| 69 | 1,344 | 11,179 |
| 70 | 1,227 | 9,889 |
| 75 | 743 | 5,006 |

Selected annuity values:

$\ddot{a}_{\overline{5}|}^{(12)} = 4.348$ $\ddot{a}_{\overline{6}|}^{(12)} = 5.076$ $\ddot{a}_{\overline{10}|}^{(12)} = 7.597$

In what range is the experience gain recognized as of 1/1/88 due to Smith's death?

(A) Less than $39,000
(B) $39,000 but less than $42,000
(C) $42,000 but less than $45,000
(D) $45,000 but less than $48,000
(E) $48,000 or more

**Problem 4 - 30**

Plan effective date: 1/1/88
Normal retirement benefit: $10 per month for each year of service.
Actuarial cost method: Entry age normal (aggregate basis).

Actuarial assumptions:

Pre-retirement terminations other than deaths: None.
Retirement age: 65

Participant data as of 1/1/88:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/53 | 1/1/43 |
| Date of hire | 1/1/78 | 1/1/78 |

Selected commutation functions and annuity values:

| Age x | $D_x$ | $\ddot{a}_{x:\overline{65-x}\rceil}$ |
|---|---|---|
| 25 | 2,441 | 15.62 |
| 35 | 1,348 | 14.22 |
| 45 | 737 | 11.84 |
| 55 | 389 | 7.92 |
| 65 | 189 | 0.00 |

$\ddot{a}_{65}^{(12)} = 9.35$

In what range is the accrued liability as of 1/1/88?

(A) Less than \$6,600
(B) \$6,600 but less than \$7,200
(C) \$7,200 but less than \$7,800
(D) \$7,800 but less than \$8,400
(E) \$8,400 or more

**Problem 4 - 31**

Normal retirement benefit: 50% of final 3-year average compensation.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: 4% per year.
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/59 |
| Date of hire | 1/1/84 |
| 1989 compensation | \$10,000 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ | ${}^sD_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 25 | 22,499 | 366,760 | 59,979 | 1,920,504 |
| 30 | 16,721 | 266,509 | 54,233 | 1,632,341 |
| 65 | 1,738 | 17,040 | 22,244 | 296,192 |

$\ddot{a}_{65}^{(12)} = 9.345$

In what range is the normal cost for 1989 as of 1/1/89?

(A) Less than \$500
(B) \$500 but less than \$600
(C) \$600 but less than \$700
(D) \$700 but less than \$800
(E) \$800 or more

**Problem 4 - 32**

Normal retirement benefit: \$15 per month for each year of service.
Early retirement benefit: Accrued benefit reduced by 2% for each year by which commencement of payments precedes age 65.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 60

Data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/30 |
| Date of hire | 1/1/68 |
| Date of retirement | 12/31/88 |

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 59 | 275 | 3,019 |
| 60 | 260 | 2,751 |
| 65 | 180 | 1,658 |

In what range is the absolute value of the experience gain or loss for 1988 as of 1/1/89 due to Smith's early retirement?

(A) Less than \$2,000
(B) \$2,000 but less than \$4,000
(C) \$4,000 but less than \$6,000
(D) \$6,000 but less than \$8,000
(E) \$8,000 or more

**Problem 4 - 33**

Normal retirement benefit: 1% of final year's compensation for each year of service.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 7%
Compensation increases: 5% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/69 |
| 1988 compensation | \$30,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 10$

In what range is the accrued liability as of 1/1/89?

(A) Less than $50,000
(B) $50,000 but less than $55,000
(C) $55,000 but less than $60,000
(D) $60,000 but less than $65,000
(E) $65,000 or more

**Problem 4 - 34**

Normal retirement benefit: $20 per month for each year of service.
Pre-retirement death benefit: None.
Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Participant data as of 1/1/88:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/58 | 1/1/38 | 1/1/28 |
| Date of hire | 1/1/83 | 1/1/68 | 1/1/78 |

Green died during 1988. Smith and Brown are still active participants as of 1/1/89.

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 25 | 2,303 | 37,539 |
| 30 | 1,711 | 27,278 |
| 31 | 1,613 | 25,567 |
| 50 | 509 | 6,723 |
| 51 | 477 | 6,214 |
| 60 | 260 | 2,870 |
| 61 | 241 | 2,610 |
| 65 | 178 | 1,745 |

$\ddot{a}_{65}^{(12)} = 9.35$

In what range is the experience gain for 1988 as of 1/1/89 due to Green's death?

(A) Less than \$19,900
(B) \$19,900 but less than \$20,100
(C) \$20,100 but less than \$20,300
(D) \$20,300 but less than \$20,500
(E) \$20,500 or more

**Problem 4 - 35**

Normal retirement benefit: 25% of final year's compensation.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: 3% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/39 |
| Date of hire | 1/1/72 |

Present value of future benefits as of 1/1/89: $110,000

In what range is the accrued liability as of 1/1/89?

(A) Less than $65,000
(B) $65,000 but less than $75,000
(C) $75,000 but less than $85,000
(D) $85,000 but less than $95,000
(E) $95,000 or more

**Problem 4 - 36**

Retirement benefit:

Before 1990: 1.00% of final average compensation for each year of service.
After 1989: 1.25% of final average compensation for each year of service.

Actuarial cost method: Individual entry age normal.

Valuation results for sole participant as of 1/1/90, before amendment:

| | |
|---|---|
| Present value of future benefits | $ 4,100 |
| Unfunded liability | 800 |
| Value of assets | 1,000 |
| Present value of future compensation | 46,000 |
| 1990 compensation | 4,000 |
| Present value of accrued benefits | 1,700 |

In what range is the normal cost for 1990 as of 1/1/90 after the amendment?

(A) Less than $220
(B) $220 but less than $230
(C) $230 but less than $240
(D) $240 but less than $250
(E) $250 or more

**Problem 4 - 37**

Normal retirement benefit:

Before 1990: $10 per month for each year of service.
After 1989: $12 per month for each year of service.

Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Data and valuation results for only participants as of 1/1/90:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/60 | 1/1/50 |
| Date of hire | 1/1/85 | 1/1/85 |
| Status | Active | Active |
| Normal cost per $1,000 of projected annual benefit | $42.13 | $89.04 |

In what range is the increase in the accrued liability as of 1/1/90 due to the amendment?

(A) Less than $325
(B) $325 but less than $425
(C) $425 but less than $525
(D) $525 but less than $625
(E) $625 or more

**Problem 4 - 38**

Actuarial cost method: Entry age normal (level dollar amount).
Assumed retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/45 |
| Date of hire | 1/1/75 |
| Status as of 1/1/90 | Active |

Projected annual benefit as of 1/1/90: B
Level annual cost from age 30: P

Consider the following expressions for the accrued liability for retirement benefits as of 1/1/90:

I. $P \times \dfrac{(N_{30} - N_{45})}{D_{45}}$

II. $B \times \dfrac{N_{65}^{(12)}}{D_{45}} - P \times \dfrac{(N_{45} - N_{65})}{D_{45}}$

III. $B \times \dfrac{N_{65}^{(12)}}{D_{45}} \times \dfrac{N_{30} - N_{45}}{N_{30} - N_{65}}$

Which, if any, of these expressions is (are) correct?

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D) above.

**Problem 4 - 39**

Plan effective date: 1/1/80
Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Entry age normal.
Assumed interest rate: 7% per year.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/55 |
| Date of hire | 1/1/80 |
| 1989 compensation | \$25,000 |
| Status as of 1/1/90 | Active |

Selected commutation functions and annuity value:

| Age x | $s_x/s_{65}$ | $D_x$ | ${}^sD_x$ | $N_x$ | ${}^sN_x$ |
|---|---|---|---|---|---|
| 25 | .1420 | 1,779,168 | 6,024,894 | 25,677,330 | 193,660,240 |
| 35 | .2314 | 894,190 | 4,932,364 | 12,364,650 | 138,500,016 |
| 45 | .3769 | 445,008 | 3,998,400 | 5,690,850 | 93,472,528 |
| 55 | .6139 | 213,953 | 3,131,334 | 2,405,025 | 57,406,892 |
| 65 | 1.0000 | 94,414 | 2,250,810 | 868,052 | 30,013,858 |

$\ddot{a}_{65}^{(12)} = 8.736$

In what range is the accrued liability as of 1/1/90?

(A) Less than \$12,500
(B) \$12,500 but less than \$17,500
(C) \$17,500 but less than \$22,500
(D) \$22,500 but less than \$27,500
(E) \$27,500 or more

**Problem 4 - 40**

Normal retirement benefit: 1% of final year's compensation for each year of service.
Actuarial cost method: Entry age normal.
Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Compensation increases before 1990: 5% per year.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/85 |
| 1989 compensation | \$25,000 |
| Status as of 1/1/90 | Active |

Value of assets as of 1/1/90: \$3,500

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.735$

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than \$1,350
(B) \$1,350 but less than \$1,500
(C) \$1,500 but less than \$1,650
(D) \$1,650 but less than \$1,800
(E) \$1,800 or more

**Problem 4 - 41**

Normal retirement benefit for all active and inactive participants:

Before 1991: \$12.00 per month for each year of service.
After 1990: \$12.50 per month for each year of service.

Actuarial cost method: Entry age normal.
Assumed interest rate: 7% per year.

Selected valuation results as of 1/1/90:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 50,000 |
| Accrued liability | 800,000 |
| Value of assets | 420,000 |

Contribution for 1990: \$80,000 paid on 7/1/90.
Benefit payments for 1990: \$15,000 paid on 7/1/90.

Selected valuation results as of 1/1/91, after amendment:

| | |
|---|---|
| Accrued liability | \$890,000 |
| Value of assets | 500,000 |

In what range is the experience gain for 1990?

(A) Less than \$8,000
(B) \$8,000 but less than \$16,000
(C) \$16,000 but less than \$24,000
(D) \$24,000 but less than \$32,000
(E) \$32,000 or more

**Problem 4 - 42**

Plan effective date: 1/1/91
Normal retirement benefit: \$10 per month for each year of service.
Preretirement death benefit: None.

Actuarial assumptions:

Preretirement terminations other than deaths: None.
Retirement age: 65

Data for all active participants as of 1/1/91 and selected commutation functions:

| Age x | Number of Participants | Past Service | $D_x$ | $N_x$ | $N_x^{(12)}$ |
|---|---|---|---|---|---|
| 30 | 0 | - | 1,336 | 18,946 | 18,334 |
| 40 | 2 | 10 | 670 | 8,953 | 8,646 |
| 50 | 0 | - | 329 | 3,974 | 3,823 |
| 60 | 2 | 10 | 153 | 1,571 | 1,501 |
| 65 | 0 | - | 100 | 919 | 873 |

There are no inactive participants as of 1/1/91.

In what range is the absolute value of the difference in the normal cost for 1991 as of 1/1/91 determined under the entry age normal method applied on an individual basis versus that determined under the entry age normal method applied on an aggregate basis?

(A) Less than \$100
(B) \$100 but less than \$200
(C) \$200 but less than \$300
(D) \$300 but less than \$400
(E) \$400 or more

**Problem 4 - 43**

Plan effective date: 1/1/92
Normal retirement benefit: 50% of final 3-year average compensation.
Actuarial cost method: Entry age normal.

Selected actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/47 |
| Date of hire | 1/1/82 |
| 1991 compensation | 28,571 |

Selected preretirement commutation functions:

| Age x | $D_x$ | ${}^sD_x$ | $N_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 35 | 894 | 4,931 | 12,3651 | 138,500 |
| 45 | 445 | 3,998 | 5,691 | 93,473 |
| 55 | 214 | 3,132 | 2,405 | 57,407 |
| 65 | 94 | 2,241 | 868 | 30,014 |

Selected post retirement annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the expected accrued liability as of 1/1/93?

(A) Less than \$32,000
(B) \$32,000 but less than \$33,000
(C) \$33,000 but less than \$34,000
(D) \$34,000 but less than \$35,000
(E) \$35,000 or more

**Problem 4 - 44**

Plan effective date: 1/1/75
Normal retirement benefit: \$10 per month for each year of service.
Early retirement reduction: None.
Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age:

| | |
|---|---|
| Before 1992 | 62 |
| After 1991 | 65 |

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of hire | 1/1/80 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.39$ $\ddot{a}_{65}^{(12)} = 8.74$

In what range is the change in the accrued liability as of 1/1/92 due to the change in the assumed retirement age?

(A) Decrease of \$3,000 or more
(B) Decrease of \$1,500 but less than \$3,000
(C) \$0 or decrease of less than \$1,500
(D) Increase of more than \$0 but less than \$1,500
(E) Increase of \$1,500 or more

**Problem 4 - 45**

Normal retirement benefit: $50 per month for each year of service.
Actuarial cost method:

Before 1992: Unit credit.
After 1991: Entry age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/42 |
| Date of hire | 1/1/87 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the change in the accrued liability as of 1/1/92 due to the change in the actuarial cost method?

(A) Decrease of $5,400 or more
(B) Decrease of $1,800 but less than $5,400
(C) Decrease of less than $1,800 or increase of less than $1,800
(D) Increase of $1,800 but less than $5,400
(E) Increase of $5,400 or more

**Problem 4 - 46**

Plan effective date: 1/1/70
Normal retirement benefit: 50% of final 5-year average compensation.
Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

Interest rate: 7%
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for all participants as of 1/1/93:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/56 | 1/1/36 |
| Date of hire | 1/1/92 | 1/1/76 |
| Monthly compensation | \$2,500 | \$1,500 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 10.0$

In what range is the accrued liability as of 1/1/93?

(A) Less than \$45,000
(B) \$45,000 but less than \$45,500
(C) \$45,500 but less than \$46,000
(D) \$46,000 but less than \$46,500
(E) \$46,500 or more

**Problem 4 - 47**

Normal retirement benefit: \$10 per month for each year of service.
Normal retirement age: 65

Early retirement benefit: Accrued benefit, reduced by 0.5% for each month by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 62

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of hire | 1/1/80 |
| Date of retirement | 12/31/92 |

Selected annuity values:

$\ddot{a}_{58}^{(12)} = 10.22 \quad \ddot{a}_{62}^{(12)} = 9.39 \quad \ddot{a}_{65}^{(12)} = 8.74$

In what range is the experience gain for 1992 due to early retirement?

(A) Less than \$1,000
(B) \$1,000 but less than \$1,025
(C) \$1,025 but less than \$1,050
(D) \$1,050 but less than \$1,075
(E) \$1,075 or more

**Problem 4 - 48**

Plan effective date: 1/1/92
Normal retirement benefit: \$500 per month.
Actuarial cost method: Entry age normal, where the entry age is the age at hire.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/52 |
| Date of hire | 1/1/82 |
| Status as of 1/1/93 | Active |

Normal cost for 1992 as of 1/1/92: $1,500

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 30 | 1,262 | 17,888 |
| 40 | 632 | 8,453 |
| 41 | 590 | |

In what range is the experience loss for 1992 due to mortality?

(A) Less than $25
(B) $25 but less than $50
(C) $50 but less than $75
(D) $75 but less than $100
(E) $100 or more

**Problem 4 - 49**

Normal retirement benefit:

Before 1993: 40% of final 5-year average compensation.
After 1992: 50% of final 3-year average compensation.

Normal form of payment:

Before 1993: Life annuity.
After 1992: Fully subsidized 100% joint and survivor annuity for married participants. Life annuity for unmarried participants.

Actuarial cost method: Entry age normal (level dollar).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 3% per year.
Preretirement deaths and terminations: None.
Retirement age: 65
Marital characteristics: 80% married; spouse same age as participant.

Valuation data for each of the plan's 100 participants as of 1/1/93:

| | |
|---|---|
| Date of birth | 1/1/53 |
| Date of hire | 1/1/80 |
| 1993 compensation | $40,000 |

Selected annuity values:

$$\ddot{a}_{65}^{(12)} = 8.736 \qquad \ddot{a}_{\overline{65:65}}^{(12)} = 10.576$$

In what range is the increase in the accrued liability as of 1/1/93 due to the changes in plan provisions?

(A) Less than $1,550,000
(B) $1,550,000 but less than $1,600,000
(C) $1,600,000 but less than $1,650,000
(D) $1,650,000 but less than $1,700,000
(E) $1,700,000 or more

**Problem 4 - 50**

Plan effective date: 1/1/90.

Normal retirement benefit:

| | |
|---|---|
| Effective 1/1/90: | $15 per month for each year of service. |
| Effective 1/1/93: | $30 per month for each year of service. |

Actuarial cost method: Entry age normal.

Assumed preretirement terminations other than deaths: None.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/55 |
| Date of hire | 1/1/90 |
| Status as of 1/1/93 | Active |

Normal cost for 1990 as of 1/1/90: $2,000

Selected commutation functions:

| Age x | $D_x$ |
|---|---|
| 35 | 894 |
| 36 | 835 |
| 37 | 779 |
| 38 | 727 |

In what range is the accrued liability as of 1/1/93?

(A) Less than $11,000
(B) $11,000 but less than $13,000
(C) $13,000 but less than $15,000
(D) $15,000 but less than $17,000
(E) $17,000 or more

**Problem 4 - 51**

Normal retirement benefit: 100% of final year's compensation.

Actuarial cost method: Entry age normal (level dollar), where the entry age is the age at hire.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/48 |

| | |
|---|---|
| Date of hire | 1/1/78 |
| Date of participation | 1/1/88 |
| 1992 compensation for 1/1/92 valuation | \$60,000 |
| 1993 compensation for 1/1/93 valuation | 66,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.736$

In what range is the experience loss for 1992 due to the change in compensation?

(A) Less than \$7,500
(B) \$7,500 but less than \$9,500
(C) \$9,500 but less than \$11,500
(D) \$11,500 but less than \$13,500
(E) \$13,500 or more

**Problem 4 - 52**

Plan effective date: 1/1/80
Normal retirement benefit: \$20 per month for each year of service.

Vesting eligibility: 100% after one year of service.

Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/88 |
| Date of termination | 12/31/92 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the experience gain or loss for 1992 due to Smith's termination?

(A) Gain of \$3,000 or more
(B) Gain of \$1,000 but less than \$3,000
(C) Gain or loss of less than \$1,000
(D) Loss of \$1,000 but less than \$3,000
(E) Loss of \$3,000 or more

**Problem 4 - 53**

Normal retirement benefit:

Before 1994: 50% of final 5-year average compensation.
After 1993: 60% of final 5-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Compensation increases: 5% per year.
Preretirement terminations other than death: None.
Retirement age: 65

Valuation data for sole participant

| | |
|---|---|
| Date of birth | 1/1/60 |
| Date of hire | 1/1/90 |
| 1994 valuation compensation | \$25,000 |

Selected commutation functions:

| Age x | $D_x$ | ${}^sD_x$ | $N_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 30 | 1,262 | 5,454 | 17,888 | 164,704 |
| 34 | 958 | 5,033 | 13,323 | 143,532 |
| 65 | 94 | 2,241 | 868 | 30,013 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.776$

In what range is the change in the accrued liability as of 1/1/94 due to the plan amendment?

(A) Less than \$1,350
(B) \$1,350 but less than \$1,450
(C) \$1,450 but less than \$1,550
(D) \$1,550 but less than \$1,650
(E) \$1,650 or more

**Problem 4 - 54**

Vesting: 100% after 5 years of service.

Actuarial cost method: Individual entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 4% per year.
Preretirement deaths: None.
Retirement age: 65

Valuation data for only participants (both active as of 1/1/94):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/59 | 1/1/56 |
| Date of hire | 1/1/94 | 1/1/91 |

Normal cost for 1994 as of 1/1/94:

For Smith: \$17,000
For Brown: \$16,000

Selected probabilities of termination:

| Age x | $q_x^{(w)}$ |
|---|---|
| 35 | 0.25 |
| 36 | 0.20 |
| 37 | 0.15 |
| 38 | 0.10 |
| 39 | 0.05 |

In what range is the accrued liability as of 1/1/94?

(A) Less than $65,000
(B) $65,000 but less than $80,000
(C) $80,000 but less than $95,000
(D) $95,000 but less than $110,000
(E) $110,000 or more

**Problem 4 - 55**

Normal retirement benefit:

Effective 1/1/93: $15 per month for each year of service.
Effective 1/1/94: $18 per month for each year of service.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate:

Before 1994: 7% per year.
After 1993: 6% per year.

Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/53 |
| Date of hire | 1/1/80 |

Selected annuity values:

| | 6% | 7% |
|---|---|---|
| $12\ddot{a}_{65}^{(12)}$ | 112.14 | 104.83 |

As of 1/1/94, the increase in the accrued liability due to the change in the assumed interest rate is determined before the increase in the accrued liability due to the plan amendment.

In what range is the absolute value of the difference between (a) the increase in the accrued liability as of 1/1/94 due to the change in the assumed interest rate, and (b) the increase in the accrued liability due to the plan amendment?

(A) Less than $100
(B) $100 but less than $200
(C) $200 but less than $300
(D) $300 but less than $500
(E) $500 or more

**Problem 4 - 56**

Normal retirement benefit: $50 per month for each year of service.

Termination benefit: Accrued benefit payable at normal retirement date.

Vesting: Full and immediate.
Preretirement death benefit: None.
Actuarial cost method: Entry age normal (level dollar amount).
Actuarial assumptions:

Interest rate: 7% per year.
Pretermination deaths: None.
Post-termination deaths: Included in commutation functions below.
Preretirement terminations: 30% at age 50 only (terminations are assumed to occur at beginning of year).
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/54 |
| Date of hire | 1/1/84 |

Selected commutation functions based on post-termination assumptions:

| Age x | $D_x$ |
|---|---|
| 30 | 1,262 |
| 40 | 632 |
| 50 | 311 |
| 65 | 94 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.7$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $950
(B) $950 but less than $1,000
(C) $1,000 but less than $1,050
(D) $1,050 but less than $1,100
(E) $1,100 or more

**Problem 4 - 57**

Normal retirement benefit: 50% of final 5-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Compensation increases: 6% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/84 |
| 1994 valuation compensation | \$53,000 |

Selected commutation functions:

| Age x | $D_x$ | ${}^sD_x$ | $N_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 40 | 49,876 | 513,015 | 666,789 | 15,607,843 |
| 50 | 24,505 | 451,387 | 295,992 | 10,748,428 |
| 65 | 7,448 | 328,780 | 68,476 | 4,770,425 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability as of 1/1/94?

(A) Less than \$60,000
(B) \$60,000 but less than \$62,500
(C) \$62,500 but less than \$65,000
(D) \$65,000 but less than \$67,500
(E) \$67,500 or more

**Problem 4 - 58**

Normal retirement benefit:

Before 1995: \$150 per year for each year of service.
After 1994: \$200 per year for each year of service.

Actuarial cost method: Individual entry age normal.
Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65.

Valuation data and selected valuation results for only participants (both active as of 1/1/95):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/65 | 1/1/40 |
| Date of hire | 1/1/90 | 1/1/80 |
| Normal cost per $1,000 of projected annual benefit | $43.04 | $135.85 |

In what range is the increase in the accrued liability as of 1/1/95 due to the plan amendment?

(A) Less than $4,500
(B) $4,500 but less than $5,000
(C) $5,000 but less than $5,500
(D) $5,500 but less than $6,000
(E) $6,000 or more

**Problem 4 - 59**

Normal retirement benefit:

Before 1995: 30.0% of final year's compensation.
After 1994: 37.5% of final year's compensation.

Actuarial cost method: Individual entry age normal.
Assumed compensation increases: None.

Selected valuation results for sole participant (active as of 1/1/95), before plan amendment:

| | |
|---|---|
| Present value of future benefits | $41,000 |
| Unfunded accrued liability | 8,000 |
| Value of assets | 10,000 |
| Present value of future compensation | 460,000 |

Increase in annual projected retirement benefit due to plan amendment: $3,000.

In what range is the normal cost for 1995 as of 1/1/95 after the plan amendment?

(A) Less than $2,200
(B) $2,200 but less than $2,700
(C) $2,700 but less than $3,200
(D) $3,200 but less than $3,700
(E) $3,700 or more

**Problem 4 - 60**

Actuarial cost method: Entry age normal (level dollar amount).
Assumed retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/50 |
| Date of hire | 1/1/80 |

Projected monthly benefit as of 1/1/95: $2,500.

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 30 | 1,261,611 | 17,887,840 |
| 45 | 445,008 | 5,690,850 |
| 65 | 94,414 | 868,052 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability for retirement benefits as of 1/1/95?

(A) Less than $35,000
(B) $35,000 but less than $37,000
(C) $37,000 but less than $39,000
(D) $39,000 but less than $41,000
(E) $41,000 or more

**Problem 4 - 61**

| | |
|---|---|
| Normal retirement benefit: | \$10 per month for each year of service. |
| Early retirement benefit: | Accrued benefit, reduced by 0.5 % for each month by which the benefit commencement date precedes the normal retirement date. |
| Normal form of payment: | Fully subsidized 100% joint and survivor annuity for married participants; life annuity for unmarried participants. |
| Actuarial cost method: | Individual entry age normal. |

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 62.
Marital characteristics: 80% of participants at the assumed retirement age are married with a spouse the same age.

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of hire | 1/1/85 |
| Date of retirement | 12/31/94 |
| Date of benefit commencement | 1/1/95 |
| Spouse's date of birth | 1/1/35 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 10.0$ $\quad$ $\ddot{a}_{62}^{(12)} = 9.0$ $\quad$ $\ddot{a}_{65}^{(12)} = 8.0$

$\ddot{a}_{60:60}^{(12)} = 8.0$ $\quad$ $\ddot{a}_{62:62}^{(12)} = 7.5$ $\quad$ $\ddot{a}_{65:65}^{(12)} = 7.0$

In what range is the absolute value of the experience gain or loss as of 1/1/95 due to Smith's early retirement?

(A) Less than \$450
(B) \$450 but less than \$900
(C) \$900 but less than \$1,350
(D) \$1,350 but less than \$1,800
(E) \$1,800 or more

**Problem 4 - 62**

Normal retirement benefit: $25 per month for each year of service.

Eligibility for early retirement: Age 55.

Early retirement benefit: Accrued benefit, reduced for commencement of payments before age 65.

Early retirement adjustment factor at age 55: 0.412.

Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for participant Smith (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/41 |
| Date of hire | 1/1/70 |
| Date of retirement | 12/31/95 |
| Date of benefit commencement | 1/1/96 |

Selected annuity values:

$$\ddot{a}_{55}^{(12)} = 10.78 \qquad \ddot{a}_{65}^{(12)} = 8.74$$

In what range is the decrease in the accrued liability as of 1/1/96 due to Smith's retirement?

(A) Less than $9,000
(B) $9,000 but less than $18,000
(C) $18,000 but less than $27,000
(D) $27,000 but less than $36,000
(E) $36,000 or more

**Problem 4 - 63**

Normal retirement benefit: $20 per month for each year of service up to 25 years.

Actuarial cost method:

| | |
|---|---|
| Before 1996: | Entry age normal. |
| After 1995: | Unit credit. |

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/36 |
| Date of hire | 1/1/56 |

Selected annuity value:

$\ddot{a}_{62}^{(12)} = 9.24$

In what range is the absolute value of the change in the normal cost for 1996 as of 1/1/96 due to the change in the actuarial cost method?

(A) Less than $500
(B) $500 but less than $1,000
(C) $1,000 but less than $1,500
(D) $1,500 but less than $2,000
(E) $2,000 or more

**Problem 4 - 64**

Vesting eligibility: 0% if less than 5 years of service; 100% if 5 or more years of service.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: None.
Pre-retirement decrements other than withdrawals: None.
Selected withdrawal rates:

| $x$ | $q_x^{(w)}$ |
|---|---|
| 35 | .5 |
| 36 | .4 |
| 37 | .3 |
| 38 | .2 |
| 39 | .1 |
| 40 and over | 0 |

Retirement age: 65.

Valuation data for sole participant Smith (active as of 1/1/96):

| | |
|---|---|
| Entry age | 35 |
| Attained age | 38 |

Normal cost for Smith for 1996 as of 1/1/96: $10,000.

In what range is the accrued liability as of 1/1/96?

(A) Less than $30,000
(B) $30,000 but less than $50,000
(C) $50,000 but less than $70,000
(D) $70,000 but less than $90,000
(E) $90,000 or more

**Problem 4 - 65**

Normal retirement benefit:

Before 1996: 1.25% of final 5-year average compensation for each year of service.
After 1995: 1.75% of final 3-year average compensation for each year of service.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Pre-retirement decrements: None.
Retirement age: 65.

Valuation data for sole participant Smith (active as of 1/1/96):

Date of birth 1/1/50
Date of hire 1/1/88

Normal cost for Smith for 1988 as of 1/1/88: $6,500.

In what range is the increase in the accrued liability for Smith as of 1/1/96 due to the plan amendment?

(A) Less than $34,000
(B) $34,000 but less than $36,000
(C) $36,000 but less than $38,000
(D) $38,000 but less than $40,000
(E) $40,000 or more

**Problem 4 - 66**

Normal retirement benefit: 60% of final 3-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Pre-retirement decrements: None.
Retirement age: 65.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/50 |
| Date of hire | 1/1/80 |
| 1995 valuation compensation for 1/1/95 valuation | $50,000 |
| 1996 valuation compensation for 1/1/96 valuation | 50,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.736$

In what range is the experience gain in 1995 due to a compensation increase other than assumed?

(A) Less than $4,000
(B) $4,000 but less than $4,700
(C) $4,700 but less than $5,400
(D) $5,400 but less than $6,100
(E) $6,100 or more

**Problem 4 - 67**

Plan effective date: 1/1/76.

Normal retirement benefit:

Before 1997: 50% of final 3-year average compensation.
After 1996: 60% of final 3-year average compensation.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | None. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Valuation data and results for all participants as of 1/1/97 (before amendment):

| | Smith | Brown |
|---|---|---|
| Date of hire | 1/1/96 | 1/1/79 |
| Normal cost as of 1/1 | \$1,840 | \$1,580 |

In what range is the increase in the accrued liability as of 1/1/97 due to the plan amendment?

(A) Less than \$12,000
(B) \$12,000 but less than \$13,000
(C) \$13,000 but less than \$14,000
(D) \$14,000 but less than \$15,000
(E) \$15,000 or more

**Problem 4 - 68**

Normal retirement benefit: 50% of final 3-year average compensation, payable monthly.

Actuarial cost method: Entry age normal (level percentage of pay).

Actuarial assumptions:

| | |
|---|---|
| Compensation increases: | 5% per year. |
| Preretirement terminations other than deaths: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/97):

| | |
|---|---|
| Date of birth | 1/1/57 |
| Date of hire | 1/1/87 |
| 1997 valuation compensation | \$42,000 |

Selected commutation functions:

| $x$ | $D_x$ | $N_x$ | ${}^sD_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 30 | 1,261,611 | 17,887,840 | 5,452,611 | 164,704,000 |
| 40 | 632,274 | 8,452,729 | 4,451,205 | 114,813,792 |
| 64 | 103,139 | 971,191 | 2,341,728 | 32,355,586 |
| 65 | 94,414 | 868,052 | 2,250,810 | 30,013,858 |

In what range is the accrued liability as of 1/1/97?

(A) Less than $28,000
(B) $28,000 but less than $30,000
(C) $30,000 but less than $32,000
(D) $32,000 but less than $34,000
(E) $34,000 or more

**Problem 4 - 69**

Termination benefit: None.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | None. |
| Preretirement decrements other than terminations: | None. |
| Selected probabilities of termination: | |

| $x$ | $q_x^{(w)}$ |
|---|---|
| 37 | .50 |
| 38 | .40 |
| 39 | .30 |
| 40 | .20 |
| 41 | .10 |

Retirement age: 65.

Valuation data for all participants and selected valuation results as of 1/1/97:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/59 | 1/1/58 | 1/1/56 |
| Date of participation | 1/1/96 | 1/1/96 | 1/1/94 |
| Normal cost as of 1/1 | $1,000 | $2,500 | $4,500 |

In what range is the accrued liability as of 1/1/97?

(A) Less than $34,000
(B) $34,000 but less than $38,000
(C) $38,000 but less than $42,000
(D) $42,000 but less than $46,000
(E) $46,000 or more

## 4.6 Solutions to Problems

### Problem 4 - 1

Part A - Determination of Normal Cost

| | | Jack | Jill |
|---|---|---|---|
| (1) | Total years of service at NRA | 16 | 27 |
| (2) | Benefit (30% of Salary) | \$9,000 | \$6,000 |
| (3) | Values at Retirement ((2) × 12.50) | 112,500 | 75,000 |
| (4) | Present Value of Benefits at EA (3) × $v^{y\text{-}ea}$ | 38,108 | 12,070 |
| (5) | $\ddot{a}_{\overline{y\text{-}ea}\rceil}$ | 10.11 | 12.83 |
| (6) | Normal Cost ((4) ÷ (5)) | 3,769 | 941 |
| (7) | Total Normal Cost 3,769 + 941 = 4,710 | | |

Part B - Determination of Accrued Liability (Retrospective Method)

| | | | |
|---|---|---|---|
| (8) | Past years of service | 6 | 2 |
| (9) | $\ddot{a}_{\overline{x\text{-}ea}\rceil} \times (1+i)^{x\text{-}ea}$ | 7.65 | 2.21 |
| (10) | Accrued Liability ((9) × (6)) | 28,833 | 2,080 |
| (11) | Total Accrued Liability 28,833 + 2,080 = 30,913 | | |

Part C - Determination of Accrued Liability (Prospective Method)

| | | | |
|---|---|---|---|
| (12) | Present Value Benefits at Att. Age ((3) × $v^{y\text{-}x}$) | 57,189 | 13,819 |
| (13) | $\ddot{a}_{\overline{y\text{-}x}\rceil}$ | 7.52 | 12.47 |
| (14) | Present Value Future Normal Costs ((6) × (13)) | 28,343 | 11,734 |
| (15) | Accrued Liability ((12) - (14)) | 28,846 | 2,085 |
| (16) | Total Accrued Liability 28,846 + 2,085 = 30,931 | | |

### Problem 4 - 2

Step I: Determine the expected unfunded accrued liability.

$$e\text{UAL}_{86} = (\text{UAL}_{85} + \text{Normal Cost}_{85})(1.065) - \text{Contribution}$$

where $\text{UAL}_{85} = \text{AL}_{85} - \text{Assets}_{85}$

$$eUAL_{86} = (450{,}000 - 200{,}000 + 50{,}000)(1.065) - 75{,}000$$
$$= 319{,}500 - 75{,}000 = 244{,}500$$

Step II: Determine the actuarial gain or loss.

$$UAL_{86} = AL_{86} - Assets_{86}$$
$$UAL_{86} = 530{,}000 - 290{,}000 = 240{,}000$$
$$\text{Gain/Loss} = eUAL_{86} - UAL_{86}$$
$$= 244{,}500 - 240{,}000 = 4{,}500 \text{ Gain}$$

**Problem 4 - 3**

| As of 1/1/86: | Winken | Blinken | Nod |
|---|---|---|---|
| (1) Attained Age | 55 | 40 | 30 |
| (2) Total years of service at NRA | 25 | 30 | 40 |
| (3) Monthly Benefit $25 × (2) | $625 | $750 | $1,000 |
| (4) Normal Cost $[(3) \times 10.50 \times 12 \times D_{65}]/(N_{ea} - N_{ra})$ | 1,059 | 854 | 548 |

Normal Cost = 1,059 + 854 = 1,913 (since Nod has terminated).

**Problem 4 - 4**

Since the effective date of the plan was 1/1/85, the Unfunded Accrued Liability was equal to the Accrued Liability on that date.

Step I: Determine the Accrued Liability as of 1/1/85.

$$AL = \sum \left(\text{Normal Cost} \times (N_{ea} - N_{aa-1})/D_{aa-1}\right)$$

$$AL(\text{Winken}) = (1{,}059 \times 7{,}195) \div 297 = 25{,}655$$
$$AL(\text{Blinken}) = (854 \times 3{,}835) \div 814 = 4{,}023$$
$$AL(\text{Nod}) = (548 \times 7{,}286) \div 1{,}548 = 2{,}579$$

Accrued Liability at 1/1/85 = 25,655 + 4,023 + 2,579 =32,257

Step II: Determine the Accrued Liability as of 1/1/86.

$$AL(\text{Winken}) = (1{,}059 \times 7{,}492) \div 277 = 28{,}643$$
$$AL(\text{Blinken}) = (854 \times 4{,}650) \div 763 = 5{,}205$$
$$AL(\text{Nod}) = \text{PV Accrued Benefit} = 833$$

Accrued Liability at 1/1/86 = 28,643 + 5,205 + 833 = 34,681

Step III: Determine Expected Accrued Liability at 1/1/86.

$$\begin{aligned} eUAL_{86} &= (UAL_{85} + NC_{85})(1.06) - \text{Contribution} \\ &= (32{,}257 + 1{,}913 + 548)(1.06) - 7{,}200 \\ &= (34{,}718)(1.06) - 7{,}200 = 29{,}601 \end{aligned}$$

Step IV: Determine Actual Unfunded Liability and Gain/Loss.

$$UAL_t = AL_t - \text{Assets}_t$$

$$UAL_{86} = 34{,}681 - 7{,}200 = 27{,}481$$

$$\text{Gain/Loss}_t = eUAL_t - UAL_t$$
$$\text{Gain/Loss}_{86} = 29{,}601 - 27{,}481 = 2{,}120 \text{ (Gain)}$$

**Problem 4 - 5**

Key Concepts: 1) Calculate the total gain or loss.
2) Determine the gain or loss based upon the assumption that plan assets increase at 8% per year.
3) Calculate the difference between results of Steps 1 and 2, which represents the gain or loss attributable to "other factors".

Step I: Calculate the total gain/loss.

$$\begin{aligned} UAL_{85} &= AL_{85} - \text{Assets}_{85} \\ &= 435{,}000 - 300{,}000 = 135{,}000 \end{aligned}$$

$$UAL_{86} = 495{,}000 - 365{,}000 = 130{,}000$$

$$\begin{aligned} eUAL_{86} &= (UAL_{85} + NC_{85})(1 + i) - \text{Contribution} \\ &= (135{,}000 + 34{,}000)(1.08) - 42{,}000 = 140{,}520 \end{aligned}$$

$$\begin{aligned}\text{Total Gain} &= e\text{UAL} - \text{UAL}\\ &= 140{,}520 - 130{,}000 = 10{,}520 \ \text{(gain)}\end{aligned}$$

Step II: Calculate the gain based on the expected Assets.

$$\begin{aligned} e\text{Assets}_{86} &= (300{,}000)(1.08) + 42{,}000 = 366{,}000\\ \text{Gain based upon } e\text{Assets}_{86} &= e\text{Assets} - \text{Assets}\\ &= 366{,}000 - 365{,}000 = 1{,}000 \ \text{(loss)}\end{aligned}$$

Step III: Compute Gain/Loss from "other sources".

$$\begin{aligned}\text{Total Gain} &= 10{,}520\\ \text{Gain from investment earnings} &= -1{,}000 \ \text{(loss)}\\ \text{Gain from "other sources"} &= 10{,}520 - (-1{,}000) = 11{,}520\end{aligned}$$

**Problem 4 - 6**

Statement I:

First, the present value of vested accrued benefits is independent of the cost method used. Second, because the statement refers to vested benefits, the present value of a participant's benefit does not depend upon any withdrawal decrement that might be used. The whole concept of vesting is that the participant will receive the accrued benefit even if he withdraws.

In valuation X, if the vested accrued benefits are first valued using $D_x^{(T)}$ functions (which reflect a withdrawal assumption), then the "cost of vesting" associated with the vested accrued benefits must be added to the initial value. The result is then equal to that obtained by assuming no withdrawals (as in valuation Y). Statement I is true.

Statement II:

Assume that we use a withdrawal decrement from Entry Age to Attained Age only. At Attained Age, the Present Value of Future Benefits will be identical under valuations X and Y. However, the Normal Cost at Entry Age will usually be less under valuation X and hence, the Present Value of Future Normal Costs at Attained Age will be less. Therefore, since Accrued Liability = Present Value of Future Benefits less the Present Value of Future Normal Costs, the Accrued Liability at Attained Age will be greater under valuation X than under valuation Y. Statement II is true.

Statement III:

This is the usual situation with a moderate withdrawal assumption. However, for purposes of illustration, consider the results if one were to assume that all participants withdraw prior to becoming vested. The Present Value of Future Benefits is zero at Entry Age and Attained Age. Therefore, the Accrued Liability is zero under the prospective formula for valuation X. Valuation Y would, of course, develop a positive Accrued Liability. Statement III is true.

Answer is D.

**Problem 4 - 7**

| Part A - Determine Normal Cost: | Tinker | Bell |
|---|---|---|
| (1) Salaries at attained age | \$40,000 | \$25,000 |
| (2) Projected Salary at retirement (1) × $(1.04)^{ra-aa}$ | 72,038 | 98,652 |
| (3) Projected Benefit | 36,019 | 71,029 |
| (4) Present Value of Benefits at entry age $(3) \times \ddot{a}_{ra}^{(12)} \times (D_{ra}/D_{ea})$ | 57,404 | 55,795 |
| (5) Salaries at entry age (1) × $(1/1.04)^{aa-ea}$ | 27,023 | 24,038 |
| (6) Present Value of Future Salaries at entry age $(5) \times ({}^sN_{ea} - {}^sN_{ra})/{}^sD_{ea}$ | 488,716 | 563,922 |
| (7) Normal Cost % at Entry Age (4) ÷ (6) | 11.75 | 9.89 |
| (8) Normal Cost at Attained Age (7) × (1) | 4,700 | 2,473 |

(9) Total Normal Cost = 4,700 + 2,473 = 7,173

| Part B - Determine Accrued Liability: | | |
|---|---|---|
| (10) Present Value of Benefits $(3) \times \ddot{a}_{ra}^{(12)} \times (D_{ra}/D_{aa})$ | 111,448 | 59,484 |
| (11) Present Value of Future Normal Costs $(8) \times ({}^sN_{aa} - {}^sN_{ra})/{}^sD_{aa}$ | 56,646 | 56,946 |
| (12) Accrued Liability at Attained Age (10) - (11) | 54,802 | 2,538 |

(13) Total Accrued Liability 54,802 + 2,538 = 57,340

**Problem 4 -8**

Key Concept: Problem demonstrates that the salary is not necessary to determine the Normal Cost Percentage. The interest assumption of 7% and Salary Increase assumption of 4% is the equivalent of 2.8846% [(1.07 ÷ 1.04) = 1.028846].

Determine Present Value of Future Benefits at entry age:

$$\begin{aligned} PVFB_{ea} &= (.10 \times \text{Salary})(10.50)(1.04)^{10}(v^{18}) \\ &= (.10 \times \text{Salary})(10.50)(1.480)(.29586) \\ &= .45977 \times \text{Salary} \end{aligned}$$

Determine Present Value of Future Salaries at entry age:

$$\begin{aligned} PVFS_{ea} &= \text{Salary} \times 1.04^{-8} \times \ddot{a}_{\overline{18}|j} \qquad j = 2.8846\% \\ &= \text{Salary} \times .73069 \times 14.2893 \\ &= 10.441 \times \text{Salary} \end{aligned}$$

$$\begin{aligned} \text{Normal Cost Percentage} &= (PVFB_{ea} \div PVFS_{ea}) \\ &= (.45977 \times \text{Salary}) \div (10.441 \times \text{Salary}) = .044 \ \ (4.4\%) \end{aligned}$$

**Problem 4 - 9**

In this problem, we are asked to determine a new Normal Cost without knowing the Entry Age of the Participant. However, since we know the original Normal Cost, we can solve for the value of the Commutation function N for the entry age. Using Retirement Age of 65:

$$\begin{aligned} PVFB_{ea} &= \text{Benefits} \times 120 \times (D_{65} / D_{ea}) \\ &= 200 \times 120 \times (256/D_{ea}) \\ &= 6{,}144{,}000/ D_{ea} \\ PVFS_{ea} &= \text{Salary} \times (N_{ea} - N_{65})/D_{ea} \end{aligned}$$

$$\begin{aligned} NC\% \text{ at EA} &= PVFB \div PFVS \\ &= 6{,}144{,}000 \div [\text{Salary} \times (N_{ea} - N_{65})] \end{aligned}$$

$$\begin{aligned} \text{Normal Cost} &= \text{Salary} \times (NC\% \text{ at EA}) \\ &= 6{,}144{,}000 \div (N_{ea} - N_{65}) \\ 2{,}084 &= 6{,}144{,}000 \div (N_{ea} - N_{65}) \\ 2{,}948 &= N_{ea} - N_{65} \\ N_{ea} &= 2{,}948 + N_{65} = 2{,}948 + 2{,}677 = 5{,}625 \end{aligned}$$

Using Retirement Age of 62:

$$\begin{aligned}
PVFB &= 200 \times 126 \times (D_{62} / D_{ea}) \\
&= 200 \times 126 \times (328 / D_{ea}) \\
&= 8{,}265{,}600 \div D_{ea}
\end{aligned}$$

$$\begin{aligned}
PVFS &= Salary \times [(N_{ea} - N_{62}) / D_{ea}] \\
&= Salary \times [(5{,}625 - 3{,}585) / D_{ea}] \\
&= (Salary \times 2{,}040) / D_{ea}
\end{aligned}$$

$$NC\% = (8{,}265{,}600 \div D_{ea}) \div (Salary \times 2{,}040 \div D_{ea})$$

$$\begin{aligned}
\text{Normal Cost} &= 8{,}265{,}600 \div 2{,}040 = 4{,}052 \\
\text{Increase in Normal Cost} &= 4{,}052 - 2{,}084 = 1{,}968
\end{aligned}$$

**Problem 4 - 10**

$$\begin{aligned}
UAL_{85} &= 600{,}000 - 370{,}000 = 230{,}000 \\
UAL_{86} &= 625{,}000 - 415{,}000 = 210{,}000 \\
eUAL_{86} &= (UAL_{85} + NC_{85})(1 + i) - \text{Contribution} \\
&= (230{,}000 + NC_{85})(1.08) - \text{Contribution} \\
&= 248{,}400 + (1.08)(NC_{85}) - [(1.08)(NC_{85}) + 40{,}000] = 208{,}400 \\
\text{Gain/Loss} &= eUAL - UAL \\
&= 208{,}400 - 210{,}000 = (1{,}600) \quad \text{(loss)}
\end{aligned}$$

Since answer is negative, there is a loss of $1,600.

**Problem 4 - 11**

Step I: Determine Present Value of Future Benefits at Entry Age.

$$\begin{aligned}
PVFB &= \text{Projected Benefit} \times \ddot{a}_{60}^{(12)} \times (D_{60}/D_{ea}) \\
&= (12)(50\%)(Salary)(s_{59}/s_{aa}) \times (D_{60}/D_{ea})(\ddot{a}_{60}^{(12)})
\end{aligned}$$

$$\begin{aligned}
PVFB(Smith) &= (12)(.50)(3{,}000)(2) \times 1 = 36{,}000 \\
PVFB(Brown) &= (12)(.50)(1{,}000)(2.5) \times .5 = 7{,}500
\end{aligned}$$

Step II: Determine Present Value of Future Salaries at Entry Age.

$$\text{Entry Age Salary} = (\text{Sal}) \times (s_{59} / s_{aa}) \div (s_{59} / s_{ea})$$
$$\text{Salary}_{ea}(\text{Smith}) = 36{,}000 \times (2 \div 2.5) = 28{,}800$$
$$\text{Salary}_{ea}(\text{Brown}) = 12{,}000 \times (2.5/3.5) = 8{,}571$$
$$\text{PVFS}_{ea} = \text{Salary}_{ea} \times ({}^{s}N_{ea} - {}^{s}N_{60}) \div {}^{s}D_{ea}$$
$$\text{PVFS(Smith)} = 28{,}800 \times (3{,}000 \div 125) = 691{,}200$$
$$\text{PVFS(Brown)} = 8{,}571 \times (4{,}500 \div 100) = 385{,}695$$

Step III: Determine the Normal Cost.

$$\text{Normal Cost \%} = \text{PVFB} \div \text{PVFS}$$
$$\text{Normal Cost \%(Smith)} = 36{,}000 \div 691{,}200 = 5.21\%$$
$$\text{Normal Cost \%(Brown)} = 7{,}500 \div 385{,}695 = 1.94\%$$
$$\text{Normal Cost(Smith)} = 36{,}000 \times 5.21\% = 1{,}876$$
$$\text{Normal Cost(Brown)} = 12{,}000 \times 1.94\% = 233$$
$$\text{Normal Cost} = 1{,}876 + 233 = 2{,}109$$

Answer is D.

**Problem 4 - 12**

To determine the Gain or Loss, we must first determine the Accrued Liability as of 1/1/84. To do this, we must first determine the Normal Cost also as of 1/1/84.

Step I: Determine Normal Cost at Entry Age and Attained Age.

$$\text{Projected Benefit} = 40\% \times 24{,}000 \times (1.06)^{15} = 23{,}007$$
$$\text{PVFB}_{45} = 23{,}007 \times \ddot{a}_{65}^{(12)} \times (1.07)^{-20}$$
$$= 1{,}917 \times 115 \times .2584 = 56{,}966$$
$$\text{NC}_{45} = \text{PVFB}_{45} \div [({}^{s}N_{45} - {}^{s}N_{65}) / {}^{s}D_{45}]$$
$$= 56{,}966 \div 18.320 = 3{,}109$$
$$\text{NC}_{50} = 3{,}109 \times (1.06)^{5} = 4{,}161$$

Step II: Determine the Accrued Liability at 1/1/84.

$$\text{AL}_{84} = \text{PVFB}_{84} - \text{PVFNC}_{84}$$
$$\text{PVFB}_{84} = (23{,}007/12)(115)(1.07)^{-15}$$
$$= 1{,}917 \times 115 \times .3624 = 79{,}893$$
$$\text{PVFNC}_{84} = \text{Normal Cost} \times [({}^{s}N_{50} - {}^{s}N_{65}) / {}^{s}D_{50}]$$
$$= 4{,}161 \times 14.057 = 58{,}491$$

$$AL_{84} = 79{,}893 - 58{,}491 = 21{,}402$$

Step III: Determine the Experience Gain or Loss.

$$\begin{aligned} eUAL_{85} &= (21{,}402 + 4{,}161)(1.07) - \text{Contribution} \\ &= (25{,}563)(1.07) - 7{,}000 = 20{,}352 \\ \text{Gain/Loss} &= eUAL - UAL \\ &= 20{,}352 - 23{,}000 = (2{,}648) \quad \text{(loss)} \end{aligned}$$

Since the number is negative, there is an experience loss of \$2,648.

Answer is C.

**Problem 4 -13**

To determine the experience gain or loss, we must first determine the Accrued Liability and the Unfunded Accrued Liability as of 1/1/84.

The Unfunded Accrued Liability as of 1/1/83 is easily determined to be:

$$\begin{aligned} UAL_{83} &= AL_{83} - \text{Plan Assets} \\ &= 304{,}300 - 235{,}000 = 69{,}300 \end{aligned}$$

Reminder: Present Value of Salaries at age x = Salary × $\ddot{a}_{\overline{65-x}|}$ and Present Value of Future Normal Costs at age x = NC × $\ddot{a}_{\overline{65-x}|}$. To determine the Present Value of Future Normal Costs:

$$\begin{aligned} PVFNC &= NC \times \ddot{a}_{\overline{65-x}|} \\ \text{but, } \ddot{a}_{\overline{65-50}|} &= PVFS \div \text{Salary} \\ &= 250{,}000 \div 15{,}500 = 16.13 \text{ (Age 50)} \\ \text{and } \ddot{a}_{\overline{65-55}|} &= 180{,}000 \div 18{,}000 = 10 \text{ (Age 55)} \end{aligned}$$

$$\begin{aligned} PVFNC_{50} &= 16.13 \times 800 = 12{,}904 \\ PVFNC_{55} &= 10 \times 1{,}025 = 10{,}250 \end{aligned}$$

To determine the Accrued Liability on 1/1/84:

$$\begin{aligned} AL_{84} &= PVFB_{84} - PVFNC_{84} \\ &= 350{,}000 - 12{,}904 - 10{,}250 = 326{,}846 \end{aligned}$$

To determine the Gain/Loss:

$$\begin{aligned} UAL_{84} &= AL_{84} - Assets_{84} \\ &= 326{,}846 - 275{,}000 = 51{,}846 \end{aligned}$$

$$\begin{aligned} eUAL_{84} &= (UAL_{83} + NC_{83})(1.07) - (\text{Contributions} + \text{Interest}) \\ &= (69{,}300 + NC_{83})(1.07) - (\text{Contributions} + \text{Interest}) \\ &= 74{,}151 + 1.07(NC) - [21{,}000 + 1.07(NC)] = 53{,}151 \\ \text{Gain/Loss} &= eUAL_{84} - AL_{84} \\ &= 53{,}151 - 51{,}846 = 1{,}305 \text{ (Gain)} \end{aligned}$$

Answer is E.

**Problem 4 - 14**

Key Concept: This problem is similar to problem 4 - 9 in that we are given a normal cost and are asked to determine the commutation function N at entry age which is not known.

There is an additional consideration, however, in that we need to convert the $N_x^{(12)}$ functions at retirement ages to $N_x$. This is done through the relationships:

$$N_{65} = N_{65}^{(12)} + (\tfrac{11}{24})D_{65}$$

$$\text{and } N_{66} = N_{65} - D_{65}$$

From these formulae,

$$\begin{aligned} N_{65} &= 1200 + (\tfrac{11}{24})(100) = 1245.833 \\ \text{and } N_{66} &= 1245.833 - 100 = 1145.833 \end{aligned}$$

Step I: Determine $N_{ea}$ for the old normal cost.

$$\begin{aligned} NC = 800 &= [(\text{Ben} \times 12 \times N_{65}^{(12)}) \div D_{ea}] / [(N_{ea} - N_{65})/D_{ea}] \\ &= [(100 \times 12 \times 1200) / D_{ea}] \div [(N_{ea} - N_{65} / D_{ea}] \\ &= (1{,}440{,}000 / D_{ea}) \div [(N_{ea} - 1{,}245.833)/D_{ea}] \\ &= 1{,}440{,}000 \div (N_{ea} - 1245.833) \\ \text{and } N_{ea} &= (1{,}440{,}000 \div 800) + 1245.833 = 3045.833 \end{aligned}$$

Step II: Determine the new Normal Cost:

$$\begin{aligned} \text{New } NC_{ea} &= (\text{Benefit} \times 12 \times N_{66}^{(12)})/(N_{ea} - N_{66}) \\ &= (1{,}200 \times 1{,}103) \div (3{,}045.833 - 1145.833) \\ &= 1{,}323{,}600 \div 1{,}900 = 696.63 \end{aligned}$$

Answer is A.

**Problem 4 - 15**

Key Concept: This problem calls for conversion of commutation functions $D_x$ to ${}^sD_x$.

$${}^sD_x = D_x \times (1.06)^x \qquad {}^sN_x = \sum {}^sD_x$$

| Age x | $D_x$ | ${}^sD_x$ |
|---|---|---|
| 61 | 385 | 13,462.277 |
| 62 | 354 | 13,120.999 |
| 63 | 326 | 12,808.171 |
| 64 | 299 | 12,452.214 |
| 65 | 274 | 12,095.722 |

Entry Age Normal Cost % = (PVFB ÷ PVFS)

To determine the Present Value of Future Benefits at age 61:

$$\begin{aligned} PVFB &= (\text{Ben} \times 12 \times \ddot{a}_{65}^{(12)})(D_{65} / D_{61}) \\ &= (700 \times 12 \times 10)(274/385) = 59{,}782 \end{aligned}$$

To determine the Present Value of Future Salaries at age 61:

$$\begin{aligned} PVFS &= \text{Salary} \times ({}^sN_{61} - {}^sN_{65}) / {}^sD_{61} \\ &= \text{Salary} \times (51{,}843.661) / 13{,}462.277 \\ &= \text{Salary} \times 3.851 \end{aligned}$$

(Note that ${}^sN_{61} - {}^sN_{65} = {}^sD_{61} + {}^sD_{62} + {}^sD_{63} + {}^sD_{64}$)

To determine the Normal Cost:

Normal Cost % = 59,782 ÷ (Salary × 3.851)

$$\text{Normal Cost} = (1.06)^2\ (\text{Salary})(59{,}782)\ /(\text{Salary} \times 3.851) = 17{,}442.50$$

Answer is D.

**Problem 4 - 16**

Key Concepts: Care should be exercised in determining the number of salary increases from valuation to retirement date. In this particular problem, there are only 8 assumed salary increases even though there are 9 years remaining until retirement.

Interest assumption of 7% and salary increase assumption of 5% is the equivalent of an interest assumption of 1.90476% [(1.07/1.05) - 1].

$$\text{Projected Benefit(Age 65)} = 60\% \times 100{,}000 \times (1.05)^8 = 88{,}647$$

$$\begin{aligned} PVFB_{55} &= (88{,}647) \times \ddot{a}_{65}^{(12)} \times v_{.07}^{10} \\ &= 7{,}387 \times 100 \times .5083 = 375{,}481 \end{aligned}$$

$$\begin{aligned} PVFS_{55} &= 100{,}000 \times \ddot{a}_{\overline{10}|j} \quad \text{where } j = 1.90476\% \\ &= 100{,}000 \times 9.1995 = 919{,}950 \end{aligned}$$

$$\begin{aligned} NC_{55} &= 100{,}000 \times (375{,}481 \div 919{,}950) = 40{,}815 \\ NC_{56} &= 40{,}815 \times 1.05 = 42{,}856 \end{aligned}$$

Answer is C.

**Problem 4 - 17**

Key Concept: The total Gain/Loss is determined, then the Gain/Loss attributable to investment earnings is determined. Since the only variants are investment earnings and salary increases, the difference must represent the Gain/Loss attributable to salary increases.

Step I: Determine total Gain/Loss.

$$\begin{aligned} e\text{UAL (85)} &= (UAL_{84} + NC_{84})(1 + i) - C_{84} \\ &= (300{,}000 + 55{,}000)(1.075) - 90{,}000 = 291{,}625 \end{aligned}$$

Actual UAL = 330,696
Gain/Loss = 291,625 - 330,696 = (39,071) (loss)

Step II: Determine Gain/Loss from investment earnings.

Expected Value of Assets

$$eAssets_{85} = Assets_{84} \times 1.075 + \text{Contribution}$$
$$= (300{,}000 \times 1.075) + 90{,}000 = 412{,}500$$

Gain/Loss from investment earnings = $e$Assets - Assets
$$= 412{,}500 - 400{,}000 = 12{,}500 \quad \text{(loss)}$$

Step III: Determine Gain/Loss from salary increases.

Loss from Salary Increases = 39,071 - 12,500 = 26,571

Answer is D.

**Problem 4 - 18**

Key Concepts: Since the problem involves the calculation of the present value of future benefits, the fact that the stated funding method is entry age normal is irrelevant.

The result needed is the increase in the cost of vesting. This can be approached by calculating the cost based on the old schedule and the new and then the difference, or by calculating the present value of the difference directly. The information given dictates the second approach.

The definition of $M_x^{(w)}$ is consistent with the assumption of the timing of terminations.

$$\text{Cost of vesting} = \sum_{x=aa}^{ra-1} (AB_x)(\text{Vesting }\%)\,(C_x^{(w)}/D_{aa})$$

where $C_x^{(w)} = l_x\, v^x\, q_x^{(w)}\,{}_{65-x|}\ddot{a}_x^{(12)} = M_x^{(w)} - M_{x+1}^{(w)}$, as defined in the problem.

The amendment changes the results at ages 58 and 59 only.

$$\text{Therefore the increase} = \sum_{x=58}^{59} (AB_x) \times (C_x^{(w)}/D_{50})$$

Increase for age 58 = $[(12 \times 8 \text{ years} \times 10) \times (45 - 40)]/250$
$$= (960 \times 5)/250 = 19.20$$

Increase for age 59 = [12 × 9 years × 10) × (40 - 38)] /250
= (1,080 × 2) /250 = 8.64

Total Increase = 19.20 + 8.64 = 27.84

Answer is B.

Additional Considerations for Problem 4 - 18:

Problem 4-18 involves the determination of the value of the ancillary benefit of vesting which has not been discussed in this or previous chapters. So far in our discussion when commutation functions were involved, there has been the implicit assumption that the only benefits that ever will be provided are those at normal retirement. If a participant dies or terminates employment, he would lose all benefits that have accrued to his benefit to date. In the real world, however, this may not occur so provision should be made for the cost of the contingent liability of pre-retirement terminations.

The item that is affected when we provide for termination benefits is the value of future benefits. We begin with the expression for the Present Value of Future Benefits as we have learned it so far, [Ben × $12\ddot{a}_{65}$ × $(D_{65}/D_x)$], then add an amount that reflects the probability of withdrawal of that portion of benefits becoming payable during the year of valuation, the year following the year of valuation, the year after that, and so on until normal retirement date. The amount of increase depends upon two quantities as follows:

(1) The Present Value of the Vested Accrued Benefit on the valuation date and all subsequent expected valuation dates;

(2) The probability that termination will occur during the year of valuation and any subsequent year.

The Present Value of Vested Accrued Benefit for each Participant may be expressed as:

$$PV\,(AB) = (\%\ \text{Vested}) \times (AB_x) \times 12({}_{y-x-1|}\ddot{a}^{(12)}_{x+1})$$

where AB is the accrued benefit and $({}_{y-x-1|}\ddot{a}^{(12)}_{x+1})$ is an annuity involving interest and mortality only since the participant cannot lose his Accrued Benefit once he is vested.

The probability of withdrawal due to termination can be expressed as:

$$C^{(w)}_x / D_x \qquad \text{where } C^{(w)}_x = q^{(w)}_x\, l_x\, v^{x+1}$$

The amount of increase is calculated by applying the probability of termination to the present value of vested accrued benefit for each year until the year preceding retirement and then summing the result.

For each participant then, the present value of future benefits can be expressed as:

$$\text{PVFB} = (\text{B})12\ddot{a}_y^{(12)}\,(D_y/D_{ea})$$

$$+\ 1/D_{ea} = \sum_{t=ea}^{y-1}(C^{(w)}{}_t \times (\%\text{ vested}) \times (AB_{t+1}) \times 12\ddot{a}_y^{(12)}\,(D'_y/D'_{x+1})$$

where $D'$ is composed of interest and mortality only, and in the problem, $M_x = M_{x+1} + C_x$.

**Problem 4 - 19**

Statement I: Normal Cost under the aggregate cost method.

$$\text{NC} = (700\ddot{a}_{65}^{(12)}\,v^{35} + 300\ddot{a}_{65}^{(12)}\,v^{15} + 200\ddot{a}_{65}^{(12)}\,v^{10}) \div [(1/3)(\ddot{a}_{\overline{35|}} + \ddot{a}_{\overline{15|}} + \ddot{a}_{\overline{10|}})]$$

$$= 300\ddot{a}_{65}^{(12)}\ (7v^{35} + 3v^{15} + 2v^{10}) \div (\ddot{a}_{\overline{35|}} + \ddot{a}_{\overline{15|}} + \ddot{a}_{\overline{10|}})$$

$$= 300\ddot{a}_{65}^{(12)}\,[7(.094) + 3(.362) + 2(.508)] \div [(13.85 + 9.75 + 7.52)]$$

$$= 300\ddot{a}_{65}^{(12)}\,(.658 + 1.086 + 1.016) \div 31.12$$

$$= 300\ddot{a}_{65}^{(12)} \times .0887 = 100\ddot{a}_{65}^{(12)} \times .2661$$

Normal Cost under the entry age normal method.

$$\text{NC} = (700\ddot{a}_{65}^{(12)}\,v^{35} \div \ddot{a}_{\overline{35|}}) + (300\ddot{a}_{65}^{(12)}\,v^{15} \div \ddot{a}_{\overline{15|}}) + (200\ddot{a}_{65}^{(12)}\,v^{10} \div \ddot{a}_{\overline{10|}})$$

$$= 100\ddot{a}_{65}^{(12)} \times [(7)(.094)/(13.85) + (3)(.362/9.75) + (2)(.508)/7.52)]$$

$$= 100\ddot{a}_{65}^{(12)} \times (.048 + .111 + .135) = 100\ddot{a}_{65}^{(12)} \times .294$$

Normal cost under aggregate method is smaller, so statement is true.

Statement II: An experience gain from investment return under the Aggregate Cost Method generates a decrease in normal cost. Statement II is true.

Statement III: An experience gain from investment return under the Entry Age Normal Cost Method is amortized over later years and does not affect the normal cost. Statement III is false.

Answer is C.

**Problem 4 - 20**

Key Concept: The expression $\ddot{a}_{65}^{(12)}$ with a one year setback is equivalent to $N_{64}^{(12)}/D_{64}$.

$$\ddot{a}_{65}^{(12)} = 9{,}626 \div 1{,}080 = 8.91$$

Determine Present Value of Future Benefits at Entry Age:

$$\begin{aligned} PVFB_{ea} &= (100)(12)(8.91)(D_{65}/D_{36}) \\ &= (1{,}200)(8.91)(990 \div 8{,}595) = 1{,}232 \end{aligned}$$

Determine Normal Cost at Entry age:

$$\begin{aligned} NC_{ea} &= (PVFB)[D_{36}/(N_{36} - N_{65})] \\ &= 1{,}232[8{,}595 \div (118{,}701 - 9{,}041)] \\ &= 1{,}232(8{,}595 \div 109{,}660) = 96.56 \end{aligned}$$

Determine Accrued Liability at attained age:

$$\begin{aligned} AL_{aa} &= NC \times (N_{36} - N_{41})/D_{41} \\ &= 96.56\ (118{,}701 - 81{,}084) \div 6{,}085 \\ &= 96.56\ (37{,}617 \div 6{,}085) = 597 \end{aligned}$$

Answer is D.

**Problem 4 - 21**

Key Concept: Under Entry Age Normal applied on an aggregate basis, the total present value of future benefits is divided by the average temporary annuity. To determine the average temporary annuity:

$$ATA = (1/2)[(N_{45} - N_{65})/D_{45} + (N_{50} - N_{65})/D_{50}]$$

| | Employees A | Employees B |
|---|---|---|
| (1) Attained Age | 50 | 55 |
| (2) Entry age | 45 | 50 |
| (3) Years of service to NRD | 20 | 15 |
| (4) Present Values at Retirement | 24,000 | 18,000 |
| (5) PVFB at Entry age = (4) × ($D_{65}$ / $D_{ea}$) | 3,528 | 4,523 |
| (6) Total PVFB = 3,528 + 4,523 = 8,051 | | |

$$ATA = (1/2)[(9{,}012 \div 1{,}000) + (4{,}945 \div 585)]$$
$$= (1/2)(17.465) = 8.7325$$

$$NC = PVFB \div ATA$$
$$= 8{,}051 \div 8.7325 = 922$$

Answer is C.

**Problem 4 - 22**

Total years of service at Normal Retirement: 20 years.
Retirement benefit: \$20 × 20 years = \$400 per month

Accrued Liability = Unfunded Accrued Liability + Assets
= 9,700 + 8,000 = 17,700

Present Value of Future Normal Costs = Normal Cost × 7.90
= 1,300 × 7.90 = 10,270

Present Value of Future Benefits = Present Value of Future Normal Costs + Accrued Liability
PVFB = 17,700 + 10,270 = 27,970

Accrued benefit cost method is now considered:

$$\text{Accrued benefit} = 10 \text{ Years of Service} \times 20 = 200$$

If the Present Value of Future Benefits of $400 per month is $27,970, then the Present Value of Future Benefits of $200 per month is $13,985. The Accrued Liability also equals $13,985.

$$\begin{aligned} \text{UAL} &= \text{Accrued Liability - Actuarial value of assets.} \\ &= 13{,}985 - 8000 = 5{,}985 \end{aligned}$$

Answer is A.

**Problem 4 - 23**

Key Concept: The value for $\ddot{a}_{65}^{(12)}$ is not given. However, it can be calculated using Smith's normal cost, which is provided.

| | Smith | Brown |
|---|---|---|
| Attained age | 35 | 34 |
| Past years of service | 5 | 9 |
| Total years of service | 35 | 40 |
| Retirement benefit | $350 | $400 |

Step I: Calculate $\ddot{a}_{65}^{(12)}$ using the Normal Cost for Smith.

$$\text{NC} = (350)(12\ddot{a}_{65}^{(12)})(v^{35}) \div \ddot{a}_{\overline{35|}}$$

$$\begin{aligned} &= (350)(12\ddot{a}_{65}^{(12)})(.0937) \div 13.854 \\ &= (2.367)(12\ddot{a}_{65}^{(12)}) = 320 \end{aligned}$$

$$12\ddot{a}_{65}^{(12)} = 320 \div 2.367 = 135.19$$

Step II: Calculate Normal cost for Brown.

$$\begin{aligned} \text{NC} &= (\text{benefit})(12\ddot{a}_{65}^{(12)})(v^{40}) \div \ddot{a}_{\overline{40|}} \\ &= (400)(135.19)(.0668) \div 14.2649 \\ &= 3{,}612 \div 14.2649 = 253 \end{aligned}$$

Answer is B.

**Problem 4 - 24**

Key Concept: It is difficult to keep up with the number of years in problems involving salary scales. In this particular problem, the benefit is determined by the final year's salary, where we are given the prior year's salary. In order to determine the final year's salary, the number of changes of salary should be counted.

| | |
|---|---|
| Attained age | 40 |
| Entry age | 35 |
| Current salary | \$30,000 |

$$\text{Projected salary} = 30{,}000 \times (1.06)^{25} = 128{,}756$$
$$\text{Projected benefit} = 50\% \text{ of } 128{,}756 = 64{,}378$$

Present Value of Future Benefits at Entry Age:

$$\begin{aligned} PVFB_{ea} &= (64{,}378)(10)(D_{65}/D_{35}) \\ &= (64{,}378\ )(10)(99/921) = 69{,}201 \end{aligned}$$

Present Value of Future Salaries and Normal Cost Percentage at Entry Age:

$$\begin{aligned} \text{Salary at Entry Age} &= (30{,}000)(1.06)^{-5} = 22{,}418 \\ PVFS_{ea} &= (22{,}418)({}^{s}N_{35} - {}^{s}N_{65})/({}^{s}D_{35}) \\ &= (22{,}418)(239{,}735 - 62{,}569) \div 7{,}077 = 561{,}213 \end{aligned}$$

$$\begin{aligned} \text{Normal Cost Percentage} &= PVFB_{ea} \div PVFS_{ea} \\ &= 69{,}201 \div 561{,}213 = 12.33\% \end{aligned}$$

Normal Cost at attained age:

$$NC_x = .1233 \times 30{,}000 = 3{,}699$$

Answer is C.

**Problem 4 - 25**

Key Concept: Changing the normal form of distribution from life only to life only with period certain has the effect of changing the retirement annuity rate which in turn affects (1) the present value of future benefits, (2) the normal cost at entry age and (3) the present value of future normal costs.

Step I: Calculate the ratio of retirement annuity rate under each option.

$$\text{Annuity rate (Life only)} = N_{65}^{(12)} / D_{65}$$

$$= 9{,}000 \div 990 = 9.091$$

$$\text{Annuity rate (10 C \& L)} = \ddot{a}_{\overline{10}|}^{(12)} + (N_{75}^{(12)} / D_{65})$$

$$= 7.287 + (2{,}400 \div 990) = 9.711$$

$$\text{Ratio} = 9.711 \div 9.091 = 1.068$$

Annuity rate for 10 C&L is 6.8% higher than the life only.

| | | Life only | | 10 C&L |
|---|---|---|---|---|
| (1) | PVFB - Actives | \$1,500,000 | × 1.068 = | \$1,602,000 |
| | - Retirees | 500,000 | | 500,000 |
| (2) | PVFB - Total | \$2,000,000 | | \$2,102,000 |
| (3) | Plan Assets | 600,000 | | 600,000 |
| (4) | Unfunded Accrued Liability | 800,000 | | |
| (5) | Present Value of Future Normal Costs (2) - (3) - (4) | 600,000 | × 1.068 = | 640,800 |
| (6) | Unfunded Accrued (2) - (3) - (5) | ---- | | 861,200 |
| (7) | Total Amount of increase 861,200 - 800,000 = 61,200 | | | |

Answer is C.

**Problem 4 - 26**

Key Concept: Since Smith has no past years of service, his accrued liability is zero. Hence, we need consider only Brown and Green.

| | Age | Years of Service Past | Future | Total | Benefit |
|---|---|---|---|---|---|
| Brown | 40 | 10 | 25 | 35 | \$350 |
| Green | 50 | 20 | 15 | 35 | 350 |

$$\text{Normal Cost} = (\text{Benefit})(12\ddot{a}_{65}^{(12)})(D_{65}) \,/\, (N_{ea} - N_{65})$$

Since Brown and Green have the same number of years of total service, the Normal Cost for each will be the same.

$$\text{Normal Cost} = (350)(12)(8)(55) \div (12{,}570 - 465) = 152.66$$
$$\text{Accrued Liability} = (\text{Normal Cost})(N_{ea} - N_x)/D_x$$
$$\text{AL (Brown)} = (152.66)(12{,}570 - 5{,}485) \div 450 = 2{,}404$$
$$\text{AL (Green)} = (152.66)(12{,}570 - 2{,}255) \div 200 = 7{,}873$$
$$\text{AL (Total)} = 2{,}404 + 7{,}873 = 10{,}277$$

Answer is B.

**Problem 4 - 27**

Key Concept: Since the assumed retirement age is 60, the 6% reduction must be used throughout the problem, as well as counting years of service to age 60 only.

Step I: Determine retirement benefit.

$$\begin{aligned}\text{Ben} &= (10 \times \text{years of service})[1 - (.06)(5)] \\ &= (10 \times 20)(1 - .30) = 140\end{aligned}$$

Step II: Determine Accrued Liability.

$$\begin{aligned}\text{AL} &= \text{Ben }(12\ddot{a}_{60}^{(12)}) \times (D_{60}/D_{50}) \times [(N_{40} - N_{50})/(N_{40} - N_{60})] \\ &= (140)(12 \times 9.79)(151/322)[(8{,}761 - 3{,}902) \div (8{,}761 - 1{,}547)] \\ &= (140)(117.48)(.4689)[4{,}859/7{,}214] = 5{,}194\end{aligned}$$

Answer is B.

**Problem 4 - 28**

Key Concept: In problems involving salary scales, it is necessary to count the number of salary changes. In this problem, for example, the participant's attained age is 50 but since his rate of salary for 1988 is given, there are only 14 salary changes until his normal retirement age. Note that if his actual annual salary for 1987 had been given, there would have been 15 salary changes until his normal retirement age.

(1) Projected salary at NRA = $(50{,}000)(1.05)^{14} = 98{,}997$

To determine the average salary over the final n years preceding retirement, use the formula

Final Average Compensation = (Final Compensation)($\ddot{a}_{\overline{n}|5\%}/n$) = (98,997)(.95314) = 94,358

(2) Projected Benefit at Normal Retirement = 47,179

(3) Present Value of Future Benefits at Entry Age = $(47{,}179)(9)(1.06)^{-30} = 73{,}929$

(4) Salary at Entry Age = $(50{,}000)(1.05)^{-15} = 24{,}051$

The Present Value of Future Salaries where each annual salary is discounted by *d* percent while increasing each year by *g* percent is the equivalent of a discount of $[(1+d) / (1+g)] - 1$. In this problem $(1.06 / 1.05) - 1 = .952\% = j$.

(5) PVFS = $(24{,}051)(\ddot{a}_{\overline{30}|j}) = 630{,}993$

(6) Normal Cost Percent = (3) / (5) = 11.72%

(7) Normal Cost as of 1/1/88 = 50,000 × 11.72% = 5,860

Answer is B.

**Problem 4 - 29**

Key Concept: A Life Annuity with n years certain can be regarded as consisting of two parts:

(i) an Annuity Certain of n payments; and
(ii) a Deferred Annuity commencing in n years and continuing for life.

These two components should be treated separately to determine any experience gain or loss. For the purpose of this problem, part (i) (the annuity for ten years certain) can be ignored since the monthly payments will continue if Smith is living or dead. Hence, there can be no experience gain from this source if Smith dies.

Considering part (ii) (the deferred annuity commencing at age 75) only:

Step I: Determine the actual Accrued Liability at 1/1/88.

AL (Deferred Annuity) = 0 (no payments will be made)

Step II: Determine expected Accrued Liability at 1/1/88.

$e\text{AL}_{70}$ (Deferred Annuity) = $(\text{AL}_{70})(p_{69})$

$p_{69}$ is not given; however, it can be calculated from the relationship between $D_{69}$ and $D_{70}$.

$$D_{70}/D_{69} = vp_{69}$$

$$p_{69} = (D_{70}) / (vD_{69})$$
$$= (1{,}227)(1.06)/1{,}344 = .9677$$

$$\text{AL}_{70}(\text{Deferred Annuity}) = (1{,}000)(12)(N_{75}^{(12)} / D_{70})$$
$$= (1{,}000)(12)(5{,}006 / 1{,}227) = 48{,}958$$

All information to determine the expected accrued liability at age 70 is now available.

$$e\text{AL}_{70} = (48{,}958)(.9677) = 47{,}377$$

Therefore, the experience gain = 47,377 - 0 = 47,377

Answer is D.

**Problem 4 - 30**

Key Concepts: The problem requires that a distinction be made between the Entry Age Normal Method, aggregate basis, and the Entry Age Normal, individual basis. To determine the Normal Cost under the individual basis, the present value of the projected benefit at Entry Age (or the Present Value of Future Normal Costs) is divided by $\ddot{a}_{\overline{ra-ea}|}$ for each participant. Summation of these individual normal costs provides the total Normal Cost for the plan.

Using the aggregate approach, the Present Value of Future Normal Costs at Entry Age is summed and then divided by the average of $\ddot{a}_{\overline{ra-ea}|}$. Under this approach, the

Normal Cost changes each year as the Present Value of Future Normal Costs is divided by the average of the appropriate $\ddot{a}_{\overline{ra-ea}|}$'s. The Accrued Liability is determined by the relationship:

$$PVFB = \text{Accrued Liability} + PVFNC$$

| Data as of 1/1/88: | Smith | Brown |
|---|---|---|
| Age | 35 | 45 |
| Past Service | 10 | 10 |
| Future Service | 30 | 20 |
| Projected Pension | 400 | 300 |
| Accrued Benefit | 100 | 100 |

Step 1: Calculate Present Value of Future Benefits at Entry Age.

$$PVFB_{ea} = (\text{Benefit})(12\ddot{a}_{65}^{(12)})(D_{65}/D_{ea})$$

$$\begin{aligned} PVFB(\text{Smith}) &= (400)(12)(9.35)(189 / 2{,}441) = 3{,}475 \\ PVFB(\text{Brown}) &= (300)(12)(9.35)(189 / 1{,}348) = 4{,}719 \\ \text{Total } PVFB &= 3{,}475 + 4{,}719 = 8{,}194 \end{aligned}$$

Step II: Calculate Normal Cost.

$$\text{Normal Cost} = PVFB / (\ddot{a}_{25:\overline{40}|} + \ddot{a}_{35:\overline{30}|}) \times 2$$

$$= 8{,}194/(15.62 + 14.22) \times 2 = 549.20$$

Step III: Determine Accrued Liability at 1/1/88

$$AL_{88} = PVFB_{88} - PVFNC_{88}$$

$$\begin{aligned} PVFB(\text{Smith}) &= (400)(12)(9.35)(189 / 1{,}348) = 6{,}293 \\ PVFB(\text{Brown}) &= (300)(12)(9.35)(189 / 737) = 8{,}632 \\ PVFB\ (\text{Total}) &= 6{,}293 + 8{,}632 = 14{,}925 \end{aligned}$$

$$PVFNC_{88} = (\text{Normal Cost})(\ddot{a}_{35:\overline{30}|} + \ddot{a}_{45:\overline{20}|}) / 2$$

$$\begin{aligned} &= (549.20)(14.22 + 11.84) / 2 = 7{,}156 \\ AL_{88} &= 14{,}925 - 7{,}156 = 7{,}769 \end{aligned}$$

Answer is C.

As an additional exercise, it might be helpful to solve the problem using the individual approach. In this case,

$$\text{NC (Smith)} = 3{,}475 \,/\, 15.62 = 222.47$$
$$\text{NC (Brown)} = 4{,}719 \,/\, 14.22 = 331.86$$

and

$$\text{PVFB (Smith)} = (400)(12)(9.35)(189 \,/\, 1{,}348) = 6{,}293$$
$$\text{PVFB (Brown)} = (300)(12)(9.35)(189 \,/\, 737) = 8{,}632$$

$$\text{PVFNC (Smith)} = (222.47)(14.22) = 3{,}164$$
$$\text{PVFNC (Brown)} = (331.86)(11.84) = 3{,}929$$

$$\text{Accrued Liability} = (6{,}293 + 8{,}632) - (3{,}164 + 3{,}929) = 7{,}832$$

The difference in the two answers is often insignificant. In this case, the difference is significant because it would knock you out of a correct answer.

**Problem 4 - 31**

The retirement benefit is calculated as follows:

$$\text{Final Average Salary} = 10{,}000 \times \left( \frac{(1.04)^{32} + (1.04)^{33} + (1.04)^{34}}{3} \right)$$

$$= 10{,}000 \left( \frac{3.5081 + 3.64838 + 3.7943}{3} \right) = 36{,}503$$

$$\text{Benefit} = (.5)(36{,}503) = 18{,}252$$

The Normal Cost can now be calculated:

$$NC_{ea} = \frac{PVFB_{25}}{PVFS_{25}} = \text{Benefit} \times \ddot{a}_{65}^{(12)} \times \frac{\left(\dfrac{D_{65}}{D_{25}}\right)}{\left(\dfrac{{}^sN_{25} - {}^sN_{65}}{{}^sD_{25}}\right)}$$

$$= \frac{(18{,}252) \times (1{,}738/22{,}499) \times (9.345)}{(1{,}920{,}504 - 296{,}192)/(59{,}979)} = \frac{13{,}176}{27.0813} = 487$$

Since the Normal Cost must remain the same percentage of salary, the Normal Cost at entry age 25 can be accumulated for five years to age 30.

$$NC_{89} = NC_{ea} \times (1.04)^5 = 487 \times (1.04)^5 = 593$$

Answer is B.

**Problem 4 - 32**

Key Concept: The gain or loss due to early retirement is equal to the difference between the actual liability and the accrued liability had the participant not retired.

Step I: Determine the actual liability.

$$\text{Early Retirement Benefit} = (15)(21 \text{ years of service})(\text{early retirement factor})$$
$$= (15)(21)[1-(.02)(6 \text{ years})] = 277.20$$

$$\text{Actual AL} = 277.20 \times 12\ddot{a}_{59}^{(12)} = 277.20 \times 12 \times \frac{N_{59}^{(12)}}{D_{59}}$$

$$= 277.20 \times 12 \times \left(\frac{3{,}019}{275}\right) = 36{,}518$$

Step II: Determine the expected liability.

Assumed Benefit at age 60 = (15) × (22 years of service) × (early retirement factor)

$$= (15) \times (22) \times [1 - (.02)(5 \text{ years})] = 297$$

Under Entry Age Normal,

$$NC = \frac{PVFB_{ea}}{\ddot{a}_{\overline{ra-ea}|}} = \frac{(297)(12\ddot{a}_{60}^{(12)})v^{22}}{\ddot{a}_{\overline{22}|}} = 820$$

$$AL = NC \times \ddot{s}_{\overline{aa-ea}|} = 820 \times \ddot{s}_{\overline{21}|} = 34{,}762$$

Step III: Calculate the Gain/Loss.

$$\text{Loss} = \text{Actual AL-Expected AL}$$
$$= 36{,}518\text{-}34{,}762 = 1{,}756$$

Answer is A.

**Problem 4 - 33**

$$\text{Final Salary} = (30{,}000)(1.05)^{10} = 48{,}867$$
$$\text{Projected Benefit} = (48{,}867)(.01)(30 \text{ years of service}) = 14{,}660$$
$$PVFB_{ea} = (14{,}660)(\ddot{a}_{65}^{(12)})(v^{30}) = 19{,}258$$

Since the salary scale is 5% per year, assume the salary decreases 5% per year back to entry age.

$$\text{Salary}_{ea} = 30{,}000/(1.05)^{20} = 11{,}307$$

$$PVFS_{ea} = 11{,}307\ddot{a}_{\overline{30}|j}, \text{ where } j = \frac{1+i}{1+s} - 1 = \frac{1.07}{1.05} - 1 = .0190476$$

$$PVFS_{ea} = 261{,}472$$

$$NC_{ea} = \left(\frac{PVFB_{ea}}{PVFS_{ea}}\right) \times \text{Salary}_{ea} = \left(\frac{19{,}258}{261{,}472}\right) \times 11{,}307 = 832$$

Since Normal Cost increases as salary increases,

$$NC_{aa} = NC_{ea} \times (1+s)^{(aa-ea)}$$
$$= 832 \times (1.05)^{20} = 2{,}208$$

$$PVFB_{aa} = PVFB_{ea} \times (1.07)^{20} = 19{,}258 \times (1.07)^{20} = 74{,}522$$

$$\begin{aligned} AL &= PVFB_{aa} - PVFNC \\ &= 74{,}522 - (2{,}208)(\ddot{a}_{\overline{10}|j}) \\ &= 74{,}522 - (2{,}208)(9.1994) = 54{,}210 \end{aligned}$$

Answer is B.

**Problem 4 - 33 (Alternative Solution)**

A retrospective approach can be used as follows:

$$AL = NC_{aa} \times \ddot{s}_{\overline{aa-ea}|} = (2{,}208)(24.5267) = 54{,}155$$

The difference is due to rounding.

**Problem 4 - 34**

Key Concept: Since there is no pre-retirement death benefit, the actual liability for Green is zero. Therefore, the experience gain is equal to the expected Accrued Liability under the Entry Age Normal Method.

Projected Monthly Benefit for Green = (20)(15 Years of Service) = 300

$$\begin{aligned} EAN\ NC_{88} &= (300)(12\ddot{a}_{65}^{(12)})/[(N_{50} - N_{65})/D_{65}] \\ &= (300(12)(9.35)/[(6{,}723 - 1{,}745)/178] = 1{,}204 \end{aligned}$$

$$\begin{aligned} EAN\ AL_{88} &= 1{,}204 \times (N_{50} - N_{60})/D_{60} \\ &= 1{,}204 \times (6{,}723 - 2{,}870)/260 = 17{,}842 \end{aligned}$$

$$\begin{aligned} eAL_{89} &= (AL_{88} + NC_{88})(1.06) \\ &= (17{,}842 + 1{,}204)(1.06) = 20{,}189 \end{aligned}$$

Answer is C.

**Problem 4 - 35**

Step I: Calculate the Normal Cost at Entry Age.

$$PVFB_{ea} = 110{,}000 \times v^{17} = 40{,}850$$

$$NC_{ea} = 40{,}850/\ddot{a}_{\overline{32}|j} \quad \text{where } j = 1.06/1.03 - 1 = .0291262$$
$$= 1{,}924$$

Step II: Calculate the Normal Cost as of 1/1/89.

$$NC_{89} = (NC_{ea})(1.03)^{17} = (1{,}924)(1.03)^{17} = 3{,}180$$

Step III: Calculate the Accrued Liability as of 1/1/89.

$$AL_{89} = NC_{89} \times \ddot{s}_{\overline{17}|j} = 3{,}180 \times \ddot{s}_{\overline{17}|j} = 70{,}694$$

Answer is B.

**Problem 4 - 36**

Key Concept: The Normal Cost after the amendment is equal to the Normal Cost before the amendment increased by 25%.

Before the amendment,

$$NC = (PVFB - AL)/(PVFS/Salary)$$
$$= [4{,}100 - (800 + 1{,}000)]/(46{,}000/4{,}000) = 200$$

After the amendment,

$$NC = 200 \times 1.25 = 250$$

Answer is E.

**Problem 4 - 37**

Step I: Calculate the benefit increases.

Smith: $2 \times 12 \times 40$ years of service = 960 per year
Brown: $2 \times 12 \times 30$ years of service = 720 per year

Step II: Calculate the normal cost increases.

Smith: $(960/1{,}000) \times 42.13 = 40.44$
Brown: $(720/1{,}000) \times 89.04 = 64.11$

Step III: Calculate the accrued liability increases.

Smith: $40.44 \times \ddot{s}_{\overline{5}|} = 40.44 \times 6.1533 = 248.84$

Brown: $64.11 \times \ddot{s}_{\overline{5}|} = 64.11 \times 6.1533 = 394.49$

Total Increase $= 248.84 + 394.49 = 643.33$

Answer is E.

**Problem 4 - 38**

I. The retrospective formula for Accrued Liability under Entry Age Normal is:

$$NC \times (N_{ea} - N_{aa})/D_{aa} = P \times (N_{30} - N_{45})/D_{45}$$

True.

II. The prospective formula for Accrued Liability under Entry Age Normal is:

$$\text{PVFB}_{aa} - \text{PVFNC}_{aa} = \text{B}\left(\frac{D_{65}}{D_{45}}\right)\left(\frac{N_{65}^{(12)}}{D_{65}}\right) - \text{P}\left(\frac{N_{45} - N_{65}}{D_{45}}\right)$$

$$II$$

$$= \text{B}\,\frac{N_{65}^{(12)}}{D_{45}} - \text{P}\left(\frac{N_{45} - N_{65}}{D_{45}}\right)$$

True.

III. The Normal Cost under Entry Age Normal is:

$$\text{NC} = (\text{Benefit})(\ddot{a}_{ra}^{(12)})(D_{ra}/D_{ea})/[(N_{ea} - N_{ra})/D_{ea}]$$

$$\text{Substituting,}\quad \text{P} = \frac{\text{B}\ \dfrac{D_{65}}{D_{30}} \times \dfrac{N_{65}^{(12)}}{D_{65}}}{\dfrac{N_{30} - N_{65}}{D_{30}}} = \frac{\text{B} \times N_{65}^{(12)}}{N_{30} - N_{65}}$$

Substituting into statement I,

$$\text{AL} = \text{B}\left(\frac{N_{65}^{(12)}}{N_{30} - N_{65}}\right)\left(\frac{N_{30} - N_{45}}{D_{45}}\right)$$

$$= \text{B}\left(\frac{N_{65}^{(12)}}{D_{45}}\right)\left(\frac{N_{30} - N_{45}}{N_{30} - N_{65}}\right)$$

True.

Answer is D.

**Problem 4 - 39**

Key Concept: Use commutation functions without salary scale for calculating present values and commutation functions with salary scale for amortizing.

Final year's compensation = 25,000/.2314 = 108,038

Benefit = .5 × 108,038 = 54,019

Under Entry Age Normal with salary scale, the normal cost rate per participant is a <u>rate</u> which is applied to future expected or actual salary. The equation of value is:

NC Rate × PVFS = PVFB

NC Rate at Entry Age = $PVFB_{ea}/PVFS_{ea}$

$$= \frac{54{,}019 \times \ddot{a}_{65}^{(12)} \times \dfrac{D_{65}}{D_{25}}}{\text{Salary}_{25} \times \dfrac{{}^sN_{25} - {}^sN_{65}}{{}^sD_{25}}}$$

$$= \frac{54{,}019 \times 8.736 \times \dfrac{94{,}414}{1{,}779{,}168}}{25{,}000 \times (\dfrac{.142}{.2314})\left[\dfrac{193{,}660{,}240 - 30{,}013{,}858}{6{,}024{,}894}\right]} = \frac{25{,}043}{416{,}699} = .0600986$$

$$AL_{90} = \text{Salary} \times \text{NC rate} \times \ddot{s}^{(s)}_{25:\overline{10|}}$$

$$= 25{,}000 \times .0600986 \times \left[\frac{{}^sN_{25} - {}^sN_{35}}{{}^sD_{35}}\right]$$

$$= 1{,}502.47 \times \left[\frac{193{,}660{,}240 - 138{,}500{,}016}{4{,}932{,}364}\right] = 16{,}803$$

Answer is B.

**Problem 4 - 39 (Alternative Solution)**

$$\text{PVFB} = 54{,}019 \times \ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{35}} = 49{,}827$$

$$\text{NC}_{25} = 54{,}019 \times \ddot{a}_{65}^{(12)} \times \frac{\dfrac{D_{65}}{D_{25}}}{\dfrac{{}^{s}N_{25} - {}^{s}N_{65}}{{}^{s}D_{25}}} = 922$$

$$\text{NC}_{35} = 922 \times \frac{S_{35}}{S_{25}} = 1{,}502$$

$$\text{PVFNC} = 1{,}502 \times \frac{{}^{s}N_{35} - {}^{s}N_{65}}{{}^{s}D_{35}} = 33{,}036$$

$$\text{AL} = \text{PVFB} - \text{PFVNC} = 49{,}827 - 33{,}036 = 16{,}791$$

**Problem 4 - 40**

$$\text{Final salary} = 25{,}000 \times (1.05)^{15} = 51{,}973$$

$$\text{Benefit} = .01 \times 20 \times 51{,}973 = 10{,}395$$

$$\text{PVFB}_{\text{ea}} = 10{,}395 \times \ddot{a}_{65}^{(12)} \times v^{20} = 10{,}395 \times 8.735 \times .258419 = 23{,}465$$

$$\text{NC}_{\text{ea}} = \frac{\text{PVFB}_{\text{ea}}}{\ddot{a}_{\overline{20|}j}}, \text{ where } 1+j = \frac{1+i}{1+s} = \frac{1.07}{1.05} = 1.019$$

$$= \frac{23{,}465}{16.824} = 1{,}395$$

The Normal Cost increases 5% per year to remain a level percentage of salary.

$$NC_{90} = NC_{ea} \times (1.05)^{aa-ea}$$
$$= 1{,}395 \times (1.05)^5 = 1{,}780$$

Answer is D.

**Problem 4 - 41**

Key concept: Experience gains or losses are calculated before the effects of plan amendments or assumption changes are taken into account.

$$eUAL_{91} = (AL_{90} - Assets_{90} + NC_{90})(1 + i) - Contribution_{90}\ (1 + \tfrac{1}{2}i)$$

$$= (800{,}000 - 420{,}000 + 50{,}000)\ (1.07) - 80{,}000 \left(1 + \frac{.07}{2}\right) = 377{,}300$$

Note that since the contribution was deposited midway through the year, it is credited with only one-half year's interest.

$AL_{91}$ (Post Amendment) = 890,000

$AL_{91}$ (Pre-Amendment) = (12/12.5) × (890,000) = 854,400

Note that the ratio (12/12.5) can be used since the benefit was changed for all participants.

$$UAL_{91} = AL_{91} - Assets_{91}$$

$$= 854{,}400 - 500{,}000 = 354{,}400$$

Experience Gain = 377,300 - 354,400 = 22,900

Answer is C.

Note: The benefit payments for 1990 were given as information but not used in solving this problem. It would have been used if the question were to ask about the <u>actual</u> return on plan assets during the year. The relationship is as follows:

$$Assets_{91} = Assets_{90} + Contribution_{90} - Benefit\ Payments_{90} + Interest_{90}$$

$$500{,}000 = 420{,}000 + 80{,}000 - 15{,}000 + Interest_{90}$$

$\text{Interest}_{90} = 15{,}000$

This is to be compared to the expected interest, in determining the investment gain/loss.

$$\begin{aligned}\text{Expected Interest} &= \text{Expected Prior Asset Interest} \\ &\quad + \text{Expected Contribution Interest} \\ &\quad - \text{Expected Benefit Payment Interest} \\ &= 420{,}000\,(.07) + 80{,}000(.07/2) - 15{,}000(.07/2) = 31{,}675\end{aligned}$$

Therefore, investment loss = 31,675 - 15,000 = 16,675
and Non-investment gain = Total Gain + Investment Loss = 22,900 + 16,675 = 39,575

**Problem 4 - 42**

Step I: Calculate the individual EAN Normal Cost

For the participants at age 40, with retirement benefits of \$350 per month

$$NC_{EAN} = 2 \times \frac{PVFB_{30}}{\ddot{a}_{30:\overline{35|}}} = 2 \times \frac{\left(350 \times 12 \times \ddot{a}_{65}^{(12)} \times \dfrac{D_{65}}{D_{30}}\right)}{\dfrac{N_{30} - N_{65}}{D_{30}}}$$

$$= 2 \times \frac{12 \times 350 \times \left(\dfrac{873}{1336}\right)}{\dfrac{18946 - 919}{1336}} = 2 \times \frac{(2744.46)}{(13.493)} = 407$$

For the participants at age 60, with retirement benefits of $150 per month

$$NC_{EAN} = 2 \times \frac{PVFB_{50}}{\ddot{a}_{50:\overline{15|}}} = 2 \times \frac{\left(150 \times 12 \times \ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{50}}\right)}{\frac{N_{50} - N_{65}}{D_{50}}}$$

$$= 2 \times \frac{\left(12 \times 150 \times \left(\frac{873}{329}\right)\right)}{\left(\frac{3{,}974 - 919}{329}\right)} = 2 \times \frac{(4776.29)}{(9.2857)} = 1{,}029$$

Total Individual $NC_{EAN} = 407 + 1{,}029 = 1{,}436$

Step II: Calculate the Aggregate EAN Normal Cost.

$$NC_{EAN} = \text{Number of Participants} \times \frac{\sum PVFB_{ea}}{\sum \ddot{a}_{ea:\overline{ra-ea|}}}$$

$$= 4 \times \left(\frac{2 \times 2744.46 + 2 \times 4776.29}{2 \times 13.493 + 2 \times 9.2857}\right) = 1{,}321$$

Step III: Calculate the difference in the Normal Costs.

Difference = 1,436 - 1,321 = 115

Answer is B.

**Problem 4 - 43**

$$\text{Benefit} = (.5)(28{,}571)\left[\frac{(1.05)^{18} + (1.05)^{19} + (1.05)^{20}}{3}\right] = 36{,}127$$

Salary at age 35 = $28{,}571/(1.05)^{10} = 17{,}540$

$$\text{Normal Cost Rate at 35} = \frac{36{,}127 \times \ddot{a}_{65}^{(12)} \times \dfrac{D_{65}}{D_{35}}}{17{,}540 \times \dfrac{{}^{s}N_{35} - {}^{s}N_{65}}{{}^{s}D_{35}}}$$

$$= \frac{33{,}200}{385{,}894} = .0860$$

$$\text{Thus, } AL_{92} = (.0860)\ (28{,}571) \left[\frac{{}^{s}N_{35} - {}^{s}N_{45}}{{}^{s}D_{45}}\right] = 27{,}673$$

$$NC_{92} = (.0860)\ (28{,}571) = 2{,}457$$

$$e\text{AL}_{93} = \left(AL_{92} + NC_{92}\right)(1.07) = (27{,}673 + 2{,}457)\ (1.07) = 32{,}239$$

Answer is B.

**Problem 4 - 44**

Step I: Under the old basis, the Normal Retirement Age is age 62.

Benefit = 10 × 17 years of service = 170

$$NC_{ea} = \frac{PVFB_{45}}{\ddot{a}_{\overline{17|}}} = 170 \times 12\ddot{a}_{62}^{(12)} \times \frac{v^{17}}{\ddot{a}_{\overline{17|}}}$$

$$= \frac{6{,}064.17}{10.4466} = 580$$

$$AL_{92} = NC_{ea} \times \ddot{s}_{\overline{12|}} = 580 \times 19.1406 = 11{,}102$$

Step II: Under the new basis, the Normal Retirement Age is age 65.

Benefit = 10 × 20 years of service = 200

$$NC_{ea} = \frac{200 \times 12\ddot{a}_{65}^{(12)} \times v^{20}}{\ddot{a}_{\overline{20|}}} = \frac{5{,}420.60}{11.3356} = 478$$

$$AL_{92} = NC_{ea} \times \ddot{s}_{\overline{12|}} = 478 \times 19.1406 = 9{,}149$$

Step III: Calculate the change in the Accrued Liability.

AL decrease = 11,102 - 9,149 = 1,953

Answer is B.

**Problem 4 - 45**

Step I: Calculate the Accrued Liability under the Unit Credit Cost Method.

$$AL = PVAB = 5 \text{ years of service} \times 50 \times 12 \times \ddot{a}_{65}^{(12)} \times v^{15} = 9{,}503$$

Step II: Calculate the Accrued Liability under the Entry Age Normal Cost Method.

Retirement Benefit = 50 × 20 years of service = 1,000

$$NC_{EAN} = \frac{PVFB_{ea}}{\text{Temporary annuity ea to ra}} = \frac{1000 \times 12\ddot{a}_{65}^{(12)} \times v^{20}}{\ddot{a}_{\overline{20|}}}$$

$$AL_{EAN} = NC_{EAN} \times \ddot{s}_{\overline{aa-ea|}} = \frac{1{,}000 \times 12\ddot{a}_{65}^{(12)} \times \ddot{s}_{\overline{5|}}}{\ddot{s}_{\overline{20|}}}$$

$$= \frac{1000 \times 12 \times 8.74 \times 6.15329}{43.86517} = 14{,}712$$

Step III: Calculate the change in the Accrued Liability.

Increase in AL = 14,712 - 9,503 = 5,209

Answer is D.

**Problem 4 - 46**

Step I: Determine benefits.

| | |
|---|---|
| Smith: | Benefit = (.5)(2,500) = 1,250 |
| Brown: | Benefit = (.5)(1,500) = 750 |

Step II: Determine Normal Costs.

Normal Cost = (Benefit)($12\ddot{a}_{65}^{(12)}$)/$\ddot{s}_{\overline{ra-ea|}}$

$$\text{Smith NC} = (1{,}250)(12)(10)/\ddot{s}_{\overline{29}|} = 1{,}605$$

$$\text{Brown NC} = (750)(12)(10)/\ddot{s}_{\overline{25}|} = 1{,}329$$

Step III: Determine Accrued Liability

$$\text{Accrued Liability} = \text{NC} \times \ddot{s}_{\overline{aa\text{-}ea}|}$$

$$\text{Smith AL} = 1{,}605 \times \ddot{s}_{\overline{1}|} = 1{,}717$$

$$\text{Brown AL} = 1{,}329 \times \ddot{s}_{\overline{17}|} = 43{,}856$$

$$\text{Total AL} = 1{,}717 + 43{,}856 = 45{,}573$$

Answer is C.

**Problem 4 - 47**

Key Concept: Gains/losses due to early retirement are computed by comparing the actual PVFB with the Accrued Liability under the method had early retirement not been elected.

Projected normal retirement benefit = 10 × 17 years of service = 170
Projected early retirement benefit = 170 × [1 - (.06)(3 years)] = 139.40
Accrued normal retirement benefit = 10 × 13 years of service = 130
Accrued early retirement benefit = 130 × [1 - (.06)(7 years)] = 75.40

$$\text{Expected Accrued Liability} = (139.40)(12\ddot{a}_{62}^{(12)}) \times \frac{\ddot{s}_{\overline{13}|}}{\ddot{s}_{\overline{17}|}} = 10{,}258$$

$$\text{Actual Accrued Liability} = (75.40)(12\ddot{a}_{58}^{(12)}) = 9{,}247$$

There is a gain since the expected liability exceeds the actual liability.

$$\text{Gain} = 10{,}258 - 9{,}247 = 1{,}011$$

Answer is B.

**Problem 4 - 48**

Key Concept: The mortality gain/loss is determined by comparing the expected Accrued Liability with the actual Accrued Liability.

$$AL_{92} = 1{,}500 \times \ddot{s}_{30:\overline{10|}} = 1{,}500 \times (\frac{N_{30} - N_{40}}{D_{40}}) = 22{,}393$$

$$eAL_{92} = (AL_{92} + NC_{92}) \times (1.07) = (22{,}393 + 1{,}500) \times (1.07) = 25{,}566$$

Since the sole participant is active as of 1/1/93, the actual Accrued Liability must include an accumulation from 1992 for mortality. Therefore,

$$\begin{aligned} \text{Actual } AL_{93} &= (AL_{92} + NC_{92}) \times D_{40}/D_{41} \\ &= (22{,}393 + 1{,}500) \times D_{40}/D_{41} = 25{,}594 \end{aligned}$$

Mortality Loss = 25,594 - 25,566 = 28

Answer is B.

**Problem 4 - 49**

Final 5-year average compensation:

$$= (40{,}000) \times [(1.03)^{20} + (1.03)^{21} + (1.03)^{22} + (1.03)^{23} + (1.03)^{24}]/5 = 76{,}711$$

Final 3-year average compensation:

$$= (40{,}000) \times [(1.03)^{22} + (1.03)^{23} + (1.03)^{24}]/3 = 78{,}966$$

Since this Entry Age Normal method spreads Normal Cost as a level dollar amount, the salary scale is used <u>only</u> to determine the projected benefit.

Old plan Accrued Liability

$$= (100 \text{ participants})(76{,}711)(.4) \times \ddot{a}_{65}^{(12)} \times \frac{\ddot{s}_{\overline{13|}}}{\ddot{s}_{\overline{38|}}} = 3{,}128{,}678$$

The new plan provides a fully subsidized 100% Joint & Survivor benefit for married participants. Since it is assumed that 80% of the participants are married, this is the same as assuming that 80% of each participant is married and 20% is unmarried.

The maturity value associated with this assumption is:

$$= (.2)(\ddot{a}_{65}^{(12)}) + (.8)(\ddot{a}_{\overline{65:65}}^{(12)})$$

The Accrued Liability under the new plan is

$$= (100 \text{ participants})(78{,}966)(.5)[.2\ddot{a}_{65}^{(12)} + .8\ddot{a}_{\overline{65:65}}^{(12)}] \times \frac{\ddot{s}_{\overline{13|}}}{\ddot{s}_{\overline{38|}}} = 4{,}704{,}153$$

Increase = 4,704,153 - 3,128,678 = 1,575,475

Answer is B.

**Problem 4 - 50**

Key Concept: Accrued Liability under the Entry Age Normal Cost Method depends only upon the benefit formula in effect for the current plan year.

Accrued Liability under the 1/1/90 formula,

$$= (2{,}000)[(N_{35} - N_{38})/D_{38}]$$

$$= (2{,}000)((D_{35}+D_{36}+D_{37})/D_{38}) = 6{,}900$$

Accrued Liability under new formula,

$$= 6{,}900 \times (30/15) = 13{,}800$$

Answer is C.

**Problem 4 - 51**

Key Concept: Any increase in salary in excess of that expected will increase the Accrued Liability in the Entry Age Normal method by the percentage increase in compensation in excess of that expected.

$$\text{Expected Final Salary} = (60{,}000)(1.05)^{20} = 159{,}198$$

$$e\text{AL} = (159{,}198)(\ddot{a}_{65}^{(12)}) \times \left(\frac{\ddot{s}_{\overline{15|}}}{\ddot{s}_{\overline{35|}}}\right) = 252{,}814$$

Actual Salary Increase Percentage = 66,000/60,000 = 1.1

$$\text{Actual AL} = 252{,}814 \times (1.1/1.05) = 264{,}853$$

$$\text{Experience Loss} = e\text{AL} - \text{AL} = 252{,}814 - 264{,}853 = (12{,}039) \quad \text{(loss)}$$

Answer is D.

**Problem 4 - 51 (Alternative Solution)**

Calculate the increase in Accrued Liability associated with the increase in final salary.

$$\text{Salary}_{93} = (66{,}000)(1.05)^{19} = 166{,}779$$

Calculate the (gain)/loss associated with the increase in final salary.

$$(166{,}779 \quad -159{,}198) \times \ddot{a}_{65}^{(12)} \times \left(\frac{\ddot{s}_{\overline{15|}}}{\ddot{s}_{\overline{35|}}}\right) = 12{,}039 \ (\mathit{loss})$$

**Problem 4 - 52**

Key Concept: The experience gain or loss is the difference between the Accrued Liability under the funding method had Smith remained active and the actual liability due to Smith's termination.

$$\text{EAN AL} = (20)(17 \text{ years of service})(12\ddot{a}_{65}^{(12)})\left(\frac{\ddot{s}_{\overline{5|}}}{\ddot{s}_{\overline{17|}}}\right) = 6{,}649$$

$$\text{Actual AL} = (20)(5 \text{ years of service})(12\ddot{a}_{65}^{(12)})v^{12} = 4{,}657$$

Since the expected liability under the funding method exceeds the actual liability, there is a gain.

Gain = 6,649 - 4,657 = 1,992

Answer is B.

It is interesting to note that had the funding method been Unit Credit, there would have been no gain or loss since the Accrued Liability under the Unit Credit method is the present value of the Accrued Benefit.

**Problem 4 - 53**

Key Concept: In problems involving salary scales, it is necessary to count the number of salary changes. In this problem, the participant's attained age is 34. Since his 1994 valuation compensation is given, there will only be 30 salary changes before retirement, with the last change occurring on the valuation date as of 1/1/2024.

Step I: Calculate the Final Average Salary.

$$\text{Final Average Salary} = (25{,}000)\left[(1.05)^{30} + (1.05)^{29} + (1.05)^{28} + (1.05)^{27} + (1.05)^{26}\right]/5$$
$$= 98{,}237$$

Note that other formulas could be used to determine Final Average Salary. For example,

$$\text{Final Average Salary} = (25{,}000)(\ddot{s}_{\overline{30|}} - \ddot{s}_{\overline{25|}})/5 = 98{,}237$$

$$\text{Final Average Salary} = (\text{Final Salary})(\ddot{a}_{\overline{5|}}/5)$$
$$= (25{,}000)(1.05)^{30}(\ddot{a}_{\overline{5|}}/5) = 98{,}237$$

Step II: Calculate the increase in the retirement benefit.

$$\text{Increase} = (.1)(98{,}237) = 9{,}823.70$$

Step III: Determine the increase in the Normal Cost at entry age.

$$\begin{aligned} PVFB_{30} &= (9{,}823.70)(\ddot{a}_{65}^{(12)})(D_{65}/D_{30}) \\ &= (9{,}823.70)(8.776)(94/1{,}262) = 6{,}422 \end{aligned}$$

$$\begin{aligned} NC_{30} &= PVFB_{30} \div [({}^sN_{30} - {}^sN_{65})/{}^sD_{30}] \\ &= 6{,}422 \div [(164{,}704 - 30{,}013)/5{,}454] = 260 \end{aligned}$$

Step IV: Determine the increase in the Accrued Liability.

$$\begin{aligned} AL_{34} &= NC_{30} \times (1.05)^4 \times [({}^sN_{30} - {}^sN_{34})/{}^sD_{34}] \\ &= 260 \times 1.2155 \times [(164{,}704 - 143{,}532)/5{,}033] = 1{,}329 \end{aligned}$$

Answer is A.

**Problem 4 - 54**

Key Concept: The Accrued Liability is defined as either the accumulation of prior Normal Costs or the difference between the Present Value of Future Benefits and the Present Value of Future Normal Costs. Since Normal Cost is given in this problem, the Accrued Liability should be calculated by accumulating the prior Normal Costs. Note that Smith was just hired on 1/1/94 and therefore has no prior Normal Cost.

Step I: Calculate the Normal Cost for Brown for each prior year.

Since the Normal Costs increase by the 4% salary scale rate each year,

$$\begin{aligned} NC_{91} &= 16{,}000 \div (1.04)^3 = 14{,}224 \\ NC_{92} &= 16{,}000 \div (1.04)^2 = 14{,}793 \\ NC_{93} &= 16{,}000 \div (1.04) = 15{,}385 \end{aligned}$$

Step II: Calculate the Accrued Liability.

$$AL = NC_{91} \times (1.07)^3 \times (1/{}_3p_{35}^{(w)}) + NC_{92} \times (1.07)^2 \times (1/{}_2p_{36}^{(w)}) + NC_{93} \times (1.07) \times (1/p_{37}^{(w)})$$

$$= (14{,}224)(1.225043)[1/\,[(.75)(.8)(.85)]] + (14{,}793)(1.1449)[1/\,[(.8)(.85)]] + (15{,}385)(1.07)(1/.85)$$
$$= 34{,}167 + 24{,}906 + 19{,}367 = 78{,}440$$

Answer is B.

**Problem 4 - 55**

Step I: Calculate the Accrued Liability based upon the original plan provisions and assumptions.

Monthly Retirement Benefit = (15)(38 years of service) = 570

$$NC = (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38|}} \qquad (i = 7\%)$$
$$= (570)(104.83)/184.64 = 323.62$$

$$AL = (\text{Normal Cost})(\ddot{s}_{\overline{14|}})$$
$$= (323.62)(24.129) = 7{,}809$$

Step II: Calculate the Accrued Liability with the new 6% interest rate assumption.

$$NC = (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38|}} \qquad (i = 6\%)$$
$$= (570)(112.14)/144.058 = 443.71$$

$$AL = (NC)(\ddot{s}_{\overline{14|}})$$
$$= (443.71)(22.276) = 9{,}884$$

Step III: Calculate the Accrued Liability with both the new benefit formula and the new interest rate.

Monthly Retirement Benefit = (18)(38 years of service) = 684

$$NC = (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38|}} \qquad (i = 6\%)$$
$$= (684)(112.14)/144.058 = 532.45$$

$$AL = (NC)(\ddot{s}_{\overline{14|}})$$
$$= (532.45)(22.276) = 11{,}861$$

Step IV: Calculate the difference between the increase in the Accrued Liability due to the change in the interest rate and the increase due to the plan amendment.

Increase due to interest rate = 9,884 - 7,809 = 2,075

Increase due to plan amendment = 11,861 - 9,884 = 1,977

Difference = 2,075 - 1,977 = 98

Answer is A.

**Problem 4 - 56**

Key Concept: In situations where decrements are assumed to occur preretirement, the Present Value of Future Benefits is equal to the present value of each type of benefit (retirement, termination, death, for example).

Step I: Calculate the present value of the termination benefit at entry age.

Termination Benefit = (50)(20 years of service) = 1,000

$$\begin{aligned} PVFB_{30} &= (1{,}000)(12\ddot{a}_{65}^{(12)})(q_{50}^{(w)})(D_{65}/D_{50})(v^{20}) \\ &= (1{,}000)(12)(8.7)(.3)(94/311)(.2584) = 2{,}446 \end{aligned}$$

Note that commutation functions are used to discount only from age 65 to age 50, since preretirement mortality is assumed to occur only <u>after</u> termination.

Step II: Calculate the present value of the retirement benefit at entry age.

Retirement Benefit = (50)(35 years of service) = 1,750

$$\begin{aligned} PVFB_{30} &= (1{,}750)(12\ddot{a}_{65}^{(12)})(p_{50}^{(w)})(v^{35}) \\ &= (1{,}750)(12)(8.7)(.7)(.0937) = 11{,}983 \end{aligned}$$

Step III: Calculate the value of the temporary annuity from entry age to retirement age.

$$\begin{aligned} \ddot{a}_{30:\overline{35|}} &= (1 + v + v^2 + \ldots + v^{19}) + (p_{50}^{(w)}v^{20} + \ldots + p_{50}^{(w)}v^{34}) \\ &= (1 + v + \ldots + v^{19}) + p_{50}^{(w)}v^{20}(1 + v + \ldots + v^{14}) \\ &= \ddot{a}_{\overline{20|}} + (.7)v^{20}\,\ddot{a}_{\overline{15|}} \\ &= 11.3356 + (.7)(.2584)(9.7455) = 13.0984 \end{aligned}$$

Step IV: Calculate the Normal Cost.

$$
\begin{aligned}
NC &= PVFB_{30} \div \ddot{a}_{30:\overline{35}|} \\
&= (2{,}446 + 11{,}983) \div 13.0984 = 1{,}101
\end{aligned}
$$

Answer is E.

**Problem 4 - 57**

Step I: Calculate the final five year average salary. See the discussion in Problem 4 - 53 concerning the calculation of Final Average Salary.

$$
\begin{aligned}
\text{Final Average Salary} &= (53{,}000)\left[(1.06)^{14} + (1.06)^{13} + (1.06)^{12} + (1.06)^{11} + (1.06)^{10}\right]/5 \\
&= 107{,}009
\end{aligned}
$$

Step II: Calculate the Accrued Liability.

$$
\begin{aligned}
PVFB_{40} &= (107{,}009)(.5)(\ddot{a}_{65}^{(12)})(D_{65}/D_{40}) \\
&= (107{,}009)(.5)(8.74)(7{,}448/49{,}876) = 69{,}831
\end{aligned}
$$

$$
\begin{aligned}
NC_{40} &= (PVFB_{40})/[({}^sN_{40} - {}^sN_{65})/{}^sD_{40}] \\
&= (69{,}831)/[(15{,}607{,}843 - 4{,}770{,}425)/513{,}015] = 3{,}306
\end{aligned}
$$

$$
\begin{aligned}
AL_{50} &= (NC_{40})(1+s)^{10}[({}^sN_{40} - {}^sN_{50})/{}^sD_{50}] \\
&= (3{,}306)(1.06)^{10}[(15{,}607{,}843 - 10{,}748{,}428)/451{,}387] \\
&= 63{,}738
\end{aligned}
$$

Answer is C.

**Problem 4-57 (Alternative Solution):**

An alternative way to calculate the Accrued Liability is to determine the Normal Cost as a rate of salary at entry age. The Present Value of Future Benefits at entry age is calculated as shown in Step II above. Then the Normal Cost rate at entry age is determined as follows:

$$
\begin{aligned}
\text{NC Rate}_{40} &= (PVFB_{40}/[({}^sN_{40} - {}^sN_{65})/{}^sD_{40}] \div [(53{,}000)/(1+s)^{10}] \\
&= (69{,}831/21.12495) \div (53{,}000/1.79085) = .1117
\end{aligned}
$$

The Normal Cost rate can now be multiplied by the current salary, and accumulated from entry age to attained age to determine the Accrued Liability.

$$AL_{50} = \text{NC Rate}_{40} \times 53{,}000 \times ({}^{s}N_{40} - {}^{s}N_{50})/{}^{s}D_{50}$$
$$= (.1117)(53{,}000)\left[(15{,}607{,}843 - 10{,}748{,}428)/451{,}387\right]$$
$$= 63{,}733$$

Answer is C.

**Problem 4 - 58**

Step I: Calculate the Accrued Liability per $1,000 of projected annual benefit for each participant.

Smith: $43.04\ \ddot{s}_{\overline{5}|} = (43.04)(6.1533) = 264.84$
Brown: $135.85\ \ddot{s}_{\overline{15}|} = (135.85)(26.8881) = 3{,}652.75$

Step II: Calculate the increase in projected annual benefit for each participant. Note that the benefit increases by $50 per year for each year of service.

Smith: $50 \times 40$ years of service = 2,000
Brown: $50 \times 25$ years of service = 1,250

Step III: Calculate the increase in the Accrued Liability.

$$\text{Increase in AL} = (264.84)(2{,}000/1{,}000) + (3{,}652.75)(1{,}250/1{,}000)$$
$$= 529.68 + 4{,}565.94 = 5{,}095.62$$

Answer is C.

**Problem 4 - 59**

Key Concept: Since the Entry Age Normal Normal Cost is independent of the plan assets, the Normal Cost after the amendment is proportionally increased over the Normal Cost before the amendment.

We can use the relationship PVFB = AL + PVFNC to determine the Normal Cost.

Since PVFNC = NC × (PVFS/Salary), we need to find the salary for the 1995 valuation. The benefit formula increased by 25%, so the benefit before the increase was 12,000 (3,000 × 4).

$$\text{Salary}_{95} = 12{,}000/.3 = 40{,}000$$

So, $\text{PVFNC} = \text{NC} \times (460{,}000/40{,}000)$

Also $AL = UAL + Assets$

Therefore, $PVFB = AL + PVFNC$

$$41{,}000 = (8{,}000 + 10{,}000) + NC \times (460{,}000/40{,}000)$$

$$\begin{aligned} NC \text{ (before amendment)} &= 2{,}000 \\ \text{and } NC \text{ (after amendment)} &= 2{,}000 \times 37.5/30 = 2{,}500 \end{aligned}$$

Answer is B.

**Problem 4 - 60**

Step I: Calculate the Normal Cost.

$$\begin{aligned} NC &= (2{,}500)(12\ddot{a}_{65}^{(12)})/\ddot{s}_{30:\overline{35}|} \\ &= (2{,}500)(12\ddot{a}_{65}^{(12)})/[(N_{30} - N_{65})/D_{65}] \\ &= (2{,}500)(12)(8.74)/[(17{,}887{,}840 - 868{,}052)/94{,}414] = 1{,}454.50 \end{aligned}$$

Step II: Calculate the Accrued Liability.

$$\begin{aligned} AL &= NC \times \ddot{s}_{30:\overline{15}|} \\ &= 1454.50 \times [(N_{30} - N_{45})/D_{45}] \\ &= 1454.50 \times [(17{,}887{,}840 - 5{,}690{,}850)/445{,}008] = 39{,}866 \end{aligned}$$

Answer is D.

**Problem 4 - 60 (Alternative Solution)**

Use the relationship $PVFB = AL + PVFNC$, or alternatively, $AL = PVFB - PVFNC$.

$$\begin{aligned} PVFB &= (2{,}500)(12\ddot{a}_{65}^{(12)})(D_{65}/D_{45}) \\ &= (2{,}500)(12)(8.74)(94{,}414/445{,}008) = 55{,}629 \end{aligned}$$

$$\begin{aligned} PVFNC &= NC \times \ddot{a}_{45:\overline{20}|} \\ &= 1{,}454.50 \times [(N_{45} - N_{65})/D_{45}] \\ &= 1{,}454.50 \times [(5{,}690{,}850 - 868{,}052)/445{,}008] = 15{,}763 \end{aligned}$$

$$\begin{aligned} AL &= PVFB - PVFNC \\ &= 55{,}629 - 15763 = 39{,}866 \end{aligned}$$

Answer is D.

**Problem 4 - 61**

Key Concept: The experience gain (loss) is determined by calculating the difference between the Accrued Liability under the funding method to the actual liability due to early retirement.

Step I: Calculate the Entry Age Normal Accrued Liability.

Projected Retirement Benefit = (10)(12 years of service)(1 - (.005)(36)) = 98.40

Value at Retirement of the
Projected Retirement Benefit = $(98.40)(12)[.8(\ddot{a}^{(12)}_{62} + \ddot{a}^{(12)}_{62} - \ddot{a}^{(12)}_{62:62}) + .2(\ddot{a}^{(12)}_{62})] = 12,044$

The value of the Projected Retirement Benefit is determined using the assumption that 80% of the participants receive a joint and survivor annuity and 20% receive a life annuity.

$$NC = 12,044/\ddot{s}_{\overline{12}|}$$
$$= 12,044/19.1406 = 629.24$$

$$AL = 629.24\ \ddot{s}_{\overline{10}|}$$
$$= (629.24)(14.7836) = 9,302$$

Step II: Calculate the actual liability.

Early Retirement Benefit = (10)(10 years of service)(1 - (.005)(60)) = 70

$$\text{Value of ERB} = (70)(12)(\ddot{a}^{(12)}_{60} + \ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:60}) = 10,080$$

Step III: Calculate the experience loss.

$$\text{Loss} = \text{Value of ERB} - AL$$
$$= 10,080 - 9,302 = 778$$

Answer is B.

**Problem 4 - 62**

Key Concept: The decrease in the accrued liability is determined by calculating the difference between the accrued liability under the funding method had early retirement not been elected and the actual liability due to early retirement.

Step I: Calculate the Entry Age Normal Accrued Liability.

Projected Retirement Benefit = (25)(36 years of service) = 900.00

$$\text{Value at Retirement} = (900.00)(12)(\ddot{a}_{65}^{(12)}) = 94{,}392$$

$$\text{Normal Cost} = 94{,}392/\ \ddot{s}_{\overline{36|}} = 94{,}392/159.3374 = 592$$

$$\text{Accrued Liability} = (592) \times \ddot{s}_{\overline{26|}} = (592)(73.4838) = 43{,}502$$

Step II: Calculate the actual liability.

Early Retirement Benefit = (25)(26 years of service)(.412) = 267.80

$$\text{Value of ERB} = (267.80)(12)(\ddot{a}_{55}^{(12)}) = (267.80)(12)(10.78) = 34{,}643$$

Step III: Calculate the decrease in the accrued liability

$$\begin{aligned}\text{Decrease} &= \text{Accrued Liability - Value of ERB}\\ &= 43{,}502 - 34{,}643\\ &= 8{,}859\end{aligned}$$

Answer is A.

**Problem 4 - 63**

Key concept: The Unit Credit Normal Cost is $0 since the participant has more than 25 years of past service. So, the change in the Normal Cost will just be equal to the Entry Age Normal Normal Cost.

Projected Retirement Benefit = (20)(25 years of service) = 500

$$\begin{aligned}\text{EAN Normal Cost} &= (500)(12)(\ddot{a}_{65}^{(12)})/\ \ddot{s}_{\overline{45|}}\\ &= (6{,}000)(9.24)/(305.75176)\\ &= 181\end{aligned}$$

Answer is A.

**Problem 4 - 64**

Key Concept: The Entry Age Normal Accrued Liability is equal to the accumulated value of the past Normal Costs. Since the Normal Cost each year was \$10,000, each Normal Cost must be accumulated with interest and by the withdrawal decrement (the probability that withdrawal did not occur).

$$\begin{aligned}\text{Accrued Liability} &= (10{,}000)((1.08)^3(1/p_{35}^{(w)})(1/p_{36}^{(w)})(1/p_{37}^{(w)}) \\ &\qquad + (1.08)^2(1/p_{36}^{(w)})(1/p_{37}^{(w)}) + (1.08)(1/p_{37}^{(w)})) \\ &= (10{,}000)((1.08)^3(1/.5)(1/.6)(1/.7) \\ &\qquad + (1.08)^2(1/.6)(1/.7) + (1.08)(1/.7)) \\ &= (10{,}000)(5.9986 + 2.7771 + 1.5429) \\ &= 103{,}186\end{aligned}$$

Answer is E.

**Problem 4 - 65**

Key Concept: There is only one participant in the plan. Therefore, the Accrued Liability will increase in proportion to the increase in the normal retirement benefit.

Step I: Calculate the benefit per dollar of salary and year of service under the old formula.

$$\begin{aligned}\text{Unit of Benefit (Old)} &= (.0125)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17} + (1.05)^{16} + (1.05)^{15}]/5 \\ &= .0287185\end{aligned}$$

Step II: Calculate the benefit per dollar of salary and year of service under the new formula.

$$\begin{aligned}\text{Unit of Benefit (New)} &= (.0175)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17}]/3 \\ &= .0421493\end{aligned}$$

Step III: Calculate the Accrued Liability under the old benefit formula. Note that since there have been no gains or losses (this must be assumed since there is no information to the contrary), the normal cost would have increased at the salary scale rate each year.

$$\text{Normal Cost}_{96} = (6{,}500)(1.05)^8 = 9{,}603$$

$$\text{Accrued Liability}_{96} = (9{,}603)(\ddot{s}_{\overline{8}|j}) \quad \text{where } j = (1.07/1.05)\text{-}1$$

$= 83{,}714$

Step IV: Calculate the increase in the Accrued Liability.

$$\text{AL Increase} = (83{,}714)[(.0421493 - .0287185)/.0287185)]$$
$$= 39{,}151$$

Answer is D.

**Problem 4 - 66**

Key Concept: The experience gain can be calculated from the decrease in the projected benefit due to the actual compensation experience.

Step I: Calculate the projected Final Average Compensation (FAC) for each of the 1/1/95 and 1/1/96 valuations.

$$FAC_{95} = (50{,}000)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17}]/3$$
$$= 120{,}426$$

$$FAC_{96} = (50{,}000)[(1.05)^{18} + (1.05)^{17} + (1.05)^{16}]/3$$
$$= 114{,}692$$

Note that $FAC_{96}$ could have been calculated by dividing $FAC_{95}$ by 1.05 since the compensation remained the same in 1996 as it was in 1995.

Step II: Calculate the benefit decrease.

$$\text{Benefit Decrease} = (120{,}426 - 114{,}692)(.6) = 3{,}440$$

Step III: Calculate the Accrued Liability decrease.

$$\text{decrease in } PVFB_{30} = (3{,}440)(\ddot{a}_{65}^{(12)})(v^{35}) = 2{,}815$$

$$\text{decrease in } NC_{30} = (2{,}815)/\ddot{a}_{\overline{35}|j} = 109 \qquad \text{where } j = (1.07/1.05) - 1$$

$$\text{decrease in } AL_{46} = (109)(1.05)^{16}(\ddot{s}_{\overline{16}|j}) = 4{,}486$$

Answer is B.

**Problem 4 - 67**

Key Concept: The retirement benefit has increased by 20% for each participant since the benefit formula has increased from 50% of average salary to 60% of average salary.

The increase in the Accrued Liability for each participant is equal to the increase in the normal cost accumulated over past years of service.

$$\text{AL Increase (Smith)} = (.2)(1{,}840)\,\ddot{s}_{\overline{1}|} = (.2)(1{,}840)(1.0700) = 394$$

$$\text{AL Increase (Brown)} = (.2)(1{,}580)\ddot{s}_{\overline{18}|} = (.2)(1{,}580)(36.3790) = 11{,}496$$

$$\text{Total AL Increase} = 394 + 11{,}496 = 11{,}890$$

Answer is A.

**Problem 4 - 68**

Key Concept: Use commutation functions **without** salary scale for calculating present values of benefits and commutation functions **with** salary scale for amortizing.

Step I: Calculate the Final Average Salary.

$$\begin{aligned}\text{Final Average Salary} &= (42{,}000)[(1.05)^{24} + (1.05)^{23} + (1.05)^{22}]/3 \\ &= 129{,}106\end{aligned}$$

Note that since retirement is assumed to occur on 1/1/2022 (age 65), the final salary used will be in the 1/1/2021 valuation, 24 years from now.

Step II: Calculate the Normal Cost at entry age.

$$\begin{aligned}PVFB_{EA} &= (129{,}106)(.5)(\ddot{a}_{65}^{(12)})(D_{65}/D_{30}) \\ NC_{EA} &= PVFB_{EA}/[({}^sN_{30} - {}^sN_{65})/{}^sD_{30}] \\ &= (129{,}106)(.5)(\ddot{a}_{65}^{(12)})(D_{65}/D_{30})/[({}^sN_{30} - {}^sN_{65})/{}^sD_{30}] \\ &= 1{,}708\end{aligned}$$

Note that $\ddot{a}_{65}^{(12)} = [N_{65} - (11/24)D_{65}]/D_{65} = 8.7358$

Step III: Calculate the Accrued Liability.

$$AL = (1{,}708)(1.05)^{10}\ [({}^{s}N_{30} - {}^{s}N_{40})/{}^{s}D_{40}] = 31{,}183$$

Note that the Normal Cost must be increased by 5% for each of the 10 years from entry age to attained age.

Answer is C.

**Problem 4 - 68 (Alternative Solution)**

Calculate the normal cost rate. This is equal to the normal cost as a percentage of annual salary and is constant each year.

The annual salary at entry age is:

$$Salary_{EA} = 42{,}000/(1.05^{10}) = 25{,}784$$

The normal cost rate is:

$$\begin{aligned} NC\% &= NC_{EA}/Salary_{EA} \\ &= 1{,}708/25{,}784 \\ &= .066243 \end{aligned}$$

The accrued liability is:

$$\begin{aligned} AL &= (NC\%)(Salary_{AA})\ [({}^{s}N_{30} - {}^{s}N_{40})/{}^{s}D_{40}] \\ &= (.066243)(42{,}000)[({}^{s}N_{30} - {}^{s}N_{40})/{}^{s}D_{40}] \\ &= 31{,}183 \end{aligned}$$

Answer is C.

**Problem 4 - 69**

Key Concept: The Entry Age Normal Accrued Liability for each participant is equal to the accumulated value of the past Normal Costs. Each Normal Cost must be accumulated with interest and by the probability of remaining an active participant.

$$AL_{SMITH} = (1{,}000)(1.07)(1/p_{37}^{(T)})$$

$$= (1{,}000)(1.07)(1/.5)$$

$$= 2{,}140$$

$$AL_{BROWN} = (2{,}500)(1.07)(1/p_{38}^{(T)})$$

$$= (2{,}500)(1.07)(1/.6)$$

$$= 4{,}458$$

$$AL_{GREEN} = (4{,}500)((1.07)^3(1/p_{38}^{(T)})(1/p_{39}^{(T)})(1/p_{40}^{(T)}) + (1.07)^2(1/p_{39}^{(T)})(1/p_{40}^{(T)}) + (1.07)(1/p_{40}^{(T)}))$$

$$= (4{,}500)((1.07)^3(1/.6)(1/.7)(1/.8) + (1.07)^2(1/.7)(1/.8) + (1.07)(1/.8))$$

$$= (4{,}500)(3.6460 + 2.0445 + 1.3375)$$

$$= 31{,}626$$

$$\text{Total AL} = 2{,}140 + 4{,}458 + 31{,}626 = 38{,}224$$

Answer is C.

Note: $p_x^{(T)} = 1 - q_x^{(w)}$ since there are no other preretirement decrements.

# Chapter 5

# Individual Level Premium Method

## 5.1 Normal Cost

As you have probably surmised by this time, there is no funding method that is perfect and which represents the ideal method for all plans. Each has its own particular advantages and disadvantages. For example, one of the characteristics of the Entry Age Normal Method is that salary increases in excess of those assumed generate experience losses that are amortized over a period of time. Hence, if a highly paid participant receives a salary increase near his normal retirement age, there could be a substantial experience loss and insufficient time to fund the increased amount required prior to his retirement. The Individual Level Premium Method will correct this problem to a large extent. It is similar to the Entry Age Normal Method in that Normal Cost is computed assuming level dollar amounts (or percentages) for future years and for each active participant. It differs from the Entry Age Normal Method, however, in two important respects:

(1) The original entry age is the age at the effective date of the Plan or at actual participation, if later;

(2) Changes in benefits due to salary increases or decreases are treated as increments with the normal cost attributable to these changes computed at attained age.

For example, suppose a plan provides a benefit of 50% of salary at age 65. The plan has only one participant age 45 with salary of \$4,000 per month. Assuming investment earnings of 6% and $12\ddot{a}_{65}^{(12)} = 120$, valuation results would be:

| | |
|---|---:|
| (1) Benefit (50% of \$4,000) | 2,000 |
| (2) Amount needed at NRD (\$120 × (1)) | 240,000 |
| (3) Pres. Value of Benefits (2) × $v^{20}$ | 74,833 |
| (4) Normal Cost ((3) ÷ $\ddot{a}_{\overline{20}\rvert}$) | 6,155 |

The following year he receives a salary increase to \$4,200 per month. The normal cost is computed on the increment of increase or \$200 per month.

| | |
|---|---|
| (5) Benefit increase (50% of \$200) | \$100 |
| (6) Years to Normal Retirement | 19 |
| (7) Additional amount needed at NRD (5) × \$120 | 12,000 |
| (8) Present value of increase (7) × $v^{19}$ | 3,966 |
| (9) Normal Cost increase (8) ÷ $\ddot{a}_{\overline{19}|}$ | 335 |
| (10) Normal Cost on second valuation date (4) + (9) | 6,490 |

As with the Entry Age Normal Method, if other assumptions are used, commutation functions are obtained from a service table.

If salary increases are assumed, the Normal Cost percentage is obtained which is, as with the Entry Age Normal Method, the Present Value of Future Benefits divided by Present Value of Future Normal Costs. The increments of salary to be used in subsequent years then is the amount of deviation from the expected rather than the actual salary.

As indicated above, the Individual Level Premium Method provides for funding of benefits attributable to salary changes through Normal Costs. Other gains or losses (e.g. investment return, withdrawals, etc.) generate actuarial gains and losses which must be determined and amortized. The methodology is similar to that under Entry Age Normal.

$$\begin{aligned} AL &= \text{Present Value of Future Benefits less Present Value of Future Normal Costs} \\ UAL &= AL - \text{Assets} \\ eUAL_{t+1} &= (UAL_t + NC_t)(1 + i) - C - I_c \\ \text{Gain(Loss)} &= eUAL - UAL \end{aligned}$$

The student should note that there is no Accrued Liability, and therefore no Unfunded Accrued Liability, at plan inception. Also note that the Individual Level Premium Cost Method is identical to the Individual Aggregate Cost Method for the first Plan Year. Furthermore, if all assumptions (other than salary changes) were exactly realized, the two methods would continue to be identical. The only real difference between the two methods is in their treatment of actuarial gains and losses. The Individual Level Premium Cost Method separately calculates and amortizes gains or losses whereas the Individual Aggregate Cost Method spreads them over future Normal Costs. Because of the similarity, the Individual Aggregate Cost Method is sometimes referred to as the Individual Level Premium with Spread Gains and Losses Cost Method.

## 5.2 Characteristics of the Individual Level Premium Method

1. The amount available at retirement for each participant should approximate the amount needed to fund the benefit.

2. Maintenance of the increments of salary and benefit increases or decreases tend to become burdensome after several years.

3. Salary increases (or decreases) do not generate experience losses (or gains) as with Entry Age Normal but are handled through increments of Normal Costs.

4. Experience gains or losses from other sources such as terminations, investment earnings, and deaths need to be amortized. In the case of a highly paid participant in a small plan, the amount of money needed at retirement may still be more or less than that accumulated.

## 5.3 Problems

### Problem 5 - 1

Effective date: 1/1/85
Benefit: 30% of final salary

Assumptions: Interest only at 7%
Normal retirement is age 65
$\ddot{a}_{65}^{(12)} = 9.50$

Data for sole participant is:

Date of Birth: 1/1/35
Date of Hire: 1/1/80
Salary at 1/1/85: $50,000
Salary at 1/1/86: $60,000

As of 1/1/86, determine the difference in normal cost between the Entry Age Normal and Individual Level Premium methods.

### Problem 5 - 2

Refer to the data in problem 5-1.

Compute the Accrued Liability under each method as of 1/1/86. What is the increase in Accrued Liability due to the salary increase under each method?

### Problem 5 - 3

Refer to the data in problem 5-1.

Assumptions remain the same except that a salary increase of 4% is assumed. Recompute the Normal Cost as of 1/1/85 and 1/1/86 using the Individual Level Premium Method.

**Problem 5 - 4**

Plan effective date: 1/1/75
Actuarial cost method: Individual Level Premium
Assumed retirement age: 65

Normal retirement benefit:

Effective 1/1/75: \$100 per month
Effective 1/1/85: \$150 per month

It is assumed that there are no pre-retirement terminations other than by death.

Data for the sole participant:

Date of Birth: 1/1/40
Date of Hire: 1/1/75

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 35 | 921 | 12,791 |
| 45 | 460 | 5,909 |
| 65 | 99 | 904 |

The normal cost as of 1/1/85 under the individual level premium cost method is \$K.
The normal cost as of 1/1/85 under the entry age normal cost method is \$L.

In what range is \$K minus \$L?

(A) Less than \$62
(B) \$62 but less than \$66
(C) \$66 but less than \$70
(D) \$70 but less than \$74
(E) \$74 or more.

**Problem 5 - 5**

Plan effective date: 1/1/84
Normal retirement benefit: $500 per month.
Actuarial cost method: Individual level premium method.
Pre-retirement death benefit: 100 times the monthly normal retirement benefit (provided by individual insurance policies.)

Actuarial assumptions for side fund:

Interest: 6%
Pre-retirement mortality and withdrawal: None.
Retirement age: 65
$\ddot{a}_{65}^{(12)} = 10$

| Participant data as of 1/1/84: | Smith | Brown |
|---|---|---|
| Current Age | 45 | 59 |
| Age at hire | 25 | 40 |
| Cash value of insurance at age 65 | $25,000 | $12,000 |

In what range is the normal cost for the side fund for 1984 as of 1/1/84?

(A) Less than $6,000
(B) $6,000 but less than $6,500
(C) $6,500 but less than $7,000
(D) $7,000 but less than $7,500
(E) $7,500 or more

**Problem 5 - 6**

Plan effective date: 1/1/77
Actuarial cost method: Individual level premium method. Benefit increases from plan amendments are funded through the normal cost rather than through adjustment of the accrued liability.

Normal retirement benefit: $10 per month for each year of service; increased by plan amendment effective 1/1/82 to $15 per month for each year of service.

Actuarial assumptions:

Interest: 7%
Pre-retirement mortality and turnover: None
$12\ddot{a}_{65}^{(12)} = 100$

Data for sole plan participant

Date of Birth: 1/1/32
Date of Hire: 1/1/72
Date of Death: 1/2/84
Death benefit paid on 1/2/84: $4,000

In what range is the mortality gain for 1984, calculated as of 12/31/84?

(A) Less than $2,500
(B) $2,500 but less than $4,000
(C) $4,000 but less than $5,500
(D) $5,500 but less than $7,000
(E) $7,000 or more.

**Problem 5 - 7**

| | |
|---|---|
| Plan Effective Date: | 1/1/84 |
| Normal Retirement Benefit: | $600 per month |
| Actuarial Cost Method: | Individual level premium method |
| Valuation Date: | 12/31 |

Actuarial assumptions:

Interest: 6%
Retirement Age: 65
Pre-retirement mortality and withdrawal: None.
$12\ddot{a}_{65}^{(12)} = 100$

The sole plan participant on 12/31/84 was born 1/2/50.

In what range is the normal cost for 1984 as of 12/31/84?

(A) Less than \$700
(B) \$700 but less than \$710
(C) \$710 but less than \$720
(D) \$720 but less than \$730
(E) \$730 or more

**Problem 5 - 8**

Plan effective date: 1/1/84
Normal retirement benefit: 50% of final 3-year average salary.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest: 7%
Salary increases: None.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for the sole participant:

Date of birth: 1/1/34
Annual salary as of 1/1/84: \$25,000

Normal cost as of 1/1/84: \$4,032
Actuarial value of assets as of 1/1/85: \$5,000

As of 1/1/85, the participant's annual salary increases to \$50,000.

In what range is the normal cost as of 1/1/85?

(A) Less than \$8,100
(B) \$8,100 but less than \$8,300
(C) \$8,300 but less than \$8,500
(D) \$8,500 but less than \$8,700
(E) \$8,700 or more.

**Problem 5 - 9**

Plan effective date: 1/1/85

Cost method: Individual level premium (split-funded). The total normal cost is equal to the side fund normal cost plus life insurance premiums.

Normal retirement benefit: 60% of final year's salary.

Death benefit: 50 times the projected monthly normal retirement benefit, provided by whole life insurance.

Actuarial assumptions for the side fund:

Interest: 8%
Salary increases: None.
Pre-retirement deaths and terminations: None
Retirement age: 65

Data for sole participant:

Date of birth: 1/1/50
1/1/85 monthly salary: \$3,000
1/1/86 monthly salary: \$3,300

Insurance policy data:

| Issue Age | Premium per \$1,000 | Cash Value at 65 per \$1,000 |
|---|---|---|
| 35 | \$30 | \$500 |
| 36 | 32 | 450 |

$12\ddot{a}_{65}^{(12)} = 100$

In what range is the excess of the total normal cost for 1986 as of 1/1/86 over the total normal cost for 1985 as of 1/1/85?

(A) Less than \$100
(B) \$100 but less than \$200
(C) \$200 but less than \$300
(D) \$300 but less than \$400
(E) \$400 or more

**Problem 5 - 10**

| | |
|---|---|
| Plan effective date: | 1/1/77. |
| Normal retirement benefit: | $500 per month. |
| Actuarial cost method: | Individual level premium, with changes due to plan amendments funded from attained age. |

Actuarial assumptions:

Interest: 8%
Retirement age: 65
Preretirement terminations other than deaths: None.

Participant data as of 1/1/87 and selected commutation functions:

| Age at Hire | Attained Age x | Number of employees | $D_x$ | $N_x - N_{65}$ | $N_x^{(12)}$ |
|---|---|---|---|---|---|
| -- | 25 | 0 | 265 | 3,390 | 3,355 |
| 25 | 35 | 1 | 125 | 1,455 | 1,485 |
| 25 | 45 | 2 | 55 | 570 | 630 |
| -- | 55 | 0 | 25 | 170 | 245 |
| -- | 65 | 0 | 10 | 0 | 80 |

Effective 1/1/87, the plan is amended to increase the normal retirement benefit to $550 per month.

In what range is the increase, due to the amendment, in the normal cost for 1987 as of 1/1/87?

(A) Less than $50
(B) $50 but less than $150
(C) $150 but less than $250
(D) $250 but less than $350
(E) $350 or more

**Problem 5 - 11**

Plan Effective Date: 1/1/86
Normal Retirement Benefit: 50% of final year's salary.
Actuarial Cost Method: Individual level premium.
Valuation Date: 12/31

Actuarial assumptions:

> Interest: 8%
> Salary increases: None.
> Retirement age: 65
> Preretirement deaths and terminations: None.

Data for sole participant:

| | |
|---|---|
| Date of birth: | 1/1/32 |
| Date of hire: | 1/1/77 |
| Salary for 1986: | \$36,000 |
| Salary for 1987: | \$30,000 |

$\ddot{a}_{65}^{(12)} = 8.33$

In what range is the normal cost for 1987 as of 12/31/87?

- (A) Less than \$7,400
- (B) \$7,400 but less than \$7,800
- (C) \$7,800 but less than \$8,200
- (D) \$8,200 but less than \$8,600
- (E) \$8,600 or more

**Problem 5 - 12**

Plan Effective Date: 1/1/86
Normal Retirement Benefit: 30% of final year's salary.
Actuarial Cost Method: Individual level premium.
Initial Valuation Date: 12/31/86

Actuarial assumptions:

> Interest: 8%
> Salary Increases: None.
> Retirement Age: 65
> Preretirement Deaths and Terminations: None.

Data for sole participant:

Date of Birth: 1/1/42
Salary for 1986: $200,000

$\ddot{a}_{65}^{(12)} = 8.40$

In what range is the normal cost for 1986 as of 12/31/86?

(A) Less than $10,000
(B) $10,000 but less than $10,050
(C) $10,050 but less than $10,100
(D) $10,100 but less than $10,150
(E) $10,150 or more

**Problem 5 - 13**

Plan effective date: 1/1/87
Normal retirement benefit: 40% of final year's compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth: | 1/1/37 |
| Compensation for 1987: | $100,000 |
| Compensation for 1988: | $120,000 |

Contribution for 1987: Normal cost as of 1/1, paid on 1/1/87.
Actuarial value of assets as of 1/1/88: $15,000.

Contribution for 1988: Normal cost as of 1/1, plus 10-year amortization of 1987 experience gain or loss, paid on 1/1/88.

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 9.333$

In what range is the contribution for 1988?

(A) Less than \$18,600
(B) \$18,600 but less than \$18,650
(C) \$18,650 but less than \$18,700
(D) \$18,700 but less than \$18,750
(E) \$18,750 or more

**Problem 5 - 14**

Plan effective date: 1/1/88
Normal retirement benefit: 40% of highest 3-year average compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 6%
Compensation increases: None.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/45 |
| Date of hire | 1/1/88 |
| Rate of compensation for 1988 | \$200,000 |
| Rate of compensation for 1989 | \$170,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 9.35$

In what range is the normal cost for 1989 as of 1/1/89?

(A) Less than \$14,100
(B) \$14,100 but less than \$14,600
(C) \$14,600 but less than \$15,100
(D) \$15,100 but less than \$15,600
(E) \$15,600 or more

**Problem 5 - 15**

Plan effective date: 1/1/88
Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate:
Before 1989 8%
After 1988 6%
Compensation increases: None.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/53 |
| Date of participation | 1/1/88 |
| Rate of compensation for 1988 | \$24,000 |
| Rate of compensation for 1989 | \$28,800 |

Selected values for $\ddot{a}_{65}^{(12)}$:

| | |
|---|---|
| Before 1989: | 8.1958 |
| After 1988: | 9.3452 |

In what range is the increase in the accrued liability as of 1/1/89 due to the change in the actuarial assumptions?

(A) Less than \$560
(B) \$560 but less than \$615
(C) \$615 but less than \$670
(D) \$670 but less than \$725
(E) \$725 or more

**Problem 5 - 16**

Plan effective date: 1/1/84
Normal retirement benefit: \$15 per month for each year of service.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/39 |
| Date of hire | 1/1/79 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 40 | 941 | 13,971 |
| 45 | 694 | 9,789 |
| 50 | 508 | 6,712 |
| 55 | 366 | 4,472 |
| 65 | 178 | 1,741 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the accrued liability as of 1/1/89?

(A) Less than \$4,000
(B) \$4,000 but less than \$6,000
(C) \$6,000 but less than \$8,000
(D) \$8,000 but less than \$10,000
(E) \$10,000 or more

**Problem 5 - 17**

Plan effective date: 1/1/85
Normal retirement benefit: 2% of final year's compensation for each year of service.
Actuarial cost method: Individual level premium (benefit increases increase the normal cost).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/60 |
| Date of hire | 1/1/85 |

Monthly compensation:

| | |
|---|---|
| 1985 | \$3,500 |
| 1986 | 3,500 |
| 1987 | 3,500 |
| 1988 | 4,000 |
| 1989 | 4,000 |
| 1990 | 4,000 |

Status as of 1/1/90 Active

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 9.345$

In what range is the accrued liability as of 1/1/90?

(A) Less than \$9,600
(B) \$9,600 but less than \$9,800
(C) \$9,800 but less than \$10,000
(D) \$10,000 but less than \$10,200
(E) \$10,200 or more

**Problem 5 - 18**

Plan effective date: 1/1/82
Normal retirement benefit: \$25 per month for each year of service.
Preretirement death benefit: None.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/50 |
| Date of hire | 1/1/81 |
| Status as of 1/1/91 | Active |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 31 | 1,540 | 25,240 |
| 32 | 1,500 | 24,000 |
| 41 | 900 | 13,050 |
| 42 | 860 | 12,150 |
| 65 | 200 | 1,792 |

$\ddot{a}_{65}^{(12)} = 8.5$

In what range is the present value of future normal costs as of 1/1/91?

(A) Less than \$8,800
(B) \$8,800 but less than \$9,100
(C) \$9,100 but less than \$9,400
(D) \$9,400 but less than \$9,700
(E) \$9,700 or more

**Problem 5 - 19**

Plan effective date: 1/1/90
Normal retirement benefit: 75% of final 3-year average compensation.
Actuarial cost method: Individual level premium (level dollar amount).

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: 4% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/48 |
| 1990 compensation | \$43,500 |
| 1991 compensation | 46,000 |
| Status as of 1/1/91 | Active |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.5$$

In what range is the increase in the normal cost for 1991 as of 1/1/91 over the normal cost for 1990 as of 1/1/90?

(A) Less than \$100
(B) \$100 but less than \$200
(C) \$200 but less than \$300
(D) \$300 but less than \$400
(E) \$400 or more

**Problem 5 - 20**

Normal retirement benefit: \$41.67 per month for each year of service.
Early retirement eligibility: Age 55.
Early retirement benefit: Accrued benefit, reduced on an actuarially equivalent basis.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

> Interest rate: 7% per year.
> Mortality: UP-84 Table.
> Preretirement terminations other than deaths: None.
> Retirement age: 65

Data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/37 |
| Date of hire | 1/1/72 |
| Date of participation | 1/1/82 |

On 12/31/91, Smith retires and elects to begin receiving benefits on 1/1/92.

Selected annuity values and probability of survival:

$\ddot{a}_{65}^{(12)} = 8.51$

$\ddot{a}_{45:\overline{20}|} = 10.79 \quad \ddot{a}_{45:\overline{10}|} = 7.37 \quad \ddot{a}_{35:\overline{30}|} = 12.82 \qquad \ddot{a}_{35:\overline{20}|} = 11.12$

${}_{10}p_{55} = .8562$

In what range is the experience gain as of 12/31/91 due to Smith's early retirement?

(A) Less than \$3,000
(B) \$3,000 but less than \$6,000
(C) \$6,000 but less than \$9,000
(D) \$9,000 but less than \$12,000
(E) \$12,000 or more

**Problem 5 - 21**

Plan effective date: 1/1/91
Normal retirement benefit: 50% of final 3-year average compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/51 |
| Date of hire | 1/1/91 |
| Compensation used in 1/1/91 valuation | \$60,000 |

Normal cost for 1992 as of 1/1/92: \$3,032

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the participant's projected 3-year average compensation used in the 1/1/92 actuarial valuation?

(A) Less than \$44,000
(B) \$44,000 but less than \$49,000
(C) \$49,000 but less than \$54,000
(D) \$54,000 but less than \$59,000
(E) \$59,000 or more

**Problem 5 - 22**

Plan effective date: 1/1/92

Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/43 |
| Date of hire | 1/1/83 |
| 1992 compensation for 1/1/92 valuation | $16,000 |
| 1993 compensation for 1/1/93 valuation | 18,000 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1993 as of 1/1/93?

(A) Less than $2,600
(B) $2,600 but less than $2,650
(C) $2,650 but less than $2,700
(D) $2,700 but less than $2,750
(E) $2,750 or more

**Problem 5 - 23**

Plan effective date: 1/1/91.
Normal retirement benefit: 50% of final 5-year average compensation.
Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/40 |
| Date of hire | 1/1/91 |
| 1991 compensation for 1/1/91 valuation | $100,000 |
| 1992 compensation for 1/1/92 valuation | 92,000 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability as of 1/1/93?

(A) Less than $37,700
(B) $37,700 but less than $38,100
(C) $38,100 but less than $38,500
(D) $38,500 but less than $38,900
(E) $38,900 or more

**Problem 5 - 24**

Normal retirement benefit: $15 per month for each year of service.

Actuarial cost method:

Before 1993: Aggregate.
After 1992: Unit credit.

Actuarial assumptions:

Interest rate: 8% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/42 |
| Date of hire | 1/1/87 |

Selected valuation results as of 1/1/92:

| | |
|---|---|
| Normal cost as of 1/1 | $898 |
| Value of assets | 1,000 |

Contribution for 1992: Normal cost for 1992 as of 1/1/92; paid on 1/1/92.
There were no noninvestment experience gains or losses for 1992.
Actual rate of investment return in 1992: 5%

In what range is the unfunded accrued liability as of 1/1/93?

- (A) Less than $700
- (B) $700 but less than $800
- (C) $800 but less than $900
- (D) $900 but less than $1,000
- (E) $1,000 or more

**Problem 5 - 25**

Plan effective date: 1/1/90
Normal retirement benefit: 50% of highest 3-year average compensation.
Compensation: Base rate of pay as of 1/1.

Actuarial cost method: Individual level premium.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/30 |
| Date of hire | 1/1/90 |
| Base rate of pay as of 1/1/90 | $60,000 |
| Base rate of pay as of 1/1/91 | 60,000 |
| Base rate of pay as of 1/1/92 | 80,000 |
| Base rate of pay as of 1/1/93 | 80,000 |
| Base rate of pay as of 1/1/94 | 65,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.786$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than \$45,000
(B) \$45,000 but less than \$47,500
(C) \$47,500 but less than \$50,000
(D) \$50,000 but less than \$52,500
(E) \$52,500 or more

**Problem 5 - 26**

Plan effective date: 1/1/95.

Funding medium: Individual whole life insurance policies and a side fund.

| | |
|---|---|
| Actuarial cost method: | Individual level premium with normal cost for side fund determined for benefits not provided by cash values of insurance policies at age 65. |

The amount of whole life insurance is 60 times the expected monthly retirement benefit.

Actuarial assumptions for side fund:

Interest rate: 6% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Date of birth for sole participant (active as of 1/1/95): 1/1/56.

Expected monthly retirement benefit: \$400.
Cash value at age 65 per \$1,000 of insurance: \$550.

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 10$

In what range is the normal cost for 1995 as of 12/31/95 for the side fund?

(A) Less than \$480
(B) \$480 but less than \$510
(C) \$510 but less than \$540
(D) \$540 but less than \$570
(E) \$570 or more

**Problem 5 - 27**

Plan effective date: 1/1/87.

Normal retirement benefit:

| | |
|---|---|
| Effective 1/1/87: | \$15 per month for each year of service. |
| Effective 1/1/96: | \$18 per month for each year of service. |

Actuarial cost method: Individual level premium.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/57 |
| Date of hire | 1/1/82 |

Value of assets as of 1/1/96: \$5,000.

Selected annuity value:

$$12\ddot{a}_{65}^{(12)} = 104.83$$

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $500
(B) $500 but less than $550
(C) $550 but less than $600
(D) $600 but less than $650
(E) $650 or more

**Problem 5 - 28**

Plan effective date: 1/1/96.

Normal retirement benefit: $1,000 per month.

Pre-retirement death benefit: $100,000, payable at end of year of death.

Actuarial cost method:

Method A: Individual level premium for all benefits other than death benefits, plus one year term cost for death benefit.

Method B: Aggregate for all benefits other than death benefits and benefits provided by cash value, plus insurance premium (split funded).

Actuarial assumptions:

Interest rate: 7% per year.
Pre-retirement decrements other than deaths: None.
Retirement age: 65.

Date of birth for sole participant (active as of 1/1/96): 1/1/56.

Provisions of life insurance policy (purchased on 1/1/96):

| | |
|---|---|
| Level annual premium | $ 1,585 |
| Projected cash value as of 1/1/2021 | 41,900 |

Selected commutation functions and annuity values:

| x | $D_x$ | $M_x$ | $N_x$ | $\ddot{a}_x^{(12)}$ |
|---|---|---|---|---|
| 40 | 632,275 | 79,292 | 8,452,729 | 12.91 |
| 41 | 589,655 | 78,036 | 7,820,454 | 12.80 |
| 65 | 94,414 | 37,625 | 868,052 | 8.74 |

In what range is the absolute value of the difference in the normal cost for 1996 as of 1/1/96 between Method A and Method B?

(A) Less than \$750
(B) \$750 but less than \$825
(C) \$825 but less than \$900
(D) \$900 but less than \$975
(E) \$975 or more

**Problem 5 - 29**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.

Actuarial cost method: Individual level premium (level dollar amount).

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 5% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/97):

| | |
|---|---|
| Date of birth | 1/1/67 |
| Date of hire | 1/1/92 |
| 1996 valuation compensation for 1/1/96 valuation | \$25,000 |
| 1997 valuation compensation for 1/1/97 valuation | \$27,500 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 9.815$$

In what range is the increase in the normal cost for 1997 as of 1/1/97 due to compensation increases from 1996 to 1997?

(A) Less than \$310
(B) \$310 but less than \$320
(C) \$320 but less than \$330
(D) \$330 but less than \$340
(E) \$340 or more

## 5.4 Solutions to Problems

**Problem 5 - 1**

| | Individual Level Premium | Entry Age Normal At 1/1/85 | Entry Age Normal At 1/1/86 |
|---|---|---|---|
| (1) Entry Age | 50 | 45 | 45 |
| (2) Retirement Benefit | \$15,000 | \$15,000 | \$18,000 |
| (3) Value at Retirement | 142,500 | 142,500 | 171,000 |
| (4) Pres. Value Benefits | 51,649 | 36,825 | 44,190 |
| (5) Normal Cost | 5,300 | 3,249 | 3,898 |

As of 1/1/86 considering salary increase only.

| | |
|---|---|
| (6) Attained Age | 51 |
| (7) Benefit Increase | 3,000 |
| (8) Value at Retirement | 28,500 |
| (9) Pres. Value of Benefit Increase | 11,053 |
| (10) Normal Cost on Increase | 1,181 |
| (11) Total Normal Cost | 5,300 + 1,181 = 6,481 |

| | 1/1/85 | 1/1/86 |
|---|---|---|
| Entry Age Normal | \$3,249 | \$3,898 |
| Individual Level Premium | 5,300 | 6,481 |

Difference = 6,481 - 3,898 = 2,583

**Problem 5 - 2**

Step I: Compute Accrued Liability as of 1/1/86.

| | | EAN | ILP |
|---|---|---|---|
| (1) | Amount needed at retirement | $171,000 | $171,000 |
| (2) | Present Value of Future Benefits (1) × $1.07^{-14}$ | 66,317 | 66,317 |
| (3) | Normal cost | 3,898 | 6,481 |
| (4) | Present Value of Future Normal Costs | 36,476 | 60,647 |
| (5) | Accrued Liability (2) - (4) | 29,841 | 5,670 |

Step II: Determine the Accrued Liability on 1/1/85 under the Entry Age Normal Method. Also determine the Accrued Liability on 1/1/86 using salaries as of 1/1/85. The Difference in Accrued Liability due to salary increases only can be calculated.

| Using Entry Age Normal Cost Method: | | 1/1/85 | 1/1/86 |
|---|---|---|---|
| (1) | Amount needed at retirement | $142,500 | $142,500 |
| (2) | Present Value of Future Benefits (1) × $v^{ra-aa}$ | 51,649 | 55,264 |
| (3) | Normal Cost | 3,249 | 3,249 |
| (4) | Present Value of Future Normal Costs | 31,663 | 30,403 |
| (5) | Accrued Liability | 19,986 | 24,861 |

Difference under Entry Age Normal Cost Method = 29,841 - 24,861 = 4,980
Difference under the Individual Level Premium Cost Method = 0.00
(Under the ILP Cost Method, these are reflected in the change in Normal Cost.)

**Problem 5 - 3**

Key Concept: Salary increase of 4% and interest of 7% is the equivalent of using an interest rate of 2.885% [(1.07÷1.04) - 1].

Step I: Compute Normal Cost as of 1/1/85.

| | |
|---|---|
| (1) Current Salary | \$50,000 |
| (2) Projected salary (1) × $1.04^{15}$ | 90,047 |
| (3) Projected Benefit (2) × .30 | 27,014 |
| (4) Value at retirement (3) × 9.50 | 256,633 |
| (5) Present Value of Future Benefits (4) × $v^{15}$ | 93,016 |
| (6) Present Value of Future Salaries (1) × $\ddot{a}_{\overline{15}\vert j}$ $j = 2.885\%$ | 619,257 |
| (7) Normal Cost Percentage (5) ÷ (6) | 15.02% |
| (8) Normal Cost (1) × (7) | 7,510 |

Step II: Compute Normal Cost % on increment.

| | |
|---|---|
| (9) Expected salary on 1/1/86 (1) × 1.04 | 52,000 |
| (10) Increase over expected 60,000 - (1) | 8,000 |
| (11) Benefit increase 30% × (2) | 2,400 |
| (12) Projection on increase to age 65 (11) × $1.04^{14}$ | 4,156 |
| (13) Present Value of Benefit increase (12) × $v^{14}$ × $\ddot{a}_{65}^{(12)}$ | 15,312 |
| (14) Present Value of Salary increase increment (10) × $\ddot{a}_{\overline{14}\vert j}$ | 93,709 |
| (15) Normal Cost % on increase (13) ÷ (14) | 16.34% |
| (16) Normal Cost increase (15) × (10) | 1,307 |
| (17) Prior Normal Cost 15.02% × 52,000 | 7,810 |
| (18) Total Normal Cost (16) + (17) | 9,117 |

**Problem 5 - 4**

Step I: Calculate the Normal Cost under the Individual Level Premium Method as of 1/1/75, then the increase of \$50 as of 1/1/85.

Entry age: 35
Attained age: 45

$\ddot{a}_{65}^{(12)} = (N_{65}/D_{65}) - (11/24) = 8.673$

$$NC_{75} = [(100)(12\ddot{a}_{65}^{(12)})(D_{65}/D_{35}] \div [(N_{35} - N_{65}) \div D_{35}]$$
$$= (1200)(8.673)(99) \div (12{,}791 - 904)$$
$$= 1{,}030{,}352 \div 11{,}887 = 87$$

$$NC_{85} = [(50)(12\ddot{a}_{65}^{(12)})(D_{65} / D_{45})] \div [(N_{45} - N_{65}) \div D_{45}]$$
$$= (50)(12)(8.673)(99) \div (5{,}909 - 904)$$
$$= 515{,}176 \div 5005 = 103$$

Therefore, Normal Cost = 87 + 103 = 190.

Step II: Compute Normal Cost under Entry Age Normal Method.

$$NC = [(150)(12\ddot{a}_{65}^{(12)})(D_{65} / D_{35})] \div [(N_{35} - N_{65})/ D_{35}]$$
$$= (150)(12)(8.673)(99) \div (12{,}791 - 904)$$
$$= 1{,}545{,}529 \div 11{,}887 = 130$$

Difference = 190 - 130 = 60

Answer is A.

**Problem 5 - 5**

Key Concept: The amount needed at retirement is offset by the cash value of insurance policies at normal retirement. The Normal Cost is then computed on the net amount needed at retirement.

| | | Smith | Brown |
|---|---|---|---|
| (1) | Attained age | 45 | 59 |
| (2) | Retirement Benefit | 500 | 500 |
| (3) | Values at age 65. (2) × 120 | 60,000 | 60,000 |
| (4) | Cash Value at age 65 | 25,000 | 12,000 |
| (5) | Side Fund at age 65 (4) - (3) | 35,000 | 48,000 |
| (6) | $\ddot{s}_{\overline{ra - aa}\rceil 6\%}$ | 38.993 | 7.394 |
| (7) | Normal Cost (5) ÷ (6) | 898 | 6,492 |

(8) Total Normal Cost = 898 + 6,492 = 7,390

Answer is D.

**Problem 5 - 6**

Key Concept: We must first determine the accrued liability as of 1/1/84, then adjust for interest to 12/31/84 (the end of the year of death). To determine the accrued liability on 1/1/84, we must first calculate the normal cost as of 1/1/77 and the addition on 1/1/82, the date of the amendment.

Step I: Determine Normal Cost.

| | | 1/1/77 | 1/1/82 |
|---|---|---|---|
| (1) | Age of Participant | 45 | 50 |
| (2) | Retirement Benefit | $10 \times 25 = 250$ | $5 \times 25 = 125$ |
| (3) | Values at Retirement (2) × 100 | 25,000 | 12,500 |
| (4) | $\ddot{s}_{\overline{ra - aa}\rceil 7\%}$ | 43.87 | 26.89 |
| (5) | Normal Cost (3) ÷ (4) | 570 | 465 |
| (6) | Total Normal Cost = 570 + 465 =1,035 | | |

Step II: Determine the Accrued Liability as of 1/1/84.

Attained age of participant = 52

Present Value of Future Benefits = $(15)(25 \text{ years})(100)(1.07)^{-13} = 15{,}561$

Present Value of Normal Costs = $(1{,}035)\ (\ddot{a}_{\overline{13}\rceil})$

$= 1{,}035 \times 8.943 = 9{,}256$

Accrued Liability on 1/1/84 = 15,561 - 9,256 = 6,305

Step III: Mortality gain on 12/31/84 is the Accrued Liability plus Normal Cost less death benefit, all adjusted for interest:

$$\text{Mortality Gain} = (6{,}305 + 1{,}035)(1.07) - (4{,}000)(1.07) = 3{,}574$$

Answer is B.

**Problem 5 - 7**

Key Concept: Since this is an end-of-year valuation, immediate annuity rather than annuity due figures should be used.

$$\text{Liability at retirement} = (\text{Benefit})(12\ddot{a}_{65}^{(12)}) \\ = 600 \times 100 = 60{,}000$$

Age of Participant on 1/1/84 = 34

$$s_{\overline{31}|} = 84.80$$

$$\text{Normal Cost} = 60{,}000 \div 84.80 = 708$$

Answer is B.

**Problem 5 - 8**

Key Concept: We are not given a value for $\ddot{a}_{65}^{(12)}$. However, since we are given the normal cost as of 1/1/84, we can compute it as follows:

$$\begin{aligned} \text{Normal Cost} &= (\text{Benefit})(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{15}|} \\ 4{,}032 &= (12{,}500)(\ddot{a}_{65}^{(12)}) \div 26.89 \\ \text{or } (12{,}500)(\ddot{a}_{65}^{(12)}) &= (4{,}032)(26.89) \\ \text{and } \ddot{a}_{65}^{(12)} &= 8.67 \end{aligned}$$

We can now calculate the normal cost for the benefit increase on 1/1/85.

$$= (\text{Benefit increase})(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{14}|} \\ = (12{,}500 \times 8.67) \div 24.13 = 4{,}491$$

$$\text{Total Normal Cost} = 4{,}491 + 4{,}032 = 8{,}523$$

Answer is D.

**Problem 5 - 9**

| | | |
|---|---|---|
| (1) | Increase in Compensation | \$300 |
| (2) | Increase in Benefits 300 × 60% | 180 |
| (3) | Increase in Insurance face amount (2) × 50 | \$9,000 |
| (4) | Increase in Insurance premium 9 × 32 | 288 |
| (5) | Increase in Value at retirement (2) × 100 | 18,000 |
| (6) | Increase in cash value at retirement 9 × 450 | 4,050 |
| (7) | Increase in side fund at retirement 18,000 - 4,050 | 13,950 |
| (8) | Increase in Pres. Value benefits 13,950 × $v^{29}$ | 1,497 |
| (9) | Increase in Normal Cost (Side fund) 1,497 ÷ $\ddot{a}_{\overline{29|}}$ | 124 |
| (10) | Total Normal Cost Increase (4) + (9) | 412 |

Answer is E.

**Problem 5 - 10**

Key Concept: Under the Individual Level Premium Method, we are concerned only with the increase in benefits and its effect upon normal cost.

$$\begin{aligned}\ddot{a}_{65}^{(12)} &= N_{65}^{(12)}/D_{65} \\ &= 80 \div 10 = 8\end{aligned}$$

$$\text{Increase in Normal Cost} = (\text{Benefit increase})(\ddot{a}_{65}^{(12)})[D_{65}/(N_x - N_{65})]$$

For employee age 35:

$$\text{Increase in Normal Cost} = (50)(12 \times 8)(10 \div 1{,}455) = 32.99$$

For employees age 45 (per employee):

$$\text{Increase in Normal Cost} = (50)(12 \times 8)(10 \div 570) = 84.21$$

Total Normal cost Increase = 32.99 + (2)(84.21) = 201.41

Answer is C.

**Problem 5 - 11**

Normal cost must be determined for both 1/1/86 and 1/1/87. The normal cost at 12/31/87 is simply the normal cost at 1/1/87 multiplied by $(1 + i)$.

Step I: Determine Normal Cost at 1/1/86.

$$\begin{aligned} NC_{86} &= (\text{Ben.})(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{n}|} \\ &= (50\% \times 36{,}000)(8.33) \div \ddot{s}_{\overline{11}|} \\ &= (18{,}000)(8.33) \div 17.9771 = 8{,}340.61 \end{aligned}$$

Step II: Determine Normal Cost at 1/1/87 by calculating Normal Cost on Benefit decrease only.

$$\begin{aligned} NC\ (\text{decrease}) &= (\text{Ben decrease})(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{10}|} \\ &= (50\% \times 6{,}000)(8.33) \div 15.6455 = 1{,}597.26 \end{aligned}$$

$$\text{Normal Cost at } 1/1/87 = 8{,}340.61 - 1{,}597.26 = 6{,}743.35$$

$$\begin{aligned} \text{Normal Cost at } 12/31/87 &= \text{Normal Cost at } 1/1/87 \times (1 + i) \\ &= 6{,}743.35 \times 1.08 = 7{,}282.82 \end{aligned}$$

Answer is A.

**Problem 5 - 12**

Key Concept: Normal Cost at 1/1/86 should be determined and then multiplied by $(1 + i)$ to determine the Normal Cost at 12/31/86.

Step I: Determine Normal Cost at 1/1/86.

$$\begin{aligned} NC_{86} &= (\text{Ben})(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{21}|} \\ &= (30\% \times 200{,}000)(8.40) \div 54.4568 = 9{,}255.04 \end{aligned}$$

Step II: Determine Normal Cost at 12/31/86.

$$\begin{aligned} NC &= NC_{86} \times 1.08 \\ &= 9{,}255.04 \times 1.08 = 9{,}995.45 \end{aligned}$$

Answer is A.

**Problem 5 - 12 (Alternative solution)**

Key Concept: Problem can be solved by using immediate annuity functions rather than annuity due functions:

$$NC_{86} = (30\% \times 200{,}000)(8.40) \div s_{\overline{27|}}$$
$$= (60{,}000)(8.40) \div 50.4229 = 9{,}995.46$$

**Problem 5 - 13**

Key Concepts: This problem consists of determining and adding the following:

1. Normal cost at 1/1/87;
2. Normal cost for benefit increase at 1/1/88;
3. Actuarial loss and amount of amortization payment.

Step I: Determine Normal Cost at 1/1/87.

$$NC_{87} = (\text{Benefit})(9.333) / \ddot{s}_{\overline{15|}} \text{ or}$$
$$= (\text{Benefit})(9.333)(1.06)^{-15}/\ddot{a}_{\overline{15|}}$$
$$= (40{,}000)(9.333) / 24.6725 = 15{,}131$$

Step II: Determine Normal Cost for benefit increase as of 1/1/88.

$$NC_{88}(\text{Increase}) = (\text{Benefit Increase})(9.333) / \ddot{s}_{\overline{14|}}$$
$$= (8{,}000)(9.333) / 22.2760 = 3{,}352$$

Step III: Determine Actuarial Loss.

$$\text{Expected Assets} = (NC_{87})(1.06)$$
$$= 15{,}131 \times 1.06 = 16{,}039$$

$$\text{Actuarial Gain} = \text{Assets - expected Assets}$$
$$= 15{,}000 - 16{,}039 = 1{,}039 \text{ (loss)}$$

$$\text{Amortization payment} = 1{,}039 \div \ddot{a}_{\overline{10}|}$$
$$= 1{,}039 \div 7.802 = 133$$

$$\text{Contribution} = 15{,}131 + 3{,}352 + 133 = 18{,}616$$

Answer is B.

**Problem 5 - 14**

Key Concept: Normal cost remains level unless the benefit changes due to salary increases (decreases) or benefit formula changes. The increase (decrease) in benefit is funded from the date of increase (decrease) in benefit.

$$\text{Benefit}_{88} = (.4)(200{,}000) = 80{,}000$$

$$NC_{88} = \frac{PVFB_{88}}{\text{Temporary Annuity}_{88}}$$

$$= (80{,}000)(v^{22})(\ddot{a}_{65}^{(12)}) \div (\ddot{a}_{\overline{22}|})$$
$$= (80{,}000)(.2775)(9.35) \div 12.76408 = 16{,}262$$

Note that the Normal Cost could also be computed:

$$= (80{,}000)(\ddot{a}_{65}^{(12)}) \div (\ddot{s}_{\overline{22}|}) = 16{,}262$$

In 1989, the Salary decreased.

$$\text{1/1/89 High 3 year average} = \frac{200{,}000 + 170{,}000 + 170{,}000}{3} = 180{,}000$$
$$\text{Benefit}_{89} = (.4)(180{,}000) = 72{,}000$$

$$\Delta\text{Benefit} = 72{,}000 - 80{,}000 = (8{,}000)$$

$$\Delta NC = (8{,}000)(\ddot{a}_{65}^{(12)})(v^{21}) \div (\ddot{a}_{\overline{21}|}) = (1{,}764)$$

Therefore, the total ILP Normal Cost = 16,262 - 1,764 = 14,498

Answer is B.

**Problem 5 - 15**

Key Concept: The Accrued Liability in the Individual Level Premium Cost Method is an accumulation of the prior normal costs. The change in interest rate requires a recomputation of those prior normal costs.

Step I: Determine 1/1/88 Normal Cost and the 1/1/89 Accrued Liability under the original assumptions.

$$\text{Prior NC}_{88} = (24{,}000)(.5)(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{30|}.08}$$
$$= (24{,}000)(.5)(8.1958)/122.3459 = 804$$

$$\text{Prior AL}_{89} = \text{NC}_{88} \times 1.08 = 804 \times 1.08 = 868$$

Step II: Determine 1/1/88 Normal Cost and 1/1/89 Accrued Liability under the new assumptions.

$$\text{New NC}_{88} = (24{,}000)(.5)(\ddot{a}_{65}^{(12)}) \div \ddot{s}_{\overline{30|}.06}$$
$$= (24{,}000)(.5)(9.3452)/83.8017 = 1{,}338$$

$$\text{New AL}_{89} = \text{NC}_{88} \times 1.06 = 1{,}338 \times 1.06 = 1{,}418$$

Step III: Determine the increase in the Accrued Liability.

$$\text{Increase} = 1{,}418 - 868 = 550$$

Answer is A.

Note: The 1989 Compensation would be used to calculate the increase in Normal Cost for the 1/1/89 valuation but is not used to calculate the Accrued Liability.

**Problem 5 - 16**

Step I: Calculate the Normal Cost.

Retirement Benefit = 15 × 25 Years of Service = 375

Since the benefit is a flat dollar amount, the Normal Cost has been level since 1984. Therefore, the Normal Cost can be calculated using the Present Value of Future Service.

$$NC_{ILP} = \frac{PVFB_{45}}{\ddot{a}_{45:\overline{20}|}}$$

$$= \frac{12 \times 375 \times \ddot{a}_{65}^{(12)} \times \dfrac{D_{65}}{D_{45}}}{\dfrac{N_{45} - N_{65}}{D_{45}}} = \frac{11542}{11.59654} = 995$$

Step II: Calculate the Accrued Liability.

$$AL_{89} = NC \times \ddot{s}_{45:\overline{5}|} = 995 \times (N_{45} - N_{50})/D_{50}$$
$$= 995 \times (9{,}789 - 6{,}712)/508 = 6{,}027$$

Answer is C.

**Problem 5 - 17**

Key Concept: Accrued Liability under the Individual Level Premium method is the accumulation of prior normal costs. Benefit increases, and corresponding Normal Cost increases, occur when salary increases.

The benefit increase in 1988 due to the 1988 salary increase is

$$\text{Benefit}_{85} = (.02) \times (3{,}500) \times (40 \text{ years of service}) = 2{,}800$$

$$NC_{85} = \frac{2{,}800 \times 12\ddot{a}_{65}^{(12)} \times v^{40}}{\ddot{a}_{\overline{40}|}} = 1{,}470$$

$$\Delta\ \text{Benefit}_{88} = (.02)(40 \text{ years of service})(4{,}000\text{-}3{,}500) = 400$$

$$\Delta NC = (400)(12\ddot{a}_{65}^{(12)})(v^{37}) \div \ddot{a}_{\overline{37}|} = 261$$

$$AL = 1{,}470\ \ddot{s}_{\overline{5}|} + 261\ \ddot{s}_{\overline{2}|} = 9{,}045 + 578 = 9{,}623$$

Answer is B.

**Problem 5 - 18**

Key Concept: Normal Cost stays level as a dollar amount from <u>participation</u> age to Normal Retirement Age unless there are plan changes.

Benefit = 25 x 34 years of service = 850

$$NC_{ILP} = \frac{PVFB_{pa}}{\ddot{a}_{pa:\overline{ra-pa}|}} \quad \text{where pa = participation age}$$

$$= \frac{12 \times 850 \times \ddot{a}_{65}^{(12)} \times \dfrac{D_{65}}{D_{32}}}{\dfrac{N_{32} - N_{65}}{D_{32}}}$$

$$= \frac{12 \times 850 \times 8.5 \times \left(\dfrac{200}{1500}\right)}{\left(\dfrac{24{,}000 - 1{,}792}{1{,}500}\right)} = 781$$

$$PVFNC_{91} = 781 \times \ddot{a}_{41:\overline{24}|}$$

$$= 781 \times \frac{N_{41} - N_{65}}{D_{41}}$$

$$= 781 \times \frac{13{,}050 - 1{,}792}{900} = 9{,}769$$

$$= 781 \times \left[\frac{N_{41} - N_{65}}{D_{41}}\right] = \left[\frac{13{,}050 - 1{,}792}{900}\right] = 9{,}769$$

Answer is E.

**Problem 5 - 19**

Key Concept: The increase in Normal Cost under the ILP method is equal to the cost of funding the increase in benefit from attained age to retirement age.

Step I: Calculate the 1990 Benefit.

$$\text{Benefit}_{90} = 43{,}500\left(\frac{(1.04)^{20} + (1.04)^{21} + (1.04)^{22}}{3}\right) \times .75 = 74{,}383$$

Step II: Calculate increased 1991 Benefit.

$$\text{Benefit}_{91} = 46{,}000\left(\frac{(1.04)^{19} + (1.04)^{20} + (1.04)^{21}}{3}\right) \times .75 = 75{,}633$$

Note: Another easier way to get the 1991 Benefit is to look at the salary increase. The salary increased 5.747% from 1990 to 1991. Thus, the change in benefit due to the change in salary can be calculated by multiplying the salary by the ratio of the actual percentage increase to the expected increase:

$$\text{Benefit}_{91} = \frac{1.05747}{1.04} \times 74{,}383 = 1.016798 \times 74{,}383 = 75{,}633$$

Increase in Benefit = 75,633 - 74,383 = 1,250

Step III: Calculate the 1991 increase in Normal Cost using the benefit increase.

$$\Delta\text{NC} = \frac{1{,}250 \times v^{22} \times \ddot{a}_{65}^{(12)}}{\ddot{a}_{\overline{22|}}} = 177$$

Answer is B.

**Problem 5 - 20**

Key Concept: The experience gain is the difference between the expected liability under the funding method if Smith had not retired and the actual value of the benefit due to early retirement. Since plan benefit is not salary based and there have been no plan amendments, NC each year is the same.

Step I: Determine Actual Liability due to Smith's early retirement.

The benefit accrued to 1/1/92 = 41.67 × 20 years of service = 833.40

The actuarial equivalent value of the benefit is:

$$= 12 \times 833.40 \times \ddot{a}_{65}^{(12)} \times v^{10} \times {}_{10}p_{55} = 37{,}043$$

Step II: Determine the Accrued Liability under ILP if Smith had not retired.

Benefit at retirement = 41.67 × 30 years of service = 1,250.10

$$NC_{92} = \frac{12 \times 1{,}250.10 \times \ddot{a}_{65}^{(12)} \dfrac{D_{65}}{D_{45}}}{\ddot{a}_{45:\overline{20|}}}$$

$$AL_{92} = PVFB_{55} - PVFNC_{55}$$

$$= 1{,}250.10 \times 12\ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{55}} - \left[ \frac{1{,}250.10 \times 12\ddot{a}_{65}^{(12)} \dfrac{D_{65}}{D_{45}}}{\ddot{a}_{45:\overline{20|}}} \right] \times \ddot{a}_{55:\overline{10|}}$$

We must manipulate some of the commutation functions given:

$$\ddot{a}_{45:\overline{20}|} = \frac{N_{45}-N_{65}}{D_{45}} = 10.79 \quad and \quad \ddot{a}_{45:\overline{10}|} = \frac{N_{45}-N_{55}}{D_{45}} = 7.37$$

$$\frac{N_{45} - N_{65}}{D_{45}} = \frac{N_{45} - N_{55}}{D_{45}} + \frac{N_{55} - N_{65}}{D_{45}} = 7.37 + \frac{N_{55} - N_{65}}{D_{45}} = 10.79$$

Solving for $N_{55}$ - $N_{65}$ gives:

$$N_{55} - N_{65} = 3.42\ D_{45}$$

or

$$\frac{N_{55} - N_{65}}{D_{55}} = 3.42\frac{D_{45}}{D_{55}}$$

$$\ddot{a}_{55:\overline{10}|} = 3.42\frac{D_{45}}{D_{55}}$$

Substituting,

$$AL_{92} = 1{,}250.10 \times 12\ddot{a}_{65}^{(12)}\ \frac{D_{65}}{D_{55}} - \left[\frac{1{,}250.10 \times 12\ddot{a}_{65}^{(12)}\left(\frac{D_{65}}{D_{45}}\right)}{\ddot{a}_{45:\overline{20}|}}\right] \times 3.42 \times \frac{D_{45}}{D_{55}}$$

$$= 1{,}250.10 \times 12\ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{55}}\left[1 - \frac{3.42}{10.79}\right] = 37{,}952$$

Step III: Calculate gain.

Gain = 37,952 - 37,043 = 909

Answer is A.

**Problem 5 - 21**

Key Concept: An increase or decrease in average compensation results in an increase or decrease in normal cost under the ILP method.

Step I: Determine the 1/1/91 Normal Cost.

Projected Annual Benefit = 60,000 × .5 = 30,000

$$NC_{91} = \frac{PVFB}{PVFS} \times \text{Salary}$$

$$= \frac{30{,}000 \times \ddot{a}_{65}^{(12)} \times v^{25}}{\ddot{a}_{\overline{25}|}} = \frac{48{,}310}{12.4693} = 3{,}874$$

Note that since there is only one participant,

$$\frac{PVFS}{\text{Salary}} = \ddot{a}_{\overline{25}|}$$

Step II: Determine the 1/1/92 change in Normal Cost.

Decrease in Normal Cost = 3,874 - 3,032 = 842

Step III: Calculate change in 1/1/92 Projected Salary.

$$(842) = \frac{(.5)(\Delta\text{Salary}) \times \ddot{a}_{65}^{(12)} \times v^{24}}{\ddot{a}_{\overline{24}|}} = \frac{(\Delta\text{Salary})(.5)(8.74)(.1971)}{12.2722}$$

$$\Delta\text{Salary} = (11{,}997)$$

Step IV: Calculate 1/1/92 Projected Salary.

$$\text{Salary}_{92} = 60{,}000 - 11{,}997 = 48{,}003$$

Answer is B.

**Problem 5 - 22**

Key Concept: An increase in compensation results in a normal cost increase associated with the compensation increase.

$$\text{ILP NC}_{92} = \frac{(.5)(16{,}000)(\ddot{a}_{65}^{(12)})}{\ddot{s}_{\overline{16|}}} = 2{,}343$$

Since 1993 compensation increases to 18,000, the compensation increase is:

$$18{,}000 - 16{,}000 = 2{,}000$$

We must determine the increase in Normal Cost that will fund for the benefit increase.

$$\Delta NC_{93} = \frac{(.5)(2{,}000)(\ddot{a}_{65}^{(12)})}{\ddot{s}_{\overline{15|}}} = 325$$

Note that $\Delta NC_{93}$ is funded only over the 15 years remaining to retirement.

$$\text{Total NC}_{93} = 2{,}343 + 325 = 2{,}668$$

Answer is C.

**Problem 5 - 23**

Key Concept: A decrease in compensation results in a normal cost decrease associated with the compensation decrease.

$$\text{ILP NC}_{91} = \frac{(100{,}000)(.5)(\ddot{a}_{65}^{(12)})}{\ddot{s}_{\overline{14}|}} = 18{,}111$$

1/1/92 Final Compensation Decrease = 100,000 - 92,000 = 8,000

Note that since the participant has more than 5 years before reaching retirement, the final 5-year average (not high consecutive 5-year average) is merely \$92,000.

$$\text{1/1/92 Normal Cost decrease} = \frac{(8{,}000)(.5)(\ddot{a}_{65}^{(12)})}{\ddot{s}_{\overline{13}|}} = 1{,}622$$

$$\text{ILP NC}_{92} = 18{,}111 - 1{,}622 = 16{,}489$$

Since the Accrued Liability is the accumulation of past Normal Costs,

$$\begin{aligned} AL_{93} &= (18{,}111)(1.07)^2 + (16{,}489)(1.07) \\ &= 20{,}735 + 17{,}643 = 38{,}378 \end{aligned}$$

Answer is C.

**Problem 5 - 24**

Key Concept: The Unfunded Accrued Liability in an immediate gain method is equal to the accrued liability less the assets.

Step I: Calculate the Accrued Liability as of 1/1/93.

$$\text{Unit Credit AL}_{93} = \text{PVAB}_{93} = (15)(6\text{ years of service})(12\ddot{a}_{65}^{(12)})v^{14}$$

We must solve for $12\ddot{a}_{65}^{(12)}$. To do so, we must look at the calculation of the Aggregate Normal Cost as of 1/1/92.

$$PVFB_{92} = (15)(20 \text{ years of service})(12\ddot{a}_{65}^{(12)})v^{15}$$

$$NC_{92} = \frac{(PVFB_{92} - Assets_{92})}{\ddot{a}_{\overline{15|}}}$$

$$= \frac{(15)(20 \text{ years of service})(12\ddot{a}_{65}^{(12)})(v^{15}) - 1{,}000}{9.244237} = 898$$

$$12\ddot{a}_{65}^{(12)} = 98.3513$$

Substituting,

$$\text{Unit Credit } AL_{93} = (15)(6)(98.3513)v^{14} = 3{,}014$$

Step II: Calculate the assets as of 1/1/93.

$$\begin{aligned} Assets_{93} &= (Assets_{92} + Contribution_{92}) \times 1.05 \\ &= (1{,}000 + 898) \times 1.05 = 1{,}993 \end{aligned}$$

Step III: Calculate unfunded accrued liability as of 1/1/93.

$$\begin{aligned} UAL_{93} &= AL_{93} - Assets_{93} \\ &= 3{,}014 - 1{,}993 = 1{,}021 \end{aligned}$$

Answer is E.

**Problem 5 - 25**

Key Concept: The Normal Cost under the Individual Level Premium method is the sum of the Normal Cost determined to fund the initial benefit at the time the plan is established, plus the Normal Costs determined to fund the increases (decreases) in the projected retirement benefit each year. If the projected retirement benefit decreases in a given year, the Normal Cost determined to "fund" the decrease will be negative, and the total Normal Cost for that participant will decrease for the year.

Step I: Calculate the Normal Cost as of 1/1/90.

$$\text{Benefit} = (60{,}000)(.5) = 30{,}000$$

$$\begin{aligned} NC_{90} &= (\text{Benefit})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{5|}} \\ &= (30{,}000)(8.786)/6.15329 = 42{,}836 \end{aligned}$$

Step II: Calculate the increase in the Normal Cost due to the increase in benefits as of 1/1/92.

$$\text{Benefit Increase} = (80{,}000 - 60{,}000)(.5) = 10{,}000$$

$$\begin{aligned} \Delta NC_{92} &= (\text{Benefit Increase})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{3|}} \\ &= (10{,}000)(8.786)/3.43994 = 25{,}541 \end{aligned}$$

Step III: Calculate the decrease in Normal Cost due to the benefit decrease in benefits as of 1/1/94.

$$\text{3 year average salary} = (80{,}000 + 80{,}000 + 65{,}000)/3 = 75{,}000$$

$$\text{Benefit Decrease} = (75{,}000 - 80{,}000)(.5) = (2{,}500)$$

$$\begin{aligned} \Delta NC_{94} &= (\text{Benefit Decrease})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{1|}} \\ &= (2{,}500)(8.786)/1.07 = (20{,}528) \end{aligned}$$

Step IV: Calculate the total Normal Cost as of 1/1/94.

$$\begin{aligned} NC_{94} &= NC_{90} + \Delta NC_{92} + \Delta NC_{94} \\ &= 42{,}836 + 25{,}541 - 20{,}528 = 47{,}849 \end{aligned}$$

Answer is C.

**Problem 5 - 26**

Key Concept: The cash value at retirement of the life insurance policy must be subtracted from the value of the benefit at retirement in order to determine the normal cost for the side fund. Alternatively, the present value of the cash value at retirement can be subtracted from the present value of future benefits.

$$\begin{aligned} \text{Face amount of insurance} &= 60 \times \text{Monthly retirement benefit} \\ &= 60 \times 400 = 24{,}000 \end{aligned}$$

$$\text{Cash Value at 65} = 24 \times 550 \text{ (per thousand of face amount)} = 13{,}200$$

$$PVFB = [(400)(12\ddot{a}_{65}^{(12)}) - 13{,}200](v^{26}) = 7{,}649$$

$$\text{Side Fund } NC_{1/1/95} = PVFB/(\ddot{a}_{\overline{26|}})$$
$$= 7{,}649/13.7834 = 555$$

$$\text{Side Fund } NC_{12/31/95} = 555 \times 1.06 = 588$$

Answer is E.

**Problem 5 - 26 (Alternative Solution)**

Since it is the first year of the plan and there are no assets, the normal cost can be calculated using the Present Value of benefits at retirement.

$$\text{Present Value of benefits at retirement} = (400)(12\ddot{a}_{65}^{(12)}) - 13{,}200 = 34{,}800$$

$$\text{Side Fund } NC_{12/31/95} = 34{,}800/s_{\overline{26|}}$$
$$= 34{,}800/59.1564 = 588$$

Note that $s_{\overline{26|}}$ was used instead of $\ddot{s}_{\overline{26|}}$ due to the end of year valuation date.

Answer is E.

**Problem 5 - 27**

Key Concept: An increase in the benefit formula results in a normal cost increase associated with the benefit increase.

Step I: Calculate the original Normal Cost.

$$\text{Original Normal Cost} = (15)(40 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})/\ \ddot{s}_{\overline{35|}}$$
$$= 425$$

Step II: Calculate the Normal Cost increase.

$$\text{Normal Cost Increase} = (3)(40 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})/\ \ddot{s}_{\overline{26|}}$$
$$= 171$$

Note that the benefit increase was \$3 per month per year of service and that the value of the benefit due to the increase was amortized over the remaining 26 years of service.

Step III: Calculate the total Normal Cost.

$$\text{Total Normal Cost} = 425 + 171 = 596$$

Answer is C.

**Problem 5 - 28**

Step I: Calculate the Individual Level Premium Normal Cost using Method A.

$$\begin{aligned}
\text{Normal Cost (other than death)} &= (1{,}000)(12)(\ddot{a}_{65}^{(12)}) / \ddot{s}_{40:\overline{25}|} \\
&= (12{,}000)(8.74)/[(N_{40} - N_{65})/D_{65}] \\
&= (12{,}000)(8.74)/[(8{,}452{,}729 - 868{,}052)/94{,}414] \\
&= 1{,}305
\end{aligned}$$

$$\begin{aligned}
\text{One Year Term Cost} &= (100{,}000)(C_{40} / D_{40}) \\
&= (100{,}000)((M_{40} - M_{41}) / D_{40}) \\
&= (100{,}000)((79{,}292 - 78{,}036)/632{,}275) \\
&= 199
\end{aligned}$$

$$\text{Total Normal Cost} = 1{,}305 + 199 = 1{,}504$$

Step II: Calculate the Aggregate Normal Cost using Method B.

$$\text{Value of Benefit at Retirement} = (1000)(12)(\ddot{a}_{65}^{(12)}) = 104{,}880$$

$$\begin{aligned}
\text{Normal Cost} &= (104{,}880 - 41{,}900) / \ddot{s}_{40:\overline{25}|} \\
&= 784
\end{aligned}$$

$$\text{Total Normal Cost} = 784 + 1{,}585 = 2{,}369$$

Step III: Calculate the difference in the Normal Costs

$$\text{Difference} = 2{,}369 - 1{,}504 = 865$$

Answer is C.

**Problem 5 - 29**

Key Concept: The increase in Normal Cost under the Individual Level Premium method is equal to the cost of funding the increase in benefit from attained age to retirement age. The amortization does not reflect the salary scale *since* the method is specified as being a level dollar amount method.

Step I: Calculate the 1996 benefit.

$$\text{Benefit}_{96} = (.02)(40 \text{ years})(25{,}000)[1.05^{35} + 1.05^{34} + 1.05^{33}]/3 = 105{,}150$$

Note that since retirement is assumed to occur on 1/1/2032 (age 65), the final salary used will be in the 1/1/2031 valuation, 35 years from 1996.

Step II: Calculate the 1997 benefit.

$$\text{Benefit}_{97} = (.02)(40 \text{ years})(27{,}500)[1.05^{34} + 1.05^{33} + 1.05^{32}]/3 = 110{,}158$$

Note: Another way to determine the 1997 benefit is to use the percentage salary increase (10% from 1996 to 1997). The 1997 benefit can be calculated by multiplying the 1996 benefit by the ratio of the actual percentage increase to the expected percentage increase:

$$\text{Benefit}_{97} = (1.10/1.05)(105{,}150) = 110{,}158$$

Step III: Calculate the benefit increase.

Benefit increase = 110,158 - 105,150 = 5,008

Step IV: Calculate the 1997 increase in Normal Cost using the benefit increase.

$$\begin{aligned} NC_{\text{INCREASE}} &= [(5{,}008) \times \ddot{a}_{65}^{(12)} \times v^{35}]/\ddot{a}_{\overline{35}|} \\ &= [(5{,}008) \times 9.815 \times .0937]/13.8540 \\ &= 332 \end{aligned}$$

Answer is D.

# Chapter 6

# Frozen Initial Liability Cost Method

## 6.1 Normal Cost and Accrued Liability

The determination of the annual contribution necessary to fund a plan under each of the funding methods described thus far involved one or more of three factors, namely, normal cost, amortization of unfunded accrued liability and amortization of experience gains and losses. Under the Aggregate Cost Method, only the normal cost determined the contribution while under the Individual Level Premium Cost Method, normal cost and experience gains and losses were involved; and under the Entry Age Normal and Unit Credit Cost Methods all three were involved.

The Frozen Initial Liability Cost Method is an aggregate method that requires the determination of normal cost plus amortization of the unfunded accrued liability, but not the separate amortization of experience gains and losses. Experience gains and losses are spread over future Normal Costs, just as under the Aggregate Method.

Determination of Normal Cost under this method requires four variables. These are:

(1) Present Value of Future Benefits, which is the liability at retirement discounted to the valuation date under the assumptions used. Have you noticed that this item is the same regardless of the funding method used?

(2) Unfunded Accrued Liability, on the first valuation date is calculated using the Entry Age Normal Method. Thereafter, it is calculated using the relationship:

$$UAL_1 = (UAL_0 + NC_0)(1 + i) - C_0 - I_c$$

where $UAL_0$ = prior year Unfunded Accrued Liability
$NC_0$ = prior year Normal Cost
$C_0$ = prior year contribution
and $I_c$ = interest earned on contribution

Notice that this is the same formula as is used to calculate the Expected Unfunded Accrued Liability under an immediate gain type method such as Entry Age Normal. Under the Frozen Initial Liability Cost Method, the actual Unfunded Accrued Liability is equal to the expected Unfunded Accrued Liability, since experience gains and losses are not separately calculated or amortized.

(3) Actuarial Value of Assets is the fund balance as evaluated by the Plan Actuary.

(4) Present Value of Future Service is the total number of years remaining to retirements of all participants discounted to the valuation date on the basis of assumed interest.

If these four variables are known, the Normal Cost can be determined as follows:

Step I: Determine the Present Value of Future Normal Costs (PVFNC), which is the Present Value of Future Benefits (PVFB) minus the Unfunded Accrued Liability (UAL) minus the Assets. This relationship can be written as:

$$\text{PVFNC} = \text{PVFB} - \text{UAL} - \text{Assets}$$

Step II: Determine the Normal Cost per participant by dividing the Present Value of Future Normal Costs (PVFNC) by the Present Value of Future Service.

$$NC_i = \text{PVFNC} \div \text{PV of Future Service}$$

Step III: Finally, the individual Normal Cost is multiplied by the number of participants in the plan to determine the Normal Cost for the plan.

$$NC = NC_i \times \text{Number of participants}$$

Normal Cost can also be determined if we know (1) the Present Value of Future Normal Costs, (2) Present Value of Future Salaries and (3) Current Salaries.

$$\begin{aligned} NC\% &= (\text{PVFNC}) \div (\text{PVFS}) \\ NC &= (\text{Salaries}) \times NC\% \\ \text{or } NC &= (\text{PVFNC}) \times (\text{Salaries}) \div (\text{PVFS}) \end{aligned}$$

This procedure is used particularly when salary increases are assumed or when benefits are based upon salary, but can be used at any time. This approach determines Normal Costs as a level percentage of salaries, whereas the approach described previously determines Normal Costs as a level dollar amount per participant.

A question also arises as to how to handle a plan amendment under the Frozen Initial Liability Method. The usual approach is to adjust the Unfunded Accrued Liability to reflect the effects of the amendment. The adjustment is made based on the change in the Entry Age Normal Accrued Liability (EAN AL) due to the amendment. Using the symbol ′ to denote values before the amendment,

$$UAL = UAL' + (EAN\ AL - EAN\ AL')$$

This adjustment has the effect of changing the Unfunded Accrued Liability to what it would have been had the amendment always been in effect.

## 6.2 Characteristics of Frozen Initial Liability Cost Method

1. The initial past service liability is determined under the Entry Age Normal Cost Method.

2. The FIL Method is an aggregate method and hence is not usually suitable for small plans.

3. Experience gains and losses are automatically spread over the remaining working years of participants.

4. When the Past Service Liability is completely amortized, the cost method reverts to the Aggregate Method.

## 6.3 Problems

### Problem 6 - 1

Valuation Date: 1/1/85
Assumptions Used: Interest: 6%
No mortality or withdrawal

There were no changes in benefits between 1/1/85 and 1/1/86.
The Plan has 4 participants.

Selected Valuation Data:

| | 1/1/85 | 1/1/86 |
|---|---|---|
| Present Value of Future Benefits | $400,000 | |
| Unfunded Accrued Liability | 95,000 | |
| Actuarial Value of Assets | 135,000 | $160,000 |
| Present Value of Future Service | 38.0 | 36.3 |

Contribution of $25,000 for 1985 was made on 12/31/85.

Determine the Normal Cost as of 1/1/86.

### Problem 6 - 2

Valuation Date: 1/1/85

Selected Valuation Results:

| | |
|---|---|
| Present Value of Future Benefits | $600,000 |
| Present Value of Future Normal Costs | 200,000 |

The Plan was amended to provide a 10% increase in benefits which generated an increase in the Entry Age Normal Accrued Liability of $20,000.

What is the Present Value of Future Normal Costs after the amendment?

**Problem 6 - 3**

Valuation Date: 1/1/86

Retirement benefit is a flat percentage of salary.
All participants are active employees.

| | |
|---|---|
| Assumptions used: | Salary increases of 4% |
| | Interest of 6% |
| | No pre-retirement mortality |
| | No withdrawals |

Valuation results as of 1/1/85:

| | |
|---|---|
| Present Value of Future Benefits | $3,000,000 |
| Actuarial Value of Assets | 1,250,000 |
| Unfunded Accrued Liability | 750,000 |
| Annual Salaries | 600,000 |
| Present Value of Future Salaries | 15,000,000 |

Contribution of $125,000 for 1985 was made on 12/31/85.
Investment earnings on Plan assets were $105,000 in 1985.
All other assumptions were exactly realized between 1/1/85 and 1/1/86.

Determine the Normal Cost as of 1/1/86.

**Problem 6 - 4**

All information is the same as in Problem 6 - 3, except that no changes in salary occurred between 1/1/85 and 1/1/86. Determine the Normal Cost as of 1/1/86.

**Problem 6 - 5**

Plan effective date: 1/1/81
Normal retirement benefit: 50% of final 5-year average salary.
Actuarial cost method: Frozen initial liability method.

Valuation results as of 1/1/84:

| | |
|---|---|
| Present Value of Future Benefits | $1,700,000 |
| Unfunded Liability | 300,000 |
| Actuarial Value of Assets | 200,000 |
| Normal cost as of 1/1 | 80,000 |

As of 1/1/84, there are no retired or terminated vested participants. All active participants are at least five years younger than the assumed retirement age.

After the above results were determined, it was discovered that all salaries were 10% higher than reported. The normal cost for 1984 was then recalculated.

In what range is the recalculated normal cost for 1984 as of 1/1/84?

(A) Less than $89,000
(B) $89,000 but less than $90,000
(C) $90,000 but less than $91,000
(D) $91,000 but less than $92,000
(E) $92,000 or more

**Problem 6 - 6**

Actuarial cost method: Frozen initial liability method.
Assumed interest rate: 7%

| Valuation results: | 1/1/83 | 1/1/84 |
|---|---|---|
| Present value of future benefits | $850,000 | $901,000 |
| Present value of future normal costs | 420,000 | |
| Actuarial value of assets (market) | 125,000 | |
| Normal cost as of 1/1 | 23,000 | |

Contributions of $50,000 for 1983 were paid in two installments of $25,000 each at 1/1/83 and 12/31/83. There were no benefits payable in 1983.

Actual investment earnings exceeded assumed investment earnings by $3,000 in 1983.

In what range is the present value of future normal costs as of 1/1/84?

(A) Less than \$413,000
(B) \$413,000 but less than \$416,000
(C) \$416,000 but less than \$419,000
(D) \$419,000 but less than \$422,000
(E) \$422,000 or more

**Problem 6 - 7**

Plan effective date: 1/1/84
Normal retirement benefit: 40% of salary in the year preceding retirement.
Actuarial cost method: Frozen initial liability.
Actuarial assumptions:

Interest: 7%
Salary increases: 5%
Pre-retirement deaths and terminations: None.
Retirement age: 65

The oldest participant on 1/1/84 was aged 63.

Valuation results as of 1/1/84:

| | |
|---|---|
| Unfunded Accrued Liability | \$ 550,000 |
| Present Value of Future Salaries | 4,011,000 |
| Normal Cost as of 1/1 | 80,000 |
| Salaries | 400,000 |

Contribution for 1984 paid at 12/31/84: \$164,000

As of 1/1/85, all participants received a 10% salary increase. There were no deaths, terminations or new entrants in 1984.

In what range is the normal cost as of 1/1/85?

(A) Less than \$86,000
(B) \$86,000 but less than \$88,000
(C) \$88,000 but less than \$90,000
(D) \$90,000 but less than \$92,000
(E) \$92,000 or more

**Problem 6 - 8**

Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Frozen initial liability.

| | |
|---|---|
| Actuarial assumptions: | Interest: 7% |
| | Pre-retirement deaths and terminations: None. |
| | Retirement age: 65 |

The sole plan participant is age 50 as of 1/1/85

Due to a plan amendment effective 1/1/85 which changed the normal retirement age from 65 to 62, the assumed retirement age is changed to 62. The unfunded liability is adjusted as of 1/1/85 to reflect the change in assumptions.

Selected valuation results as of 1/1/85:

| | Assumed Retirement Age | |
|---|---|---|
| | 65 | 62 |
| Present value of future benefits | \$15,000 | \$18,400 |
| Unfunded liability | 6,150 | |
| Actuarial value of assets | 5,000 | 5,000 |
| Entry age normal accrued liability | 13,100 | 16,350 |

In what range is the normal cost as of 1/1/85 under the revised retirement age assumption?

(A) Less than \$300
(B) \$300 but less than \$350
(C) \$350 but less than \$400
(D) \$400 but less than \$450
(E) \$450 or more

**Problem 6 - 9**

Normal retirement benefit: 1% of the final year's salary times years of service.
Actuarial cost method: Frozen initial liability.

As of 1/1/85, the assumed interest rate is changed from 5% to 7%, and the salary increase assumption is changed from 3% to 6%. The unfunded liability is adjusted as of 1/1/85 to reflect the change in assumptions.

Selected valuation results as of 1/1/85:

| | Old assumptions | New Assumptions |
|---|---|---|
| Present value of future benefits | $6,148,750 | $7,146,000 |
| Present value of future salaries | 37,250,000 | 39,100,000 |
| Unfunded Liability | 750,000 | |
| Actuarial value of assets | 3,350,000 | 3,350,000 |
| Normal cost as of 1/1 | 132,000 | |
| Entry age normal accrued liability | 4,175,000 | 4,875,000 |

In what range is the normal cost as of 1/1/85 based on the new assumptions?

(A) Less than $135,000
(B) $135,000 but less than $140,000
(C) $140,000 but less than $145,000
(D) $145,000 but less than $150,000
(E) $150,000 or more.

**Problem 6 - 10**

| | |
|---|---|
| Actuarial cost method: | Frozen initial liability. |
| Actuarial assumptions: | Interest: 7% |
| | Salary increases: 5% |
| | Pre-retirement deaths and terminations: None. |

Valuation results as of 1/1/84:

| | |
|---|---|
| Present value of future benefits | $10,000,000 |
| Actuarial value of assets | 4,000,000 |
| Unfunded liability | 2,000,000 |
| Annual salaries | 4,000,000 |
| Present value of future salaries | 40,000,000 |

Contribution for 1984 paid at 12/31/84: $618,000

During the 1984 plan year all actuarial assumptions were exactly realized except the assets earned 8%.
All participants are active employees under assumed retirement age. There have been no new entrants since 1/1/84.

In what range is the normal cost as of 1/1/85?

(A) Less than $390,000
(B) $390,000 but less than $400,000
(C) $400,000 but less than $410,000
(D) $410,000 but less than $420,000
(E) $420,000 or more

**Problem 6 - 11**

Plan effective date: 1/1/83
Actuarial cost method: Frozen initial liability method.
Assumed interest rate: 8%

Results of 1/1/83 valuation:

| | |
|---|---|
| Accrued liability | $100,000 |
| Normal cost as of 1/1 | 10,000 |

Contribution for 1983: $20,000 at 1/1/84.

Beginning in 1984, the valuation date is changed from 1/1 to 12/31.

Results of 12/31/84 valuation:

| | |
|---|---|
| Present value of future benefits | $ 265,000 |
| Present value of future salary | 2,400,000 |
| Total annual salary | 200,000 |

Contribution for 1984: $25,000 at 1/1/85.

The fund earned 8% interest in 1984. There were no benefit payments in 1983 or 1984.

In what range is the normal cost for 1984 as of 12/31/84?

(A) Less than $10,500
(B) $10,500 but less than $11,000
(C) $11,000 but less than $11,500
(D) $11,500 but less than $12,000
(E) $12,000 or more

**Problem 6 - 12**

Normal retirement benefit: Before 1/1/86: 40% of final year's salary.
Effective 1/1/86: 50% of final year's salary.

Actuarial cost method: Frozen initial liability.

Selected valuation results, as of 1/1/86, based on the 40% benefit:

| | |
|---|---|
| Present value of future benefits: | |
| For active participants | $1,200,000 |
| For retired and terminated participants | 300,000 |
| Unfunded liability | 300,000 |
| Actuarial value of assets | 500,000 |
| Present value of future salaries | 21,000,000 |
| Annual salaries | 4,000,000 |
| Entry age normal accrued liability | 700,000 |

In what range is the increase in the normal cost for 1986 as of 1/1/86 due to the plan amendment?

(A) Less than $25,000
(B) $25,000 but less than $35,000
(C) $35,000 but less than $45,000
(D) $45,000 but less than $55,000
(E) $55,000 or more

**Problem 6 - 13**

Normal retirement benefit: 50% of final year's salary.
Actuarial cost method: Frozen initial liability.

Selected valuation results as of 1/1/86:

| | |
|---|---|
| Present value of future salaries | $2,500,000 |
| Normal cost as of 1/1 | 15,000 |
| Annual salaries | 200,000 |

After the valuation was done, it was discovered that a new entrant was omitted. A revised valuation included the following information for the omitted participant as of 1/1/86:

| | |
|---|---|
| Present value of future benefits | 39,000 |
| Annual salary | 30,000 |
| Present value of future salaries | 850,000 |

In what range is the revised normal cost for 1986 as of 1/1/86?

(A) Less than $16,000
(B) $16,000 but less than $17,000
(C) $17,000 but less than $18,000
(D) $18,000 but less than $19,000
(E) $19,000 or more

**Problem 6 - 14**

Normal retirement benefit: $10 per month for each year of service.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest: 8%
Retirement age: 65
Pre-retirement deaths and terminations: None.

Valuation dates:

For 1986 plan year: 12/31/86 For 1987 plan year: 1/1/87

Selected valuation results as of 12/31/86:

| | |
|---|---|
| Present value of future benefits | $900,000 |
| Unfunded liability | 400,000 |
| Normal cost as of 12/31 | 30,000 |

Actuarial value of assets as of 12/31/86: $200,000, prior to contribution for 1986.
Contribution for 1986: $60,000, paid at 12/31/86.

There is no change in participant data between 12/31/86 and 1/1/87.

In what range is the normal cost for 1987 as of 1/1/87?

(A) Less than $27,500
(B) $27,500 but less than $28,750
(C) $28,750 but less than $30,000
(D) $30,000 but less than $31,250
(E) $31,250 or more

**Problem 6 - 15**

Employee contributions: 2% of compensation, paid at the beginning of each year.
Actuarial cost method: Frozen initial liability.
Assumed interest rate: 6%

Valuation results as of 1/1/88:

| | |
|---|---|
| Present value of future benefits (including refunds of employee contributions) | $ 2,600,000 |
| Actuarial value of assets | 650,000 |
| Unfunded liability | 300,000 |
| Present value of future compensation | 12,500,000 |
| Annual compensation | 1,000,000 |

In what range is the employer's normal cost for 1988 as of 12/31/88?

(A) Less than $114,000
(B) $114,000 but less than $119,000
(C) $119,000 but less than $124,000
(D) $124,000 but less than $129,000
(E) $129,000 or more

**Problem 6 - 16**

Normal retirement benefit: $10 per month for each year of service.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: Normal retirement age.

Date of birth of sole participant: 1/1/38

The plan is amended effective 1/1/88 to change the normal retirement age from 65 to 62 and to increase the benefit rate for all years of service from $10 to $12.

Valuation results as of 1/1/88, based on $10 benefit rate:

| | Assumed Retirement Age | |
|---|---|---|
| | 65 | 62 |
| Present value of future benefits | $15,000 | $18,400 |
| Unfunded liability | 6,150 | - |
| Actuarial value of assets | 5,000 | 5,000 |
| Accrued liability under entry age normal method | 13,100 | 16,350 |

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than $400
(B) $400 but less than $475
(C) $475 but less than $550
(D) $550 but less than $625
(E) $625 or more

**Problem 6 - 17**

Actuarial cost method: Frozen initial liability.
Assumed interest rate: 6%

Selected valuation results:

| | 1/1/87 | 1/1/88 |
|---|---|---|
| Normal cost as of 1/1 | $ 25,000 | $ - |
| Present value of future benefits | 500,000 | 565,000 |
| Present value of future normal costs | 210,000 | - |
| Actuarial value of assets | 80,000 | - |

Contributions for 1987: $25,000 paid on 6/30/87 and $20,000 paid on 12/31/87.
Benefit payments for 1987: $0
The rate of return on the actuarial value of assets during 1987 was 8.4%.

In what range Is the present value of future normal costs as of 1/1/88?

(A) Less than \$195,000
(B) \$195,000 but less than \$205,000
(C) \$205,000 but less than \$215,000
(D) \$215,000 but less than \$225,000
(E) \$225,000 or more

**Problem 6 - 18**

Normal retirement benefit: 50% of final five-year average compensation.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest: 6%
Compensation increases: 4% per year.
Pre-retirement deaths and terminations: None.
Retirement age: 65

Valuation results as of 1/1/87:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 84,000 |
| Present value of future benefits: | |
| Active participants | 2,000,000 |
| Inactive participants | 0 |
| Unfunded liability | 500,000 |
| Actuarial value of assets | 100,000 |
| Present value of future compensation | 10,000,000 |
| Annual compensation | 600,000 |

Contribution for 1987: \$150,000 paid on 1/1/88.
All participants on 1/1/87 were under age 60.
There were no new participants during 1987.

Each participant received a 7.5% compensation increase on 1/1/88. There were no experience gains or losses during 1987 from any other source.

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than \$90,000
(B) \$90,000 but less than \$92,000
(C) \$92,000 but less than \$94,000
(D) \$94,000 but less than \$96,000
(E) \$96,000 or more

**Problem 6 - 19**

Actuarial cost method:

Before 1987: Unit credit.
After 1986: Frozen initial liability.

Assumed interest rate: 6%
Assumed compensation increases: 5% per year.

Selected valuation results:

| | 1/1/87 | 1/1/88 |
|---|---|---|
| Present value of future benefits | \$ 11,000,000 | \$ 12,000,000 |
| Actuarial value of assets | 3,000,000 | 4,000,000 |
| Present value of future compensation | 112,500,000 | 115,000,000 |
| Annual compensation | 16,500,000 | - |
| Accrued liability under unit credit method | 4,325,000 | 5,000,000 |
| Accrued liability under entry age normal method | 6,500,000 | 7,600,000 |

Contribution for 1987: \$1,000,000 paid on 12/31/87.

In what range is the normal cost rate (as a percentage of compensation) for 1988 as of 1/1/88?

(A) Less than 3.970%
(B) 3.970% but less than 3.980%
(C) 3.980% but less than 3.990%
(D) 3.990% but less than 4.000%
(E) 4.000% or more

**Problem 6 - 20**

Actuarial cost method: Frozen initial liability (level dollar amount).

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data and valuation results for only participants as of 1/1/90:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/20 | 1/1/55 | 1/1/45 |
| Status | Retired | Active | Active |
| Monthly accrued benefit | $1,000 | - | - |
| Monthly projected benefit | - | $5,000 | $3,000 |

Unfunded liability as of 1/1/90: $10,000
Value of assets as of 1/1/90: $91,200

Selected annuity values:

$\ddot{a}_{65}^{(12)} = 8.74 \qquad \ddot{a}_{70}^{(12)} = 7.60$

In what range is the normal cost for 1990 as of 1/1/90?

(A) Less than $9,000
(B) $9,000 but less than $12,000
(C) $12,000 but less than $15,000
(D) $15,000 but less than $18,000
(E) $18,000 or more

**Problem 6 - 21**

Normal retirement benefit: $15 per month for each year of service.

Early retirement benefit: Accrued benefit reduced by 6% for each of the first 5 years and 3% for each of the next 5 years by which commencement of payments precedes age 65.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 8%
Preretirement deaths and terminations: None.
Retirement age: 65

Data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of hire | 1/1/72 |
| Status as of 1/1/90 | Active |

Original valuation results as of 1/1/90:

| | |
|---|---|
| Present value of future benefits | $662,000 |
| Unfunded liability | 163,250 |
| Value of assets | 142,500 |
| Normal cost per active participant | 195.34 |
| Number of active participants | 150 |

Selected annuity values:

$\ddot{a}_{55}^{(12)} = 12.33 \qquad \ddot{a}_{65}^{(12)} = 8.33$

After the valuation was done, it was discovered that Smith had retired on 12/31/89 and commenced receiving benefits. Another valuation was done to correct this error.

In what range is the increase in normal cost for 1990 as of 1/1/90 due to Smith's retirement?

(A) Less than $0
(B) $0 but less than $100
(C) $100 but less than $200
(D) $200 but less than $300
(E) $300 or more

**Problem 6 - 22**

Plan effective date: 1/1/86
Normal retirement benefit:

> Before 1991: 30% of final year's compensation.
> After 1990: 40% of final year's compensation.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

> Interest rate: 7% per year.
> Compensation increases: 5% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Selected valuation results as of 1/1/90:

| | |
|---|---|
| Normal cost as of 1/1 | $ 25,000 |
| Present value of future benefits | 700,000 |
| Unfunded liability | 275,000 |
| Present value of future compensation | 5,675,000 |
| 1990 compensation | 500,000 |

Contribution for 1990: $50,000 paid on 12/31/90.

As of 1/1/90, there were no inactive participants and no active participants were over age 63. There were no new participants, deaths, terminations, or retirements during 1990. There are no new participants on 1/1/91.

All participants received a 10% increase in compensation from 1990 to 1991.

Selected valuation results as of 1/1/91, after amendment:

| | |
|---|---|
| Value of assets | $250,000 |
| Accrued liability under entry age normal cost method | 550,000 |

In what range is the normal cost for 1991 as of 1/1/91?

(A) Less than \$30,700
(B) \$30,700 but less than \$32,700
(C) \$32,700 but less than \$34,700
(D) \$34,700 but less than \$36,700
(E) \$36,700 or more

**Problem 6 - 23**

Actuarial cost method: Frozen initial liability (level dollar amount).

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for all participants:

| | Smith | Brown | Green |
|---|---|---|---|
| Age | 45 | 55 | 65 |
| Annual projected benefit | \$20,000 | \$20,000 | \$20,000 |
| Status as of 1/1/91 | Active | Active | Retired |

Unfunded liability as of 1/1/91: \$12,000
Value of assets as of 1/1/91: \$208,800

The contribution for 1991 is equal to the normal cost for 1991 as of 1/1/91 plus a 10-year amortization payment as of 1/1/91 toward the unfunded liability as of 1/1/91.

Selected annuity values:

| Age x | $\ddot{a}_x^{(12)}$ |
|---|---|
| 45 | 12.33 |
| 55 | 10.78 |
| 65 | 8.74 |

In what range is the contribution for 1991 as of 1/1/91?

(A) Less than $9,000
(B) $9,000 but less than $10,000
(C) $10,000 but less than $11,000
(D) $11,000 but less than $12,000
(E) $12,000 or more

**Problem 6 - 24**

Plan effective date: 1/1/85
Normal retirement benefit:

Before 1991: $10 per month for each year of service.
After 1990: $15 per month for each year of service.

Preretirement death benefit: None.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/36 |
| Date of hire | 1/1/86 |
| Status as of 1/1/91 | Active |

Selected valuation results as of 1/1/91, before amendment:

| | |
|---|---|
| Present value of future benefits | $6,942 |
| Unfunded liability | 1,800 |
| Value of assets | 1,000 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 50 | 94,002 | 1,135,407 |
| 55 | 64,742 | 727,747 |
| 65 | 28,570 | 262,659 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the increase in the normal cost for 1991 as of 1/1/91 due to the amendment?

(A) Less than \$200
(B) \$200 but less than \$275
(C) \$275 but less than \$350
(D) \$350 but less than \$425
(E) \$425 or more

**Problem 6 - 25**

Normal retirement benefit: \$10 per month for each year of service.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Unfunded liability as of 1/1/92: \$10,000
Value of assets as of 1/1/92: \$41,952

Data and valuation results for the only participants:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/47 | 1/1/27 |
| Date of hire | 1/1/72 | 1/1/52 |
| Date of retirement | - | 12/31/91 |
| Present value of future benefits as of 1/1/92 | \$10,841 | \$41,952 |

Contribution for 1992: Normal cost for 1992 as of 1/1, plus an amount to amortize the unfunded liability as of 1/1/92 over 10 years; paid on 1/1/92.

Investment rate of return for 1992: 7% compounded annually.

As of 1/1/93 Smith is active and Brown is retired.

Selected annuity values:

$\ddot{a}_{65}^{(12)} = 8.74 \qquad \ddot{a}_{66}^{(12)} = 8.51$

In what range is the normal cost for 1993 as of 1/1/93?

(A) Less than $64
(B) $64 but less than $129
(C) $129 but less than $194
(D) $194 but less than $289
(E) $289 or more

**Problem 6 - 26**

Plan effective date: 1/1/82
Actuarial cost method: Frozen initial liability.

Normal retirement benefit:

Before 1993: $15 per month for each year of service.
After 1992: $20 per month for each year of service.

Asset valuation method:

Before 1993: Actuarial value.
After 1992: Market value.

Assumed interest rate:

Before 1993: 7% per year.
After 1992: 8% per year.

Selected valuation results as of 1/1/93 (based on $15 benefit level):

| | 7% | 8% |
|---|---|---|
| Present Value of Future Benefits | $450,000 | $350,000 |
| Unfunded Liability | 60,000 | |
| Actuarial Value of Assets | 225,000 | 225,000 |
| Market Value of Assets | 250,000 | 250,000 |
| Accrued Liability under entry age normal method | 270,000 | 225,000 |
| Ratio of present value of future working lifetime to number of active participants. | 11 | 10 |

There were no inactive participants as of 1/1/92 and 1/1/93.

In what range is the normal cost for 1993 as of 1/1/93?

(A) Less than $12,000
(B) $12,000 but less than $13,000
(C) $13,000 but less than $14,000
(D) $14,000 but less than $15,000
(E) $15,000 or more

**Problem 6 - 27**

Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 4% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

As of 1/1/92, all participants are active and under age 63.

Selected valuation results as of 1/1/92:

| | |
|---|---|
| Present Value of Future Benefits | \$1,200,000 |
| Value of Assets | 500,000 |
| Present Value of Future Compensation | 9,500,000 |
| Contribution for 1992 (paid on 1/1/92) | \$60,000 |

There were no deaths, terminations, or retirements during 1992, and there are no new participants on 1/1/93.

There were no investment experience gains or losses for 1992, and there was a 10% increase in compensation for all participants from 1992 to 1993.

Selected valuation results as of 1/1/93:

| | |
|---|---|
| Unfunded liability | \$400,000 |
| 1993 compensation | 750,000 |

In what range is the normal cost for 1993 as of 1/1/93?

(A) Less than \$26,100
(B) \$26,100 but less than \$27,100
(C) \$27,100 but less than \$28,100
(D) \$28,100 but less than \$29,100
(E) \$29,100 or more

**Problem 6 - 28**

Plan effective date: 1/1/94

Normal retirement benefit: \$20 per month for each year of service.
Actuarial cost method: Frozen initial liability (entry age is age at hire).

Actuarial assumptions:

Preretirement interest rate: 8% per year.
Postretirement interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/84 |

Accrued liability under unit credit method as of 1/1/94, based on a 7% rate for preretirement and postretirement interest: $14,900

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $1,100
(B) $1,100 but less than $1,200
(C) $1,200 but less than $1,300
(D) $1,300 but less than $1,400
(E) $1,400 or more

**Problem 6 - 29**

Normal retirement benefit: $15 per month for each year of service.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Date of birth for only participant ever covered by plan (active as of 1/1/94): 1/1/42

Selected valuation results:

| | 1/1/93 | 1/1/94 |
|---|---|---|
| Present value of future benefits | $122,000 | |
| Value of assets | | $27,500 |
| Unfunded liability | | 48,500 |
| Value of assets plus unfunded liability | 65,000 | 76,000 |

In what range is the investment experience gain or loss during 1993?

(A) Loss of $200 or more
(B) Loss of less than $200
(C) $0 or gain of less than $200
(D) Gain of $200 but less than $400
(E) Gain of $400 or more

**Problem 6 - 30**

Normal retirement benefit: 50% of final 5-year average compensation.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65, but not before 1/1/98.

Selected valuation results as of 1/1/93:

| | |
|---|---|
| Present value of future benefits | $ 500,000 |
| Unfunded liability | 100,000 |
| Value of assets | 100,000 |
| Present value of future compensation | 3,000,000 |
| Valuation compensation | 200,000 |

Contribution for 1993: $30,000 paid on 1/1/93.

There were no experience gains or losses from any source during 1993.

There were no terminations, deaths, or retirements during 1993, and there were no new entrants on or before 1/1/94.

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than \$20,940
(B) \$20,940 but less than \$20,980
(C) \$20,980 but less than \$21,020
(D) \$21,020 but less than \$21,060
(E) \$21,060 or more

**Problem 6 - 31**

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

As of 1/1/94, all participants were active and under age 63.

Selected valuation results as of 1/1/94:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 150,000 |
| Present value of future benefits | 2,500,000 |
| Unfunded liability | 750,000 |
| Value of assets | 250,000 |
| Total valuation compensation for 1994 | 4,000,000 |

Contribution for 1994: \$225,000 paid on 12/31/94.

There were no deaths, terminations, or retirements during 1994. There was one new entrant as of 1/1/95 who was age 50 with valuation compensation for 1995 of \$50,000.

Value of assets as of 1/1/95: \$510,000.

Experience gains or losses due to compensation increases during 1994: \$0.

In what range is the absolute value of the change in the normal cost for 1995 as of 1/1/95 due to investment experience gains or losses during 1994?

(A) Less than \$1,900
(B) \$1,900 but less than \$1,910
(C) \$1,910 but less than \$1,920
(D) \$1,920 but less than \$1,930
(E) \$1,930 or more

**Problem 6 - 32**

Plan effective date: 1/1/90.
Actuarial cost method: Frozen initial liability.
Initial accrued liability: \$150,000.
Assumed interest rate: 7 % per year.

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | \$300,000 |
| Value of assets | 200,000 |
| Present value of future compensation | 600,000 |
| Annual compensation | 60,000 |

The contribution for 1995 is paid on 12/31/95 in an amount equal to the normal cost for 1995 as of 12/31/95 plus a 10-year amortization payment as of 12/31/95 of the initial accrued liability.

Contribution for 1995: \$25,000.

In what range is the unfunded liability as of 1/1/95?

(A) Less than \$50,000
(B) \$50,000 but less than \$55,000
(C) \$55,000 but less than \$60,000
(D) \$60,000 but less than \$65,000
(E) \$65,000 or more

**Problem 6 - 33**

Plan effective date: 1/1/92.
Actuarial cost method: Frozen initial liability.
Assumed interest rate: 6% per year.
Initial accrued liability: $10,000,000.

Normal cost for 1992 as of 1/1/92: $3,000,000.
Normal cost for 1993 as of 1/1/93: $3,200,000.
Normal cost for 1994 as of 1/1/94 (after plan amendment): $3,500,000.
Increase in unfunded liability as of 1/1/94 due to plan amendment: $5,000,000.

Investment fund activity for 1992 through 1994:

| Year | Contribution | Date of Contribution | Actual Investment Return |
|---|---|---|---|
| 1992 | $4,500,000 | 1/1/92 | 8.0% |
| 1993 | 5,000,000 | 4/1/93 | 7.5% |
| 1994 | 5,500,000 | 4/1/94 | 5.5% |

In what range is the unfunded liability as of 1/1/95?

(A) Less than $11,050,000
(B) $11,050,000 but less than $11,200,000
(C) $11,200,000 but less than $11,350,000
(D) $11,350,000 but less than $11,500,000
(E) $11,500,000 or more

**Problem 6 - 34**

Plan effective date: 1/1/95.

Normal retirement benefit: $50 per month for each year of service.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/46 |
| Date of hire | 1/1/66 |

Contribution for 1995: $5,000 paid on 1/1/95.

Value of assets as of 1/1/96: $6,000.

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 9.24$$

In what range is the unfunded liability as of 1/1/96?

(A) Less than $76,000
(B) $76,000 but less than $77,000
(C) $77,000 but less than $78,000
(D) $78,000 but less than $79,000
(E) $79,000 or more

**Problem 6 - 35**

Actuarial cost method: Frozen initial liability (level dollar method).

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Selected valuation results as of 1/1/96:

| | |
|---|---|
| Normal cost as of 1/1 | $ 50,000 |
| Present value of future benefits | 1,400,000 |
| Value of assets | 390,000 |
| Unfunded liability | 300,000 |

Contribution for 1996: $75,000 paid on 7/1/96.

Benefit payments during 1996: $0

There were no experience gains or losses during 1996 from any source other than an investment gain of $6,000.

No participants terminated, died or retired during 1996, and there are no new participants as of 1/1/97.

In what range is the normal cost for 1997 as of 1/1/97?

(A) Less than $48,500
(B) $48,500 but less than $49,000
(C) $49,000 but less than $49,500
(D) $49,500 but less than $50,000
(E) $50,000 or more

**Problem 6 - 36**

Normal retirement benefit: 50% of final year's compensation.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 5% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Selected valuation results for sole participant:

| | 1/1/96 | 1/1/97 |
|---|---|---|
| Normal cost as of 1/1 | \$ 12,000 | \$12,000 |
| Present value of future benefits | 400,000 | |
| Unfunded liability | 80,000 | |
| Value of assets (before contribution) | 200,000 | |
| Valuation compensation for plan year | 80,000 | 83,200 |

Contribution for 1996: \$22,645 paid on 1/1/96.

The only experience gains or losses during 1996 were from investments and compensation increases.

The sole participant as of 1/1/96 was active and under age 63.

In what range was the rate of return on assets during 1996?

(A) Less than 6.50%
(B) 6.50% but less than 7.50%
(C) 7.50% but less than 8.50%
(D) 8.50% but less than 9.50%
(E) 9.50% or more

**Problem 6 - 37**

Plan effective date: 1/1/92.

Actuarial cost method: Frozen initial liability.

Assumed interest rates:

Valuations before 1996: 7% per year.
Valuations after 1995: 8% per year.

Normal cost for 1995 as of 1/1/95: \$10,000.

Unfunded liability as of 1/1/95: \$60,000.

Selected valuation results as of 1/1/96:

| | 7% | 8% |
|---|---|---|
| Normal cost as of 1/1 | $11,000 | $10,000 |
| Entry age normal accrued liability | 70,000 | 63,000 |
| Value of assets | 50,000 | 50,000 |

Contributions for 1995: $5,000 paid on 1/1/95 and $5,000 paid on 7/1/95.

Contributions for 1996: $4,000 paid on 1/1/96 and $4,000 paid on 7/1/96.

In what range is the unfunded liability as of 1/1/97?

(A) Less than $54,000
(B) $54,000 but less than $59,000
(C) $59,000 but less than $64,000
(D) $64,000 but less than $69,000
(E) $69,000 or more

## 6.4 Solutions to Problems

### Problem 6 - 1

Key Concept: Since there were no changes in benefits in 1985, the Present Value of Future Benefits on 1/1/86 would be:

$$(400{,}000)\,(1.06) = 424{,}000$$

The Unfunded Accrued Liability must be determined as of 1/1/86. However, one of the elements is the Normal Cost at 1/1/85 which is not given. Hence, the first step is to determine that.

Step I: Determine the Normal Cost as of 1/1/85.

| | |
|---|---|
| Present Value of Future Benefits | \$400,000 |
| Less Unfunded Liability | - 95,000 |
| Less Actuarial Value of Assets | - 135,000 |
| Present Value of Future Normal Costs | \$170,000 |
| Divide by Present Value of Future Service | ÷ 38.0 |
| Normal Cost per Participant | 4,474 |
| Multiplied by Number of Participants | × 4 |
| Plan Normal Cost | 17,896 |

Step II: 1/1/86 Unfunded Accrued Liability can now be calculated.

$$\begin{aligned} UAL_{86} &= (UAL_{85} + NC_{85})(1 + i) - C_{85} - I_c \\ &= (95{,}000 + 17{,}896)(1.06) - 25{,}000 - 0 = 94{,}670 \end{aligned}$$

Step III: Normal cost at 1/1/86 is calculated.

| | |
|---|---|
| Present Value of Future Benefits | \$424,000 |
| Less Unfunded Accrued Liability | - 94,670 |
| Less Actuarial Value of Assets | - 160,000 |
| Present Value of Future Normal Costs | \$169,330 |
| Divide by Present Value of Future Service | ÷ 36.3 |
| Normal Cost per Participant | 4,665 |
| Multiplied by Number of Participants | × 4 |
| Normal Cost of Plan | 18,660 |

**Problem 6 - 2**

Key Concept: A 10% increase in benefits will generate a 10% increase, or \$60,000, in the Present Value of Future Benefits. A \$20,000 increase in the Entry Age Normal Accrued Liability will generate a \$20,000 increase in the Unfunded Accrued Liability. Hence, the Present Value of Future Normal Costs will increase by \$60,000 and decrease by \$20,000 according to the formula:

$$PVFNC = PVFB - UAL - Assets$$

The increase in PVFNC = 60,000 - 20,000 = 40,000

After the amendment, Present Value of Future Normal Costs will be:

$$PVFNC = 200{,}000 + 40{,}000 = 240{,}000$$

**Problem 6 - 3**

Key Concept: Problem deals with relationships that exist with variables from one year to the next, particularly Present Value of Future Benefits, Unfunded Liability, Actuarial Value of Assets and Present Value of Future Salaries.

Before Unfunded Liability at 1/1/86 can be determined, the Normal Cost as of 1/1/85 must be calculated.

Step I: Determine Normal Cost at 1/1/85.

$$\begin{aligned} PVFNC_{85} &= PVFB_{85} - UAL_{85} - Assets_{85} \\ &= 3{,}000{,}000 - 750{,}000 - 1{,}250{,}000 = 1{,}000{,}000 \end{aligned}$$

$$\begin{aligned} NC\% &= PVFNC_{85} \div PVFS_{85} \\ &= 1{,}000{,}000 \div 15{,}000{,}000 = 6.6667\% \end{aligned}$$

$$\begin{aligned} NC_{85} &= Salaries_{85} \times NC\% \\ &= 600{,}000 \times .066667 = 40{,}000 \end{aligned}$$

Step II: Update Present Value of Future Benefits.

$$\begin{aligned} PVFB_{86} &= (PVFB_{85})(1 + i) \\ &= 3{,}000{,}000 \times 1.06 = 3{,}180{,}000 \end{aligned}$$

Step III: Update Unfunded Accrued Liability.

$$\begin{aligned} UAL_{86} &= (UAL_{85} + NC_{85})(1 + i) - C_{85} - I_c \\ &= (750{,}000 + 40{,}000)(1.06) - 125{,}000 - 0 = 712{,}400 \end{aligned}$$

Step IV: Update the Actuarial Value of Assets.

$$\begin{aligned} Assets_{86} &= (Assets_{85} + C_{85} + Earnings) \\ &= 1{,}250{,}000 + 125{,}000 + 105{,}000 = 1{,}480{,}000 \end{aligned}$$

Step V: Update the Present Value of Future Salaries.

$$\begin{aligned} PVFS_{86} &= (PVFS_{85} - Salaries_{85})(1 + i) \\ &= (15{,}000{,}000 - 600{,}000)(1.06) = 15{,}264{,}000 \end{aligned}$$

Step VI: Determine Salaries for 1986.

$$\begin{aligned} Salaries_{86} &= Salaries_{85} \times 1.04 \\ &= 600{,}000 \times 1.04 = 624{,}000 \end{aligned}$$

Step VII: Normal Cost for 1/1/86 can now be calculated.

$$\begin{aligned} PVFNC_{86} &= PVFB_{86} - UAL_{86} - Assets_{86} \\ &= 3{,}180{,}000 - 712{,}400 - 1{,}480{,}000 = 987{,}600 \end{aligned}$$

$$NC = (Salaries_{86})(PVFNC_{86} \div PVFS_{86})$$
$$= (624{,}000)(987{,}600) \div (15{,}264{,}000) = 40{,}374$$

**Problem 6 - 4**

Key Concept: Many values are the same as those in Problem 6 - 3. Values that are affected by salary changes are (i) Present Value of Future Benefits, (ii) Future Normal Costs, (iii) Future Salaries and (iv) Current Salaries. Following the solution to Problem 6 - 3:

Step I: Same as Problem 6 - 3, the Normal Cost as of 1/1/85 is 40,000.

Step II: Because the assumed salary increase did not occur, the projected benefits will decrease by $1.04^{-1}$ as compared to the 1985 valuation. A similar adjustment needs to be made to PVFB.

$$PVFB_{86} = PVFB_{85} \times 1.06 \div 1.04$$
$$= 3{,}000{,}000 \times 1.06 \div 1.04 = 3{,}057{,}692$$

Step III: Same as Problem 6 - 3, the Unfunded Accrued Liability as of 1/1/86 is 712,400.

Step IV: Same as Problem 6 - 3, the Assets as of 1/1/86 is 1,480,000.

Step V: Present Value of Future Salaries as computed in Problem 6 - 3 should be reduced by the factor $1.04^{-1}$.

$$PVFS_{86} = (15{,}000{,}000 - 600{,}000)(1.06) \div 1.04 = 14{,}676{,}923$$

Step VI: $Salaries_{86}$ equals $Salaries_{85}$ which is 600,000.

Step VII: 1/1/86 Normal Cost can now be calculated.

$$PVFNC_{86} = PVFB_{86} - UAL_{86} - Assets_{86}$$
$$= 3{,}057{,}692 - 712{,}400 - 1{,}480{,}000 = 865{,}292$$

$$NC_{86} = (Salaries_{86})(PVFNC_{86}) \div PVFS_{86}$$
$$= (600{,}000)(865{,}292) \div 14{,}676{,}923 = 35{,}374$$

Note: The Normal Cost was reduced by more than the factor of $1.04^{-1}$ when compared to the Normal Cost in Problem 6 - 3. Why?

Because of the "leverage" effect of the assets and the UAL on the PVFNC. A percentage change in the PVFB results in a much larger percentage change in PVFNC because the values of Assets and UAL do not change.

**Problem 6 - 5**

Key Concept: This problem deals with the effect that a change in salaries has upon the variables. The Present Value of Future Service is not provided; however, since the normal cost is known, we can calculate it from the data given.

Step I: Using the data of the original valuation, determine the Present Value of Future Service.

$$\begin{aligned} PVFNC &= PVFB - UAL - Assets \\ &= 1{,}700{,}000 - 300{,}000 - 200{,}000 = 1{,}200{,}000 \end{aligned}$$

$$NC = (PVFNC \div PV\ Future\ Service) \times n$$ (where n is the number of participants.)

or $$PV\ of\ Future\ Service \div n = PVFNC \div NC = 1{,}200{,}000 \div 80{,}000 = 15$$

Step II: Recalculate the Present Value of Future Benefits. A 10% increase in salary generates a 10% increase in the Present Value of Future Benefits.

$$\begin{aligned} PVFB_{(new)} &= PVFB_{(old)} \times 1.10 \\ &= 1{,}700{,}000 \times 1.1 = 1{,}870{,}000 \end{aligned}$$

Step III: Recalculate the Normal Cost.

$$\begin{aligned} PVFNC &= PVFB - UAL - Assets \\ &= 1{,}870{,}000 - 300{,}000 - 200{,}000 = 1{,}370{,}000 \\ NC &= (PVFNC \div PV\ of\ Future\ Service) \times n \\ &= 1{,}370{,}000 \div 15 = 91{,}333 \end{aligned}$$

Answer is D.

**Problem 6 - 6**

Key Concept: Again, the problem deals with relationships of valuation variables, particularly Unfunded Liability and Asset Value during two successive years.

Step I: Determine Unfunded Liability as of 1/1/83.

$$\begin{aligned} UAL_{83} &= PVFB_{83} - Assets_{83} - PVFNC_{83} \\ &= 850{,}000 - 125{,}000 - 420{,}000 = 305{,}000 \end{aligned}$$

Step II: Determine Unfunded Liability as of 1/1/84.

$$\begin{aligned} UAL_{84} &= (UAL_{83} + NC_{83})(1 + i) - C_{83} - I_c \\ &= (305{,}000 + 23{,}000)(1.07) - 50{,}000 - (25{,}000)(.07) \\ &= 350{,}960 - 50{,}000 - 1{,}750 = 299{,}210 \end{aligned}$$

Step III: Determine Actuarial Value of Assets on 1/1/84.

Expected Actuarial Value of Assets is:

$$\begin{aligned} eAssets_{84} &= (Assets_{83})(1 + i) + C_{83} + I_{83} \\ &= (125{,}000)(1.07) + 50{,}000 + 1{,}750 \\ &= 133{,}750 + 50{,}000 + 1{,}750 = 185{,}500 \end{aligned}$$

$$Assets_{84} = eAssets_{84} + 3{,}000 = 188{,}500$$

Step IV: Determine Present Value of Future Normal Costs at 1/1/84.

$$\begin{aligned} PVFNC_{84} &= PVFB_{84} - UAL_{84} - Assets_{84} \\ &= 901{,}000 - 299{,}210 - 188{,}500 = 413{,}290 \end{aligned}$$

Answer is B.

**Problem 6 - 7**

Key Concept: This problem, as in the previous one, deals with relationships from one year to the next.

Step I: The Present Value of Future Benefits is not given; however, we are given enough data to calculate it.

Using data from the 1/1/84 valuation,

$$PVFNC_{84} \div PVFS_{84} = NC\% = NC_{84} \div Salaries_{84}$$
$$PVFNC_{84} \div 4{,}011{,}000 = 80{,}000 \div 400{,}000$$
$$PVFNC_{84} = 802{,}200$$

Hence, $PVFB_{84} = PVFNC_{84} + UAL_{84} + Assets_{84}$
$= 802{,}200 + 550{,}000 + 0$ since UAL = AL at effective date.

$$PVFB_{84} = 1{,}352{,}200$$

Step II: Determine Present Value of Future Benefits at 1/1/85.

$$ePVFB_{85} = PVFB_{84}\,(1 + i)$$
$$= (1{,}352{,}200)(1.07) = 1{,}446{,}854$$

Adjust $ePVFB_{85}$ for salary increase:

$$PVFB_{85} = ePVFB_{85}\,(1.10 / 1.05)$$
$$= 1{,}446{,}854\,(1.10 / 1.05) = 1{,}515{,}752$$

Step III: Determine Unfunded Accrued Liability at 1/1/85.

On the effective date, 1/1/84, UAL = AL = 550,000

$$UAL_{85} = (UAL_{84} + NC_{84}](1 + i) - \text{Contribution} - I_c$$
$$= (550{,}000 + 80{,}000)(1.07) - 164{,}000 - 0 = 510{,}100$$

Step IV: Determine Present Value Future Salaries at 1/1/85.

$$ePVFS_{85} = (PVFS_{84} - Salaries_{84})(1.07)$$
$$= (4{,}011{,}000 - 400{,}000)(1.07) = 3{,}863{,}770$$

Adjust PVFS for Salary increases:

$$PVFS_{85} = ePVFS_{85} \times (1.10 / 1.05)$$
$$= 3{,}863{,}770 \times 1.10/1.05 = 4{,}047{,}759$$

Step V: Since the 1984 contribution was made on 12/31/84, the Actuarial Value of Assets on 1/1/85 is equal to the contribution of $164,000. All pieces are now known in order to calculate the Normal Cost at 1/1/85.

$$PVFNC_{85} = PVFB_{85} - UAL_{85} - Assets_{85}$$
$$= 1{,}515{,}752 - 510{,}100 - 164{,}000 = 841{,}652$$

$$NC_{85} = (PVFNC_{85} \div PVFS_{85}) \times Salaries_{85}$$
$$= 841{,}652 \div 4{,}047{,}759 \times 440{,}000 = 91{,}489$$

Answer is D.

**Problem 6 - 8**

Key Concept: When a change in benefits or assumptions occurs, the Unfunded Accrued Liability is adjusted by the amount of change in the Entry Age Normal Accrued Liability. In this case, the change in Entry Age Normal Accrued Liability is:

$$\text{Change in EAN Accrued Liability} = 16{,}350 - 13{,}100 = 3{,}250$$
$$\text{and the adjusted UAL} = 6{,}150 + 3{,}250 = 9{,}400$$

Calculate Present Value of Future Normal Costs:

$$PVFNC = PVFB - UAL - Assets$$
$$= 18{,}400 - 9{,}400 - 5{,}000 = 4{,}000$$

Present Value of Future Service is changed from $\ddot{a}_{\overline{15}|}$ to $\ddot{a}_{\overline{12}|}$ under the amendment.

$$NC = PVFNC \div \ddot{a}_{\overline{12}|}$$
$$= 4{,}000 \div 8.5 = 471$$

Answer is E.

**Problem 6 - 9**

Key Concepts: As described in the previous problem, any change of assumptions that alters the Entry Age Normal Accrued Liability alters the Unfunded Accrued Liability by the same amount.

Current year salary figures are not provided. However, they can be calculated since the normal cost under the old assumptions is given.

Step I: Determine current year salaries.

$$\begin{aligned} PVFNC &= PVFB - UAL - Assets \\ &= 6{,}148{,}750 - 750{,}000 - 3{,}350{,}000 = 2{,}048{,}750 \end{aligned}$$

$$\text{and } NC = (PVFNC \times Salaries) \div PVFS$$
$$132{,}000 = (2{,}048{,}750 \times Salaries) \div 37{,}250{,}000$$

$$\text{and } Salaries = (132{,}000 \times 37{,}250{,}000) \div 2{,}048{,}750 = 2{,}400{,}000$$

Step II: Determine Unfunded Accrued Liability under new assumptions.

$$\text{Change in EAN Accrued Liability} = 4{,}875{,}000 - 4{,}175{,}000 = 700{,}000$$

$$\begin{aligned} UAL\ (new) &= UAL\ (old) + EAN\ AL\ change \\ &= 750{,}000 + 700{,}000 = 1{,}450{,}000 \end{aligned}$$

Step III: Determine Normal Cost under new assumptions.

$$\begin{aligned} PVFNC &= PVFB - UAL - Assets \\ &= 7{,}146{,}000 - 1{,}450{,}000 - 3{,}350{,}000 = 2{,}346{,}000 \end{aligned}$$

$$\begin{aligned} NC &= (PVFNC \times Salaries) \div PVFS \\ &= (2{,}346{,}000 \times 2{,}400{,}000) \div 39{,}100{,}000 = 144{,}000 \end{aligned}$$

Answer is C.

**Problem 6 - 10**

Key Concept: This problem also deals with relationships in any year to the prior year.

Step I: In order to determine Unfunded Accrued Liability at 1/1/85, Normal Cost at 1/1/84 must be calculated.

$$\begin{aligned} PVFNC_{84} &= PVFB_{84} - UAL_{84} - Assets_{84} \\ &= 10{,}000{,}000 - 2{,}000{,}000 - 4{,}000{,}000 = 4{,}000{,}000 \end{aligned}$$

$$\begin{aligned} \text{and } NC_{84} &= (PVFNC_{84} \times Salaries_{84}) \div PVFS_{84} \\ &= (4{,}000{,}000 \times 4{,}000{,}000) \div 40{,}000{,}000 = 400{,}000 \end{aligned}$$

Step II: Determine Present Value Future Benefits at 1/1/85.

$$\begin{aligned} PVFB_{85} &= PVFB_{84} \times 1.07 \\ &= 10{,}000{,}000 \times 1.07 = 10{,}700{,}000 \end{aligned}$$

Step III: Determine Unfunded Accrued Liability at 1/1/85.

$$\begin{aligned} UAL_{85} &= (UAL_{84} + NC_{84})(1+i) - \text{Contribution}_{84} - I_c \\ &= (2{,}000{,}000 + 400{,}000)(1.07) - 618{,}000 = 1{,}950{,}000 \end{aligned}$$

Step IV: Determine Actuarial Value of Assets at 1/1/85.

$$\begin{aligned} \text{Assets}_{85} &= (\text{Assets}_{84})(1.08) + \text{Contribution}_{84} \\ &= (4{,}000{,}000)(1.08) + 618{,}000 = 4{,}938{,}000 \end{aligned}$$

Step V: Determine Salaries and Present Value of Future Salaries as of 1/1/85.

$$\begin{aligned} \text{Salaries}_{85} &= \text{Salaries}_{84} \times 1.05 \quad \text{(since salary increase was realized.)} \\ &= 4{,}000{,}000 \times 1.05 = 4{,}200{,}000 \end{aligned}$$

$$\begin{aligned} PVFS_{85} &= (PVFS_{84} - \text{Salaries}_{84})(1.07) \\ &= (40{,}000{,}000 - 4{,}000{,}000)(1.07) = 38{,}520{,}000 \end{aligned}$$

Step VI: Finally, determine the Normal Cost.

$$\begin{aligned} PVFNC_{85} &= PVFB_{85} - UAL_{85} - \text{Assets}_{85} \\ &= 10{,}700{,}000 - 1{,}950{,}000 - 4{,}938{,}000 = 3{,}812{,}000 \end{aligned}$$

$$\begin{aligned} \text{and } NC_{85} &= (PVFNC_{85} \times \text{Salaries}_{85}) \div PVFS_{85} \\ &= (3{,}812{,}000 \times 4{,}200{,}000) \div 38{,}520{,}000 = 415{,}639 \end{aligned}$$

Answer is D.

**Problem 6 - 11**

Key Concept: The change in Valuation dates changes the Unfunded Accrued Liability by interest only since no contributions have been made during 1984 for the 1984 plan year.

$$\text{Therefore, } UAL_{1/1/84} = [UAL_{1/1/83} + NC_{1/1/83}](1 + i) - C_{83}$$
$$= (100{,}000 + 10{,}000)(1.08) - 20{,}000 = 98{,}800$$

$$UAL_{12/31/84} = 98{,}800 \times 1.08 = 106{,}704$$

Next, determine the Value of Assets at 12/31/84, using the actual rate of return given of 8%.

$$Assets_{1/1/84} = 20{,}000$$
$$Assets_{12/31/84} = 20{,}000 \times 1.08 = 21{,}600$$

Determine the PVFNC at 12/31/84.

$$PVFNC_{12/31/84} = PVFB - UAL - Assets$$
$$= 265{,}000 - 106{,}704 - 21{,}600 = 136{,}696$$

Determine Normal Cost.

$$NC = (PVFNC \times Salaries) \div PVFS$$
$$= 136{,}696 \times 200{,}000 \div 2{,}400{,}000 = 11{,}391$$

Answer is C.

**Problem 6 - 12**

Key Concept: To determine the new Unfunded Accrued Liability, it is necessary to add the increase in EAN Accrued Liability due to the amendment to the old Unfunded figure. The increase applies only to the active participants.

Step I: Determine the Normal Cost for 1986 based upon benefit formula of 40% of salary.

$$PVFNC = PVFB - UAL - Plan\ Assets$$
$$= 1{,}500{,}000 - 300{,}000 - 500{,}000 = 700{,}000$$

$$NC = PVFNC \times (Annual\ Salary \div PVFS)$$
$$= 700{,}000 \times (4{,}000{,}000 \div 21{,}000{,}000) = 133{,}333$$

Step II: Adjust variables affected by Benefit increase.

An increase in benefits from 40% of salary to 50% of salary represents a 25% increase in benefits, which in turn represents a 25% increase in the Present Value of Future Benefits and Entry Age Normal Accrued Liability.

$$PVFB = (1{,}200{,}000)(1.25) + 300{,}000 = 1{,}800{,}000$$

$$\text{EAN Accrued Liability} = (700{,}000 - 300{,}000)(1.25) + 300{,}000 = 800{,}000$$

Step III: Calculate the increase in the EAN Accrued Liability due to the increase in the benefit formula.

$$\begin{aligned} \text{Change in UAL} &= \text{EAN AL(new)} - \text{EAN AL(old)} \\ &= 800{,}000 - 700{,}000 = 100{,}000 \end{aligned}$$

$$\begin{aligned} UAL &= UAL(old) + \Delta EAN\ AL \\ &= 300{,}000 + 100{,}000 = 400{,}000 \end{aligned}$$

Step IV: Compute Normal Cost based upon formula of 50% of salary.

$$\begin{aligned} PVFNC &= PVFB - UAL - Assets \\ &= 1{,}800{,}000 - (300{,}000 + 100{,}000) - 500{,}000 = 900{,}000 \end{aligned}$$

$$NC = 900{,}000 \times (4{,}000{,}000 \div 21{,}000{,}000) = 171{,}429$$

$$\text{Normal Cost Increase} = 171{,}429 - 133{,}333 = 38{,}096$$

Answer is C.

**Problem 6 - 13**

Key Concept: The Present Value of Future Benefits for the omitted participant is also the required addition to the Present Value of Future Normal Costs.

The Present Value of Future Normal Cost is not given in the valuation figures. However, it can be calculated from the data provided.

$$\begin{aligned} NC &= PVFNC \times (\text{Annual Salary} \div PVFS) \\ &= PVFNC \times (200{,}000 \div 2{,}500{,}000) = 15{,}000 \end{aligned}$$

$$PVFNC = 15{,}000 \times 2{,}500{,}000 \div 200{,}000 = 187{,}500$$

$$PVFNC_{revised} = 187{,}500 + 39{,}000 = 226{,}500$$

$$\begin{aligned} NC &= 226{,}500\ (200{,}000 + 30{,}000) \div (2{,}500{,}000 + 850{,}000) \\ &= 226{,}500 \times 230{,}000 \div 3{,}350{,}000 = 15{,}551 \end{aligned}$$

Answer is A.

**Problem 6 - 14**

Key Concept: The unique feature of this problem is the difference in the Present Value of Future Service as a result of the change in valuation dates from 12/31 to 1/1.

The components needed to calculate Normal Cost at 1/1/87 are (i) Present Value of Future Benefits, (ii) Unfunded Liability, (iii) Actuarial Value of Plan Assets, and (iv) Average Temporary Annuity.

(1) Present Value of Future Benefits remains unchanged.

$$PVFB_{87} = PVFB_{86} = 900{,}000$$

(2) $UL_{87} = UL_{86} + NC_{86} - Contribution_{86} = 400{,}000 + 30{,}000 - 60{,}000 = 370{,}000$

(3) $Assets_{87} = Assets_{86} + Contribution_{86} = 200{,}000 + 60{,}000 = 260{,}000$

(4) To calculate the Average Temporary Annuity at 12/31/86:

$$\begin{aligned} NC_{86} &= (PVFB_{86} - UL_{86} - Assets_{86}) \div ATA_{86} \\ \text{or } 30{,}000 &= (900{,}000 - 400{,}000 - 200{,}000) \div ATA_{86} \\ \text{from which } ATA_{86} &= 300{,}000 \div 30{,}000 = 10 \\ \text{or } 10 &= \sum \ddot{a}_{\overline{n}|} \div \text{Number of participants} \end{aligned}$$

To determine the $ATA_{87}$, $\sum \ddot{a}_{\overline{n-1}|}$ must be determined.

$$\begin{aligned} \sum \ddot{a}_{\overline{n-1}|} &= (\sum \ddot{a}_{\overline{n}|} - 1)(1 + i) \\ &= (10 - 1)(1.08) = 9.72 \end{aligned}$$

(5) Normal Cost can now be calculated.

$$\begin{aligned} NC_{87} &= (PVFB_{87} - UL_{87} - Assets_{87}) \div ATA_{87} \\ &= (900{,}000 - 370{,}000 - 260{,}000) \div 9.72 \\ &= 270{,}000 \div 9.72 = 27{,}778 \end{aligned}$$

Answer is B.

**Problem 6 - 14 (Alternative solution):**

Key Concept: Since Normal Cost at 12/31/87 was \$30,000 which was paid at 12/31/87, and there were no gains or losses, the Normal Cost at 1/1/87 would be:

$$\begin{aligned} NC_{86} &= NC_{86} \div 1.08 \\ &= 30{,}000 \div 1.08 = 27{,}778 \end{aligned}$$

**Problem 6 - 15**

Key Concept: Employer's normal cost is the total normal cost adjusted by the employees' contributions.

| | |
|---|---|
| Present Value of Future Benefits | \$2,600,000 |
| Less Actuarial Value of Assets | - 650,000 |
| Less Unfunded Liability | - 300,000 |
| Present Value of Future Normal Costs | \$1,650,000 |
| Divided by PV of Future Compensation | ÷12,500,000 |
| Normal Cost percentage | 13.20% |
| Times Current Compensation | × 1,000,000 |
| Total Normal Cost at 1/1/88 | 132,000 |
| Total Normal Cost at 12/31/88 | 139,920 |
| Less Employee Contributions | - 20,000 |
| Less Interest on Employee Contributions | - 1,200 |
| Employer's Normal Cost at 12/31/88 | 118,720 |

Answer is B.

**Problem 6 - 16**

Key Concept: Under the Frozen Initial Liability Cost Method, the Accrued Liability is calculated using the Entry Age Normal Cost Method. Hence, in this problem, the Unfunded Accrued Liability must be adjusted under this method.

The Present Value of Future Benefits must also be adjusted for the increase in benefits. The benefit increase from \$10 to \$12 represents a 20% increase. The Present Value of Future Benefits will also increase by 20%.

$$PVFB\ (\$12\ \text{benefit}) = 18{,}400 \times 120\% = 22{,}080$$

Under the Entry Age Normal method:

Increase in Accrued Liability due to change of normal retirement age:

$$AL_{adjustment} = 16{,}350 - 13{,}100 = 3{,}250$$

Increase in Accrued Liability due to increase in benefits:

$$AL_{adjustment} = 20\%\ \text{of}\ 16{,}350 = 3{,}270$$
$$\text{Adjusted UAL} = 6{,}150 + 3{,}250 + 3{,}270 = 12{,}670$$

We can now determine the normal cost for 1988 as of 1/1/88:

| | |
|---|---|
| Present Value of Future Benefits | \$ 22,080 |
| Less Unfunded Liability | 12,670 |
| Less Actuarial Value of Assets | 5,000 |
| PV of Future Normal Costs | \$ 4,410 |
| PV of Future Service (retirement age 62) | 8.887 |
| Normal Cost at 1/1/88 | 496.23 |

Answer is C.

**Problem 6 - 17**

Key Concept: Unfunded Accrued Liability is not given for either 1/1/87 or 1/1/88 valuations. However, enough information is provided for us to calculate it at 1/1/87 and then to build on it to determine it at 1/1/88.

Step I: Determine Unfunded Accrued Liability at 1/1/87.

$$PVFNC = PVFB_{87} - UAL_{87} - \text{Assets}$$
$$210{,}000 = 500{,}000 - UAL_{87} - 80{,}000$$
$$\text{Therefore,} \quad UAL_{87} = 210{,}000$$

Step II: Determine Unfunded Accrued Liability at 1/1/88.

$$UAL_{88} = (UAL_{87} + NC_{87})(1+i) - C_{87} - I_c$$
$$I_c = (1/2)(25{,}000)(.06) = 750$$
$$UAL_{88} = (210{,}000 + 25{,}000)(1.06) - 45{,}000 - 750 = 203{,}350$$

Step III: Determine Actuarial Asset Value at 1/1/88.

$$\text{Asset Value}_{88} = \text{Asset Value}_{87} + \text{Contribution} + \text{Interest}$$
$$= 80{,}000\,(1 + .084) + 45{,}000 + (1/2)(25{,}000)(.084) = 132{,}770$$

Step IV: Determine Present Value of Future Normal Costs at 1/1/88.

$$PVFNC_{88} = PVFB_{88} - UAL_{88} - \text{Asset Value}_{88}$$
$$= 565{,}000 - 203{,}350 - 132{,}770 = 228{,}880$$

Answer is E.

**Problem 6 - 18**

Key Concept: This problem involves the projection of future benefits and salaries with salary increase assumption. Since salaries increased by a percentage other than that assumed, the factor $(1+j) / (1+k)$ must be applied where:

$j$ = actual percentage increase
$k$ = assumed percentage increase.

Step I: Determine the Present Value of Future Benefits at 1/1/88.

$$\begin{aligned} PVFB_{88} &= (PVFB_{87})(1+i)(1+j)/(1+k) \\ &= (2{,}000{,}000)(1.06)(1.075/1.04) = 2{,}191{,}346 \end{aligned}$$

Step II: Determine Actuarial Value of Assets at 1/1/88.

Since there was no actuarial gain from investment earnings,

$$\begin{aligned} Assets_{88} &= (Assets_{87} \times 1.06) + Contribution \\ &= (100{,}000)(1.06) + 150{,}000 = 256{,}000 \end{aligned}$$

Step III: Determine Unfunded Liability at 1/1/88.

$$\begin{aligned} UAL_{88} &= (UAL_{87} + \text{Normal Cost})(1+i) - \text{Contribution} \\ &= (500{,}000 + 84{,}000)(1.06) - 150{,}000 = 469{,}040 \end{aligned}$$

Step IV: Determine Present Value of Future Salaries at 1/1/88.

$$\begin{aligned} PVFS_{88} &= (PVFS_{87} - 1987\ \text{Salaries})(1+i)(1+j)/(1+k) \\ &= (10{,}000{,}000 - 600{,}000)(1.06)(1.075/1.04) = 10{,}299{,}327 \end{aligned}$$

Step V: Use all components to determine Normal Cost at 1/1/88.

| | |
|---|---|
| Present Value of Future Benefits | \$2,191,346 |
| Less Unfunded Accrued Liability | - 469,040 |
| Less Actuarial Value of Assets | - 256,000 |
| PV of Future Normal Costs | \$1,466,306 |
| Divide by PV Future Salaries | ÷ \$10,299,327 |
| Multiply by Current Year Salaries | × 645,000 |
| Normal Cost at 1/1/88 | 91,828 |

Answer is B.

**Problem 6 - 19**

Key Concept: Since the funding method was changed to Frozen Initial Liability on 1/1/87, the Accrued Liability under the Entry Age Normal method should be used in this problem. To determine Normal Cost, we will use the following relationships:

$$PVFNC = PVFB - UAL - Assets$$
$$\text{and} \quad NC = PVFNC \div PVFS \times Salaries$$

Step I: Determine the Unfunded Accrued Liability as of 1/1/87.

| | |
|---|---|
| Present Value of Future Benefits | \$11,000,000 |
| Less Actuarial Value of Assets | - 3,000,000 |
| | 8,000,000 |
| | |
| Accrued Liability under EAN Cost Method | 6,500,000 |
| Less Actuarial Asset Value | - 3,000,000 |
| Unfunded Accrued Liability | 3,500,000 |

Step II: Determine Normal Cost at 1/1/87.

| | |
|---|---|
| Present Value of Future Normal Costs | \$4,500,000 |
| Divided by PV of Future Compensation | ÷ 112,500,000 |
| Multiplied by Current Year Compensation | × 16,500,000 |
| | |
| Normal Cost at 1/1/87 | \$ 660,000 |

Step III: Determine Unfunded Accrued Liability at 1/1/88.

$$UAL_{88} = (UAL_{87} + NC_{87})(1.06) - C_{87} - I_c$$
$$= (3{,}500{,}000 + 660{,}000)(1.06) - 1{,}000{,}000 = 3{,}409{,}600$$

Step IV: Determine Normal Cost 1/1/88 as a percentage of compensation.

| | |
|---|---|
| Present Value of Future Benefits at 1/1/88 | \$12,000,000 |
| Less Unfunded Accrued Liability | - 3,409,600 |
| Less Actuarial Value of Assets | - 4,000,000 |
| | |
| Equals Present Value of Future Normal Costs | \$ 4,590,400 |
| Divide by PV of Future Compensation | ÷115,000,000 |
| | |
| Equals Normal cost percentage | 3.9916% |

Answer is D.

Note: Since the method was changed in 1987 from an <u>immediate</u> gain method to FIL, the unfunded accrued liability is equal to the EAN accrued liability minus the assets.

**Problem 6-20**

Step I: Calculate Present Value of Future Benefits.

$$\text{Smith PVFB} = 1000 \times 12\ddot{a}_{70}^{(12)} = 91{,}200$$

$$\text{Brown PVFB} = 5000 \times 12\ddot{a}_{65}^{(12)} \times v^{30} = 68{,}889$$

$$\text{Green PVFB} = 3000 \times 12\ddot{a}_{65}^{(12)} \times v^{20} = 81{,}309$$

Total PVFB = 91,200 + 68,889 + 81,309 = 241,398

Step II: Calculate the sum of the temporary annuities of the active participants.

$$\sum \text{Temporary Annuities} = \ddot{a}_{\overline{20|}} + \ddot{a}_{\overline{30|}} = 24.61326$$

Step III: Calculate Normal Cost per Active Participant.

$$NC = \frac{\text{PVFB - (UAL + Assets)}}{\sum \text{Temporary Annuities}} = \frac{241{,}398 - (10{,}000 + 91{,}200)}{24.61326} = 5{,}696$$

Step IV: Calculate total Normal Cost.

$$NC = 5{,}696 \times 2 \text{ active participants} = 11{,}392$$

Answer is B.

**Problem 6-21**

Key Concept: The error in the Present Value of Future Service does not include Smith since he is no longer active. However, he must be included in the calculation of the Present Value of Future Benefits.

Smith was valued as follows in the original valuation:

$$\text{Benefit} = 15 \text{ x } 28 \text{ years of service} = 420$$
$$e\text{PVFB} = 420 \times 12\ddot{a}_{65}^{(12)} \times v^{10} = 19{,}446$$
$$\text{Temporary Annuity} = \ddot{a}_{\overline{10|}} = 7.2469$$

Smith is revalued as follows:

$$\text{Accrued benefit at age } 65 = 15 \text{ x } 18 \text{ years of service} = 270$$

Since Smith's age is 55, the early retirement reduction factor is

$$= (.06)(5 \text{ years}) + (.03)(5 \text{ years}) = .45$$

Accrued Benefit (reduced for early retirement) = $270 \times (1 - .45) = 148.50$

$$\text{PVAB} = 148.50 \times 12\ddot{a}_{55}^{(12)} = 21{,}972$$

$$NC_{\text{Original}} = \left( \frac{\Sigma\text{PVFB} - (\text{UL} + \text{Assets})}{\Sigma\text{Temporary Annuities}} \right)$$

$$= \frac{662{,}000 - (163{,}250 + 142{,}500)}{\Sigma\text{Temporary Annuities}} = 195.34 \text{ per active participant}$$

$$\text{where } \Sigma\text{Temporary Annuities} = 1{,}823.7432$$

Since Smith is no longer active, we can recalculate $\Sigma$ TA without future service for him:

$$\Sigma\text{Temporary Annuities} = 1{,}823.7432 - 7.2469 = 1{,}816.4963$$

The revised Present Value of Future Benefits is:

$$PVFB = 662{,}000 - 19{,}446 + 21{,}972 = 664{,}526$$

$$\text{Therefore, } NC_{New} = \frac{664{,}526 - (163{,}250 + 142{,}500)}{1{,}816.4963}$$

$$= 197.51 \text{ per active participant}$$

$$\text{Old } NC_{Total} = 195.34 \times 150 \text{ active participants} = 29{,}301$$

$$\text{New } NC_{Total} = 197.51 \times 149 \text{ active participants} = 29{,}429$$

$$\Delta\, NC = 29{,}429 - 29{,}301 = 128$$

Answer is C.

**Problem 6 - 22**

Key Concept: The problem deals with the effects of actual experience and a plan amendment upon successive years' valuations.

Step I: Calculate the Present Value of Future Benefits as of 1/1/91.

If benefits were to remain the same,

$$PVFB_{91} = PVFB_{90} \times (1+i)$$

Since salary increased 10% instead of the expected 5%,

$$PVFB_{91} = PVFB_{90} \times (1+i) \times (1.1/1.05)$$
$$= 700{,}000 \times 1.07 \times (1.1/1.05) = 784{,}667$$

Also, the plan was amended to increase benefits from 30% to 40% of final compensation. Thus, $PVFB_{91} = 784{,}667 \times (40/30) = 1{,}046{,}222$

Step II: Determine the Unfunded Accrued Liability as of 1/1/91.

The Frozen Initial Liability component for 1991 is determined as follows:

$$\begin{aligned} UAL_{91} &= (UAL_{90} + NC_{90})\,(1 + i) - C_{90} - I_c \\ &= (275{,}000 + 25{,}000)(1.07) - 50{,}000 = 271{,}000 \end{aligned}$$

We must also determine the increase in liability due to the plan amendment.

$$\text{EAN } AL_{New} = 550{,}000$$

$$\text{EAN } AL_{Old} = 550{,}000 \times (30\% \div 40\%) = 412{,}500$$

$$AL_{Increase} = 550{,}000 - 412{,}500 = 137{,}500$$

The total unfunded liability is

$$UAL_{91} = 271{,}000 + 137{,}500 = 408{,}500$$

Step III: Determine the Present Value of Future Salaries as of 1/1/91.

$$\begin{aligned} PVFS_{91} &= [PVFS_{90} - Salaries_{90}](1.07)(1.10/1.05) \\ &= (5{,}675{,}000 - 500{,}000)(1.07)\,(1.10/1.05) = 5{,}800{,}929 \end{aligned}$$

Step IV: Determine Salaries as of 1/1/91.

$$Salaries_{91} = 500{,}000\,(1.1) = 550{,}000$$

Step V: Calculate Normal Cost.

$$\begin{aligned} NC &= (PVFB - UAL - Assets) \div PVFS \times Salaries \\ &= (1{,}046{,}222 - 408{,}500 - 250{,}000) \div 5{,}800{,}929 \times 550{,}000 = 36{,}761 \end{aligned}$$

Answer is E.

**Problem 6 - 23**

Key Concept: The retired participant is included in the Present Value of Future Benefits, but the Normal Cost is calculated based upon the future working lifetimes of only the active participants.

Calculate the Present Value of Future Benefits:

| | Age x | Benefit | $v^{65\text{-}aa}$ | $PVFB_{91}$ |
|---|---|---|---|---|
| Smith | 45 | 20,000 | .2584 | 45,168 |
| Brown | 55 | 20,000 | .5083 | 88,851 |
| Green | 65 | 20,000 | 1.0000 | 174,800 |
| Total | | | | 308,819 |

At 7%, $\ddot{a}_{\overline{20|}} = 11.33559$ and $\ddot{a}_{\overline{10|}} = 7.51523$

We can now determine the Normal Cost per active participant:

$$NC_{91} = \frac{PVFB_{91} - UL_{91} - Assets_{91}}{\ddot{a}_{\overline{20|}} + \ddot{a}_{\overline{10|}}}$$

$$= \frac{308{,}819 - (12{,}000 + 208{,}800)}{11.33559 + 7.51523} = 4{,}669$$

Since there are two active participants,

Total $NC_{91} = 4{,}669 \times 2 = 9{,}338$

Amortizing the unfunded liability over 10 years,

$12{,}000 \div \ddot{a}_{\overline{10|}} = 1{,}597$

Total Contribution = 9,338 + 1,597 = 10,935

Answer is C.

**Problem 6 - 24**

Step I: Calculate Normal Cost before the amendment.

$$\text{Benefit} = 10 \text{ x } 15 \text{ years of service} = 150$$

$$NC_{Old} = \frac{PVFB - UL - Assets}{\ddot{a}_{55:\overline{10|}}}$$

$$= \frac{6{,}942 - 1{,}800 - 1{,}000}{\ddot{a}_{55:\overline{10|}}} = \frac{6{,}942 - 2{,}800}{\left(\dfrac{727{,}747 - 262{,}659}{64{,}742}\right)} = 577$$

Step II: Calculate the change in the Accrued Liability due to the amendment.

Under the original benefit formula:

$$NC_{EAN} = \frac{PVFB_{ea}}{\ddot{a}_{50:\overline{15|}}} = \frac{150 \times \dfrac{D_{65}}{D_{50}} \ddot{a}_{65}^{(12)} \times 12}{\ddot{a}_{50:\overline{15|}}}$$

$$= \frac{150 \times 12 \times \left(\dfrac{28{,}570}{94{,}002}\right) (8.74)}{\left(\dfrac{1{,}135{,}407 - 262{,}659}{94{,}002}\right)} = \frac{4{,}781.42}{9.284355} = 515$$

$$AL_{EAN} = NC \times \ddot{a}_{50:\overline{5|}}$$

$$AL_{EAN} = 515 \times \frac{1{,}135{,}407 - 727{,}747}{64{,}742} = 3{,}242$$

$$\Delta AL_{EAN} = .5 \times 3{,}242 = 1{,}621$$

The new Unfunded Liability = Old Unfunded Liability + $\Delta AL_{EAN}$ = 1,800 + 1,621 = 3,421

Step III: Calculate the Normal Cost after the amendment.

$$PVFB_{new} = 6{,}942 \times 1.5 = 10{,}413$$

$$NC_{New} = \frac{10{,}413 - (3{,}421 + 1{,}000)}{7.18371} = 834$$

Step IV: Calculate increase in Normal Cost.

$$\Delta NC = 834 - 577 = 257$$

Answer is B.

**Problem 6 - 25**

Since the benefit is not salary based,

$$NC_{FIL} = [\Sigma PVFB - (eUL + Assets)] \div ATA$$

$$1/1/92 \text{ Amortization Payment} = 10{,}000/\ddot{a}_{\overline{10|}} = 10{,}000/7.51523 = 1{,}331$$

At 1/1/92, Brown is a retiree and Smith is active, aged 45. Brown's retirement benefit is

$$= 41{,}952/\ddot{a}_{65}^{(12)} = 41{,}952/8.74 = 4{,}800$$

The Normal Cost and Unfunded Liability can be calculated

$$NC_{92} = \frac{(10{,}841 + 41{,}952) - (10{,}000 + 41{,}952)}{\ddot{a}_{\overline{20|}}} = 74$$

$$\begin{aligned} eUL_{93} &= (UL_{92} + NC_{92})(1 + i) - C_{92} - I_c \\ &= (10{,}000 + 74)(1.07) - (1{,}405)(1.07) = 9{,}276 \end{aligned}$$

Accumulated benefit payments for 1992, as of 12/31/92

$$= 12\ddot{s}_{\overline{1|}}^{(12)} \times 400 = 4{,}980$$

$$Assets_{93} = (41{,}952 + 1{,}405)(1.07) - 4{,}980 = 41{,}412$$

$$PVFB_{93} = 10{,}841\ (1.07) + 4{,}800\ (8.51) = 52{,}448$$

$$NC_{93} = \frac{52{,}448 - (9{,}276 + 41{,}412)}{\ddot{a}_{\overline{15|}}} = 159$$

Answer is C.

**Problem 6 - 26**

Key Concept: An increase in Unfunded Liability must be calculated due to the change in benefit formula and actuarial assumptions.

Step I: Calculate the increase in Unfunded Accrued Liability.

a) Decrease in UAL due to new asset valuation method.

= Actuarial value - Market value
= 225,000 - 250,000 = (25,000)

Note that this <u>decreases</u> the Unfunded Liability since the market value is greater than the actuarial value.

b) Decrease in UAL due to the new interest assumption.

= EAN AL at 8% - EAN AL at 7%
= 225,000 - 270,000 = (45,000)

c) Increase in UAL due to the new benefit formula.

= EAN AL (at 8%) under new plan - EAN AL under old plan
= (20/15)(225,000) - 225,000 = 75,000

Note that since we already computed the decrease in the UL due to the change in interest rate, we must calculate the change due to the benefit formula at the new 8% rate.

Total increase in UL = (25,000) + (45,000) + 75,000 = 5,000

Step II: Calculate the PVFB under the new formula at 8% interest.

$$PVFB = (350{,}000)(20/15) = 466{,}667$$

Step III: Calculate the Normal Cost.

$$\begin{aligned} PVFNC &= PVFB - UL - Assets \\ &= 466{,}667 - (60{,}000 + 5{,}000) - 250{,}000 = 151{,}667 \end{aligned}$$

$$\begin{aligned} NC &= PVFNC \div (\text{ratio of PV of Future Service to number of participants}) \\ &= 151{,}667/10 = 15{,}167 \end{aligned}$$

Again, note that all values are at 8% interest.

Answer is E.

**Problem 6 - 27**

Key Concept: This problem is an exercise in the development of the current year's actuarial values using the prior year's values and actual experience.

Step I: Calculate the 1/1/93 PVFB.

Since there was no change in the plan population, the PVFB should increase by the assumed interest rate. However, salary increased by 10% (rather than the assumed 4%). This results in an adjustment of 1.1/1.04.

$$\begin{aligned} PVFB_{93} &= PVFB_{92} \times 1.07 \times (1.1/1.04) \\ &= 1{,}200{,}000 \times 1.07 \times (1.1/1.04) = 1{,}358{,}077 \end{aligned}$$

Step II: Calculate the 1/1/93 Assets

Since there were no investment gains or losses,

$$\begin{aligned} Assets_{93} &= (Assets_{92} + Contribution_{92}) \times 1.07 \\ &= (500{,}000 + 60{,}000) \times 1.07 = 599{,}200 \end{aligned}$$

Step III: Calculate the 1/1/93 Present Value of Future Salaries.

The logic here is the same as in Step I.
Note that $Salary_{92} = 750{,}000/1.1 = 681{,}818$

$$PVFS_{93} = (PVFS_{92} - Salary_{92}) \times 1.07 \times (1.1/1.04)$$
$$= (9{,}500{,}000 - 681{,}818) \times 1.07 \times (1.1/1.04) = 9{,}979{,}808$$

Step IV: Calculate Normal Cost.

$$NC_{93} = (PVFB_{93} - Assets_{93} - UL_{93}) / (PVFS_{93} / Salary_{93})$$
$$= (1{,}358{,}077 - 599{,}200 - 400{,}000)/(9{,}979{,}808/750{,}000) = 26{,}970$$

Answer is B.

**Problem 6 - 28**

Key Concept: In a plan with just one participant, the first year Normal Cost calculated under the Frozen Initial Liability method is identical to the Normal Cost calculated under the Entry Age Normal method.

Step I: Calculate the annuity purchase rate at age 65, using the value of the Accrued Liability determined under the Unit Credit method. The monthly accrued benefit as of 1/1/94 is:

$$\text{Accrued Benefit}_{94} = (20)(10 \text{ years of service}) = 200$$

Since the Unit Credit Accrued Liability is equal to the present value of the accrued benefit,

$$14{,}900 = (200)(12\ddot{a}_{65}^{(12)})(v^5)$$

$$\text{and } 12\ddot{a}_{65}^{(12)} = 104.49$$

Step 2: Calculate the Normal Cost under the Entry Age Normal method.

$$\text{Monthly Normal Retirement Benefit} = (20)(15 \text{ years of service}) = 300$$

$$NC_{EAN} = 300 \times 12\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{15|}}$$

$$= 300 \times 104.49/29.3243 = 1{,}069$$

Answer is A.

**Problem 6 - 29**

Key Concepts: Since there is only one participant and no deaths or terminations were assumed, the only experience gains or losses could have come from investments. Under the Frozen Initial Liability method, gains and losses are spread over future Normal Costs. Therefore, the difference between the expected and actual Present Value of Future Normal Costs must be the experience gain or loss.

Step I: Calculate the expected Present Value of Future Normal Costs.

$$
\begin{aligned}
PVFNC_{93} &= PVFB_{93} - Assets_{93} - UL_{93} \\
&= 122{,}000 - 65{,}000 = 57{,}000
\end{aligned}
$$

$$
\begin{aligned}
NC_{93} &= 57{,}000 \div \ddot{a}_{\overline{14}|} \\
&= 57{,}000 \div 9.35765 = 6{,}091
\end{aligned}
$$

$$
\begin{aligned}
e PVFNC_{94} &= (PVFNC_{93} - NC_{93}) \times 1.07 \\
&= (57{,}000 - 6{,}091) \times 1.07 = 54{,}473
\end{aligned}
$$

Step II: Calculate the actual Present Value of Future Normal Costs.

$$
\begin{aligned}
PVFB_{94} &= PVFB_{93} \times 1.07 \\
&= 122{,}000 \times 1.07 = 130{,}540
\end{aligned}
$$

$$
\begin{aligned}
PVFNC_{94} &= PVFB_{94} - Assets_{94} - UL_{94} \\
&= 130{,}540 - 27{,}500 - 48{,}500 = 54{,}540
\end{aligned}
$$

Step III: Calculate the experience gain or loss by subtracting the actual Present Value of Future Normal Costs from the expected Present Value of Future Normal Costs.

$$
\begin{aligned}
\text{Experience Gain/Loss} &= e PVFNC_{94} - PVFNC_{94} \\
&= 54{,}473 - 54{,}540 = (67) \quad \text{Loss}
\end{aligned}
$$

Answer is B.

**Problem 6 - 30**

Key Concept: Since there were no gains or losses and the plan population has remained the same, the Normal Cost as of 1/1/94 is equal to the Normal Cost as of 1/1/93, increased by the 5% salary scale (in order to keep the Normal Cost a level percent of salary).

Step I: Calculate the Normal Cost as of 1/1/93.

$$\begin{aligned} NC_{93} &= (PVFB_{93} - Assets_{93} - UL_{93})/(PVFS_{93}/Salary_{93}) \\ &= (500{,}000 - 100{,}000 - 100{,}000)/(3{,}000{,}000/200{,}000) = 20{,}000 \end{aligned}$$

Step II: Calculate the Normal Cost as of 1/1/94.

$$\begin{aligned} NC_{94} &= NC_{93} \times (1 + s) \\ &= 20{,}000 \times (1.05) = 21{,}000 \end{aligned}$$

Answer is C.

**Problem 6 - 31**

Key Concept: The increase (decrease) in Normal Cost due to investment loss (gain) can be found by multiplying the amount of the investment loss (gain) by the ratio of Present Value of Future Salary to current Salary.

Step I: Calculate the investment gain or loss.

$$\begin{aligned} \text{Expected Assets}_{1/1/95} &= (250{,}000 \times 1.07) + 225{,}000 \\ &= 267{,}500 + 225{,}000 = 492{,}500 \end{aligned}$$

$$\text{Actual Assets}_{1/1/95} = 510{,}000$$

$$\text{Asset Gain} = 510{,}000 - 492{,}500 = 17{,}500$$

Step II: Calculate the Present Value of Future Salary as of 1/1/95. First, use the 1/1/94 valuation results to determine the PVFS as of 1/1/94.

$$\begin{aligned} NC &= (PVFB - UL - Assets)/(PVFS/Salary) \\ 150{,}000 &= (2{,}500{,}000 - 750{,}000 - 250{,}000)/(PVFS_{94}/4{,}000{,}000) \\ PVFS_{94} &= 40{,}000{,}000 \end{aligned}$$

The expected PVFS for 1995 is

$$\text{Expected PVFS}_{95} = (\text{PVFS}_{94} - \text{Salary}_{94}) \times 1.07$$
$$= (40{,}000{,}000 - 4{,}000{,}000) \times 1.07 = 38{,}520{,}000$$

The PVFS for the new entrant must be added to the expected $\text{PVFS}_{95}$ to get the actual $\text{PVFS}_{95}$.

$$\text{New Entrant PVFS} = 50{,}000\ddot{a}_{\overline{15}|j} \quad \text{where } j = 1.07/1.05 - 1 = .0190476$$
$$= (50{,}000)(13.1878) = 659{,}390$$

$$\text{PVFS}_{95} = 38{,}520{,}000 + 659{,}390 = 39{,}179{,}390$$

Step III: Calculate the total salary at of 1/1/95.

$$\text{Expected Salary}_{95} = \text{Salary}_{94} \times 1.05$$
$$= 4{,}000{,}000 \times 1.05 = 4{,}200{,}000$$

The salary for the new participant must be added.

$$\text{Actual Salary}_{95} = 4{,}200{,}000 + 50{,}000 = 4{,}250{,}000$$

Step IV: Calculate the decrease in the Normal Cost due to the investment gain.

$$\Delta\text{NC} = \text{Gain}/(\text{PVFS}_{95}/\text{Salary}_{95})$$
$$= 17{,}500/(39{,}179{,}390/4{,}250{,}000) = 1{,}898$$

Answer is A.

**Problem 6 - 32**

The 10-year amortization payment as of 12/31/95 is

$$150{,}000/a_{\overline{10}|} = 21{,}357$$

Note the use of $a_{\overline{10}|}$ instead of $\ddot{a}_{\overline{10}|}$, because the payment is made at the end of the year.

Since the total contribution paid 12/31/95 was 25,000, the Normal Cost as of 12/31/95 is:

$$25{,}000 - 21{,}357 = 3{,}643$$

and the Normal Cost as of 1/1/95 is:

$$3{,}643/1.07 = 3{,}405$$

Now we can use the formula for developing Normal Cost:

$$\begin{aligned} NC &= (PVFB - UL - Assets)/(PVFS/Salary) \\ 3{,}405 &= (300{,}000 - UL - 200{,}000)/(600{,}000/60{,}000) \\ UL &= 65{,}950 \end{aligned}$$

Answer is E.

**Problem 6 - 33**

Key Concept: The unfunded liability is developed by increasing the sum of the unfunded liability and Normal Cost as of the prior valuation date with interest to the current valuation date and subtracting the contribution made for the prior plan year with interest from the date the contribution was made.

Step I: Calculate the Unfunded Liability as of 1/1/93.

$$\begin{aligned} UL_{93} &= (UL_{92} + NC_{92}) \times (1 + i) - Contribution_{92} \times (1 + i) \\ &= (10{,}000{,}000 + 3{,}000{,}000)\times 1.06 - 4{,}500{,}000 \times 1.06 = 9{,}010{,}000 \end{aligned}$$

Step II: Calculate the Unfunded Liability as of 1/1/94.

$$\begin{aligned} UL_{94} &= (UL_{93} + NC_{93}) \times (1 + i) - Contribution_{93} \times (1 + 3/4i) \\ &= (9{,}010{,}000 + 3{,}200{,}000)\times 1.06 - 5{,}000{,}000 \times 1.045 = 7{,}717{,}600 \end{aligned}$$

Because the contribution was paid on April 1, 1993, the contribution was increased with interest for only 9 months. The interest accumulation used here was based upon simple interest, but could have reflected compound interest by using $(1.06)^{3/4}$ as the interest factor.

Step III: Calculate the Unfunded Liability as of 1/1/95.

$$\begin{aligned} UL_{95} &= (UL_{94} + UL_{\text{Plan Am.}} + NC_{94})\times(1 + i) - Contribution_{94} \times (1 + 3/4i) \\ &= (7{,}717{,}600 + 3{,}500{,}000 + 5{,}000{,}000)\times 1.06 - 5{,}500{,}000 \times 1.045 \\ &= 11{,}443{,}156 \end{aligned}$$

Answer is D.

**Problem 6 - 34**

Key Concept: The initial unfunded liability determined using the Frozen Initial Liability cost method is equal to the Accrued Liability determined using the Entry Age Normal cost method.

Step I: Calculate the initial unfunded liability as of 1/1/95.

$$\begin{aligned}\text{Entry Age Normal } AL_{95} &= (50)(45 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{29}|}/\ddot{s}_{\overline{45}|}) \\ &= 76{,}260\end{aligned}$$

Step II: Calculate the Normal Cost as of 1/1/95.

$$\begin{aligned}NC_{95} &= (50)(45 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{45}|} \\ &= 816\end{aligned}$$

Step III: Calculate the Unfunded Liability as of 1/1/96.

$$\begin{aligned}UAL_{96} &= (AL_{95} + NC_{95} - Contrib_{95})(1.07) \\ &= (76{,}260 + 816 - 5{,}000)(1.07) \\ &= 77{,}121\end{aligned}$$

Answer is C.

**Problem 6 - 35**

Key Concept: The Normal Cost for 1997 will be equal to the Normal Cost in 1996, reduced by an amortization of the investment gain.

Step I: Determine the average temporary annuity used in the 1/1/96 valuation.

$$NC_{96} = (PVFB_{96} - Assets_{96} - UL_{96})/ATA_{96}$$

$$50{,}000 = (1{,}400{,}000 - 390{,}000 - 300{,}000)/ATA_{96}$$

$$ATA_{96} = 14.2$$

Step II: Determine the average temporary annuity to be used in the 1/1/97 valuation.

The formula for determining the current annuity from the prior year annuity (in the situation where there are no preretirement decrements) is:

$$ATA_{N+1} = (ATA_N - 1)[(1+i)/(1+s)]$$

Since there is no salary scale being used as this is a level dollar amount method, the formula becomes:

$$ATA_{N+1} = (ATA_N - 1)(1+i)$$

So,

$$ATA_{97} = (14.2 - 1)(1.07) = 14.124$$

Step III: Determine the decrease in Normal Cost as of 1/1/97.

$$\begin{aligned} NC_{DECREASE} &= \text{Investment gain}/ATA_{97} \\ &= 6{,}000/14.124 \\ &= 425 \end{aligned}$$

Step IV: Determine the Normal Cost as of 1/1/97.

$$\begin{aligned} NC_{97} &= NC_{96} - NC_{DECREASE} \\ &= 50{,}000 - 425 \\ &= 49{,}575 \end{aligned}$$

Answer is D.

**Problem 6 - 36**

Step I: Determine the average temporary annuity used in the 1/1/96 valuation.

$$NC_{96} = (PVFB_{96} - Assets_{96} - UL_{96})/ATA_{96}$$

$$12{,}000 = (400{,}000 - 200{,}000 - 80{,}000)/ATA_{96}$$

$$ATA_{96} = 10$$

Step II: Determine the average temporary annuity to be used in the 1/1/97 valuation.

The formula for determining the current annuity from the prior year annuity (in the situation where there are no preretirement decrements) is:

$$ATA_{N+1} = (ATA_N - 1)[(1+i)/(1+s)]$$

So,

$$ATA_{97} = (10 - 1)(1.07/1.05) = 9.1714$$

Step III: Determine expected values as of 1/1/97.

$$\begin{aligned} eNC_{97} &= NC_{96} \times (1+s) \\ &= 12{,}000 \times (1.05) \\ &= 12{,}600 \end{aligned}$$

$$\begin{aligned} eSalary_{97} &= Salary_{96} \times (1+s) \\ &= 80{,}000 \times (1.05) \\ &= 84{,}000 \end{aligned}$$

$$\begin{aligned} ePVFB_{97} &= PVFB_{96} \times (1+i) \\ &= 400{,}000 \times (1.07) \\ &= 428{,}000 \end{aligned}$$

Step IV: Determine 1/1/97 Normal cost decrease due to salary increases being less than expected.

Since salary did not increase as expected, the expected PVFB must be adjusted by the ratio of the actual salary to expected salary to determine the actual PVFB.

$$\begin{aligned} PVFB_{97} &= ePVFB_{97} \times (Salary_{97}/eSalary_{97}) \\ &= 428{,}000 \times (83{,}200/84{,}000) \\ &= 423{,}924 \end{aligned}$$

$$\begin{aligned} NC_{DECREASE} &= (ePVFB_{97} - PVFB_{97})/ATA_{97} \\ &= (428{,}000 - 423{,}924)/9.1714 \\ &= 444 \end{aligned}$$

Step V: Determine asset gain.

Since the expected Normal Cost for 1997 was \$12,600 and the actual Normal Cost for 1997 is \$12,000, the Normal Cost decreased by \$600 due to the investment and compensation gains. It is known from step IV that the decrease due to the compensation gain was \$444. Therefore, the decrease in the Normal Cost due to investment gain is \$156 (=\$600 - \$444). The total asset gain is:

$$\begin{aligned} \text{Asset Gain} &= \text{Normal cost decrease due to investment gain} \times ATA_{97} \\ &= 156 \times 9.1714 \\ &= 1{,}431 \end{aligned}$$

Step VI: Determine the rate of return.

Assuming that the assets had earned exactly 7%, the earnings would have been:

$$\text{Earnings} = (200{,}000 + 22{,}645)(.07) = 15{,}585$$

Rate of Return = (1,431 + 15,585)/(200,000 + 22,645) = .0764, or 7.64%.

Answer is C.

**Problem 6-36 (Alternative Solution)**

The value of assets as of 1/1/97 can be solved for directly using the temporary annuity factor determined in step II above and the present value of future benefits as of 1/1/97.

$$\begin{aligned} PVFB_{97} &= PVFB_{96} \times (1+i) \times (1+s')/(1+s) \quad \text{where } s' \text{ is the actual rate of salary increase} \\ &= (400{,}000)(1.07)(83{,}200/80{,}000)/(1.05) \\ &= 423{,}924 \end{aligned}$$

The formula for determining the expected Unfunded Liability under the Frozen Initial Liability method is:

$$eUL_{N+1} = (UL_N + NC_N)(1+i) - C_N - \text{interest on } C_N \text{ from date of contribution}$$

So,

$$\begin{aligned} eUL_{97} &= (UL_{96} + NC_{96})(1.07) - (C_{1/1/96})(1.07) \\ &= (80{,}000 + 12{,}000)(1.07) - (22{,}645)(1.07) \\ &= 74{,}210 \end{aligned}$$

The normal cost as of 1/1/97 can be represented by:

$$\begin{aligned} & NC_{97} = (PVFB_{97} - eUL_{97} - Assets_{97})/ATA_{97} \\ \Rightarrow\ & 12{,}000 = (423{,}924 - 74{,}210 - Assets_{97})/9.1714 \\ \Rightarrow\ & Assets_{97} = 239{,}657 \end{aligned}$$

The actual rate of return, $i'$, can be solved from the equation:

$$\begin{aligned} & Assets_{97} = (Assets_{96} + Contribution_{96})(1+i') \\ \Rightarrow\ & 239{,}657 = (200{,}000 + 22{,}645)((1+i') \\ \Rightarrow\ & i' = .0764, \text{ or } 7.64\% \end{aligned}$$

Answer is C.

**Problem 6 - 37**

The formula for determining the expected Unfunded Liability under the Frozen Initial Liability method is:

$$eUL_{N+1} = (UL_N + NC_N)(1 + i) - C_N - \text{interest on } C_N \text{ from date of contribution}$$

So, the expected Unfunded Liability as of 1/1/96 is:

$$\begin{aligned} eUL_{96} &= (UL_{95} + NC_{95})(1.07) - (C_{1/1/95})(1.07) - (C_{7/1/95})(1.035) \\ &= (60{,}000 + 10{,}000)(1.07) - (5{,}000)(1.07) - (5{,}000)(1.035) \\ &= 64{,}375 \end{aligned}$$

As of 1/1/96 the assumed interest rate is increased from 7% to 8%. This causes a decrease in the unfunded liability equal to the difference between the Entry Age Normal Accrued Liability before and after the change.

$$\begin{aligned} UL_{DECREASE} &= EANAL_{7\%} - EANAL_{8\%} \\ &= 70{,}000 - 63{,}000 \\ &= 7{,}000 \end{aligned}$$

The total Unfunded Liability as of 1/1/96 is equal to the difference between the expected Unfunded Liability and the decrease in the Unfunded Liability due to the assumption change.

$$UL_{96} = 64{,}375 - 7{,}000 = 57{,}375$$

Now the Unfunded Liability as of 1/1/97 can be determined.

$$\begin{aligned} UL_{97} &= (UL_{96} + NC_{96})(1.08) - (C_{1/1/96})(1.08) - (C_{7/1/96})(1.04) \\ &= (57{,}375 + 10{,}000)(1.08) - (4{,}000)(1.08) - (4{,}000)(1.04) \\ &= 64{,}285 \end{aligned}$$

Answer is D.

# Chapter 7

# Attained Age Normal Cost Method

## 7.1 Normal Cost and Accrued Liability

We come to the last and probably the least often utilized of the recognized actuarial cost methods, the Attained Age Normal Method. It is easy to understand - provided that the Frozen Initial Liability Cost Method is understood. The only difference is that the Unfunded Accrued Liability is calculated using the Unit Credit Cost Method rather than the Entry Age Normal Cost Method. All other calculations proceed from that point in the same manner as the Frozen Initial Liability Cost Method.

Refer to the example problem in the previous section. If we were to use the same problem but calculate the Normal Cost under the Attained Age Normal Method, the following would be the result.

To restate the problem:

Effective date of plan: 1/1/85
Funding Method: Attained Age Normal.

Census data for plan participants as of 1/1/85:

| | Age x | Years of Service | Projected Monthly Pension |
|---|---|---|---|
| Pete | 48 | 10 | $1,000 |
| Hank | 35 | 3 | 800 |

Assumptions used: Interest - 7% No mortality or withdrawals
Retirement age - 65 $12\ddot{a}_{65}^{(12)} = 110$

Compute the Normal Cost for 1985 as of January 1, 1985.

Step I: Determine the Accrued Liability as of 1/1/85 using the Unit Credit Method.

| | Pete | Hank |
|---|---|---|
| (1) Projected Pension | $ 1,000 | $ 800 |
| (2) Accrued Pension | 370 | 73 |
| (3) Present Value of Accrued Pension $(2) \times 110 \times (1.07)^{x-65}$ | 12,885 | 1,055 |
| (4) Accrued Liability | 12,885 | 1,055 |

Step II: Determine Present Value of Future Benefits and Future Normal Cost.

| | | |
|---|---|---|
| (5) PVFB Benefit $\times 110 \times (1.07)^{x-65}$ | 34,823 | 11,560 |
| (6) PVFNC (5) - (4) | 21,938 | 10,505 |

Step III: Determine Normal Cost.

Present Value of Future Normal Costs = 21,938 + 10,505 = 32,443
Present Value of Future Service = 10.45 + 13.28 = 23.73
Normal Cost per participant = 1,367
Total Normal Cost = 2,734

The student should consider why the Attained Age Normal Cost is greater than the Frozen Initial Liability Normal Cost. The reason is that the Accrued Liabililty under Attained Age Normal is less than the Accrued Liability under the FIL Method and therefore a greater portion of the Present Value of Future Benefits (which is the same under either method) is funded through the AAN Normal Cost.

Changes due to plan amendments are handled in the same manner as under FIL, except that the change in UAL is computed as the change in Accrued Liability under the Unit Credit Method.

As under the FIL Method, Normal Costs may be computed as a level dollar amount per participant, or as a level percentage of salaries.

## 7.2 Characteristics of Attained Age Normal Method

1. The initial past service liability is determined under the Unit Credit Cost Method.

2. Method is an aggregate method and hence is not suitable for small plans.

3. Experience gains and losses are automatically spread over the remaining working years of participants.

4. Method involves a lesser past service liability than the Frozen Initial Liability method and hence a greater initial normal cost.

## 7.3 Problems

### Problem 7 - 1

Plan effective date: 1/1/82
Actuarial Cost Method: Attained age normal method.
Assumed interest rate: 6%

| Valuation results: | 1/1/82 | 1/1/83 |
|---|---|---|
| Present Value of Future Benefits | $1,000,000 | $1,200,000 |
| Accrued Liability | 400,000 | --- |
| Ratio of present value of future salaries to current salaries | 10 | 11 |

Contribution for 1982, paid at 12/31/82: $100,000
Benefits for 1982: None

In what range is the normal cost for 1983 as of 1/1/83?

(A) Less than $63,500
(B) $63,500 but less than $64,000
(C) $64,000 but less than $64,500
(D) $64,500 but less than $65,000
(E) $65,000 or more.

### Problem 7 - 2

Plan effective date: 1/1/85
Normal retirement benefit: $10 per month for each year of service.
Assumed retirement age: 65

It is assumed that there are no pre-retirement terminations other than by death.

Participant data at 1/1/85 and selected commutation functions:

| Attained Age | Age at Hire | Number of Participants | $D_x$ | $N_x - N_{65}$ |
|---|---|---|---|---|
| 35 | 35 | 40 | 92 | 1,190 |
| 45 | 35 | 60 | 46 | 500 |
| 55 | -- | 0 | 22 | 160 |
| 65 | -- | 0 | 10 | 0 |

$\ddot{a}_{65}^{(12)} = 10$

In what range is the difference between the accrued liability under the entry age normal method and the accrued liability under the attained age normal method?

(A) Less than \$100,000
(B) \$100,000 but less than \$125,000
(C) \$125,000 but less than \$150,000
(D) \$150,000 but less than \$175,000
(E) \$175,000 of more.

**Problem 7 - 3**

Plan effective date: 1/1/82
Actuarial cost method: Attained age normal.
Assumed interest rate: 7%
It is assumed that there are no deaths or other terminations of employment prior to age 65.

| Selected valuation data: | 1983 | 1984 |
|---|---|---|
| Unfunded liability as of 1/1 | \$495,000 | |
| Actuarial value of assets (market) as of 1/1 | 75,000 | |
| Normal cost as of 1/1 | 40,000 | \$45,000 |
| Contribution paid at 12/31 | 85,000 | 90,000 |

During 1983 and 1984 all assumptions were exactly realized except that plan assets earned 8% in 1983 and 9% in 1984.

In what range is the unfunded liability as of 1/1/85?

(A) Less than $467,500
(B) $467,500 but less than $472,500
(C) $472,500 but less than $477,500
(D) $477,500 but less than $482,500
(E) $482,500 or more.

**Problem 7 - 4**

Plan effective date: 1/1/85.
Normal retirement benefit:

Effective 1/1/85: $10 per month per year of service.
Effective 1/1/86: $15 per month per year of service.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest: 7%.
Pre-retirement deaths and terminations: None.
Retirement age: 65

$12\ddot{a}_{65}^{(12)} = 100$

Participant data as of 1/1/86:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/40 | 1/1/50 |
| Date of hire | 1/1/80 | 1/1/75 |

Selected valuation results as of 1/1/86, based on the $10 benefit:

| | |
|---|---|
| Unfunded liability | $2,500 |
| Actuarial value of assets | 1,100 |

In what range is the present value of future normal costs as of 1/1/86?

(A) Less than $12,800
(B) $12,800 but less than $13,800
(C) $13,800 but less than $14,800
(D) $14,800 but less than $15,800
(E) $15,800 or more

**Problem 7 - 5**

Plan effective date: 1/1/88
Normal retirement benefit: $10 per month for each year of service before 1/1/88 plus $15 per month for each subsequent year of service.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Participant data as of 1/1/88:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/43 | 1/1/28 |
| Date of hire | 1/1/73 | 1/1/68 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 9$

In what range is the normal cost for 1988 as of 1/1/88?

(A) Less than $1,800
(B) $1,800 but less than $1,900
(C) $1,900 but less than $2,000
(D) $2,000 but less than $2,100
(E) $2,100 or more

**Problem 7 - 6**

Plan effective date: 1/1/87
Normal retirement benefit: $15 per month for each year of service up to 20 years plus $20 per month for each subsequent year of service.
Death benefit: None.
Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Participant data as of 1/1/87:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/37 | 1/1/27 |
| Date of hire | 1/1/62 | 1/1/72 |

Brown dies on 12/31/87. There were no new participants during 1987.
The contribution paid for 1987 is the Normal Cost as of 1/1 plus $3,000 paid on 1/1/87.

Selected annuity value:

$12\ddot{a}_{65}^{(12)} = 112$

In what range is the unfunded liability as of 1/1/88?

(A) Less than $36,500
(B) $36,500 but less than $37,000
(C) $37,000 but less than $37,500
(D) $37,500 but less than $38,000
(E) $38,000 or more

**Problem 7 - 7**

Plan effective date: 1/1/87
Normal retirement benefit: $10 per month for each year of service.
Actuarial cost method: Attained age normal.

Actuarial assumptions:

> Interest: 6%
> Pre-retirement deaths and terminations: None.
> Retirement age: 65

Participant data as of 1/1/88:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/43 | 1/1/53 |
| Date of hire | 1/1/73 | 1/1/88 |

There were no deaths or terminations during 1987.
Contribution for 1987: $1,000 paid on 12/31/87.

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 10$$

In what range is the unfunded liability as of 1/1/88?

(A) Less than $4,800
(B) $4,800 but less than $5,600
(C) $5,600 but less than $6,400
(D) $6,400 but less than $7,200
(E) $7,200 or more

**Problem 7 - 8**

Normal retirement benefit:

> Before 1989: $10 per month for each year of service.
> After 1988: $15 per month for each year of service.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Participant data as of 1/1/89:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/59 | 1/1/49 | 1/1/39 |
| Date of hire | 1/1/89 | 1/1/79 | 1/1/69 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 30 | 16,721 | 266,509 |
| 40 | 9,205 | 136,705 |
| 50 | 4,968 | 65,680 |
| 65 | 1,738 | 17,040 |

$\ddot{a}_{65}^{(12)} = 9.345$

In what range is the increase in the unfunded liability as of 1/1/89 due to the change in the normal retirement benefit?

(A) Less than $5,000
(B) $5,000 but less than $6,000
(C) $6,000 but less than $7,000
(D) $7,000 but less than $8,000
(E) $8,000 or more

**Problem 7 - 9**

Normal retirement benefit:

Before 1989: $50 per month for each year of service.
After 1988: $54 per month for each year of service, applicable to active and inactive participants.

Actuarial cost method: Attained age normal with frozen initial liability.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement deaths and terminations: None.
Retirement age: 65

Selected valuation results:

| | 1/1/88 | 1/1/89 ($50 benefit) |
|---|---|---|
| Present value of future benefits | | |
| Active participants | $ 840,000 | $ 550,000 |
| Inactive participants | 250,000 | 570,000 |
| Unfunded liability | 300,000 | |
| Actuarial value of assets | 250,000 | 345,000 |
| Present value of future compensation | 2,340,000 | 2,090,000 |
| Annual compensation | 360,000 | 220,000 |
| Entry age accrued liability for actives | 690,000 | 410,000 |
| Unit credit accrued liability for actives | 600,000 | 320,000 |
| Present value of years of future service | 100 | 96 |
| Number of actives (all under age 64) | 10 | 8 |

Contribution for 1988: $88,000 paid on 12/31/88.

In what range is the normal cost for 1989 as of 1/1/89?

(A) Less than $38,000
(B) $38,000 but less than $40,000
(C) $40,000 but less than $42,000
(D) $42,000 but less than $44,000
(E) $44,000 or more

**Problem 7 - 10**

Plan effective date: 1/1/88
Normal retirement benefit: $25 per month for each year of service.
Actuarial cost method: Attained age normal with frozen initial liability.

Actuarial assumptions:

Interest rate: 8%
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/38 |
| Date of hire | 1/1/65 |

Contribution for 1988: $3,000 paid on 12/31/88.

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 50 | 322 | 3902 |
| 51 | 298 | 3580 |
| 65 | 99 | 904 |

$\ddot{a}_{65}^{(12)} = 8.67$

In what range is the normal cost for 1989 as of 1/1/89?

(A) Less than $1,225
(B) $1,225 but less than $1,250
(C) $1,250 but less than $1,275
(D) $1,275 but less than $1,300
(E) $1,300 or more

**Problem 7 - 11**

Plan effective date: 1/1/91
Normal retirement benefit: $10 per month for each year of service up to 30 years.
Preretirement death benefit: None.
Actuarial cost method: Attained age normal.

Actuarial assumptions:

Preretirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/41 |
| Date of hire | 1/1/71 |
| Status as of 1/1/91 | Active |

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 30 | 1,238 | 28,056 |
| 50 | 320 | 3,716 |
| 65 | 97 | 849 |

In what range is the normal cost for 1991 as of 1/1/91?

(A) Less than $150
(B) $150 but less than $350
(C) $350 but less than $550
(D) $550 but less than $750
(E) $750 or more

**Problem 7 - 12**

Plan effective date: 1/1/89
Normal retirement benefit: $35 per month for each year of service.
Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest rate: 8% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/45 |
| Date of hire | 1/1/69 |
| Status as of 1/1/91 | Active |

Selected valuation results as of 1/1/90:

| | |
|---|---|
| Normal cost as of 1/1 | \$ 1,296 |
| Unfunded liability | 12,818 |
| Value of assets | 3,000 |

Contribution for 1990: \$3,000 paid on 12/31/90.
Benefit payments for 1990: \$0

There were no experience gains or losses during 1990 from any source other than an experience gain from investments of \$500.

$\ddot{a}_{65}^{(12)} = 8.00$

In what range is the normal cost for 1991 as of 1/1/91?

(A) Less than \$1,250
(B) \$1,250 but less than \$1,265
(C) \$1,265 but less than \$1,280
(D) \$1,280 but less than \$1,295
(E) \$1,295 or more

**Problem 7 - 13**

Normal retirement benefit: \$900 per month.
Preretirement death benefit: None.
Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/48 |
| Date of hire | 1/1/83 |

Normal cost for 1993 as of 1/1/93: \$1,231
Value of assets as of 1/1/93: \$10,000

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 10.0$

In what range is the unfunded liability as of 1/1/93?

(A) Less than \$2,000
(B) \$2,000 but less than \$4,000
(C) \$4,000 but less than \$6,000
(D) \$6,000 but less than \$8,000
(E) \$8,000 or more

**Problem 7 - 14**

Plan effective date: 1/1/93
Normal retirement benefit: \$20 per month for each year of service.
Preretirement death benefit: None.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/92 |

Contribution for 1993: \$1,125 paid on 12/31/93.

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 49 | 101,241 | 1,238,268 |
| 50 | 94,135 | 1,137,027 |
| 65 | 28,610 | 263,044 |

In what range is the unfunded liability as of 1/1/94?

(A) Less than \$550
(B) \$550 but less than \$565
(C) \$565 but less than \$580
(D) \$580 but less than \$595
(E) \$595 or more

**Problem 7 - 15**

Plan effective date: 1/1/96.

Normal retirement benefit: \$10 per month for each year of service.

Actuarial assumptions:

Interest rate: 7% per year.
Pre-retirement decrements: None.
Retirement age: 65.

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/51 |
| Date of hire | 1/1/86 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the absolute value of the difference in the unfunded liability as of 1/1/96 under the attained age normal cost method and the entry age normal cost method?

- (A) Less than $1,600
- (B) $1,600 but less than $1,700
- (C) $1,700 but less than $1,800
- (D) $1,800 but less than $1,900
- (E) $1,900 or more

**Problem 7 - 16**

Plan effective date: 1/1/95.

Normal retirement benefit: 2% of final year's compensation for each year of service.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/51 |
| Date of hire | 1/1/85 |
| 1997 valuation compensation | $52,000 |

Selected valuation results as of 1/1/96:

| | |
|---|---|
| Normal cost as of 1/1 | $ 6,300 |
| Unfunded liability | 45,000 |
| Value of assets | 5,000 |

Contribution for 1996: $7,000 paid on 12/31/96.

Benefit payments for 1996: $0

There were no experience gains or losses during 1996 from any source other than an investment gain of $900.

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1997 as of 1/1/97?

(A) Less than $6,500
(B) $6,500 but less than $7,000
(C) $7,000 but less than $7,500
(D) $7,500 but less than $8,000
(E) $8,000 or more

**Problem 7 - 17**

Plan effective date: 1/1/97.

Normal retirement benefit: $20 per month for each year of service.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

| | |
|---|---|
| Preretirement terminations other than deaths: | None. |
| Retirement age: | 65. |

Valuation data for all 100 participants (all active as of 1/1/97):

| | |
|---|---|
| Date of birth | 1/1/62 |
| Date of hire | 1/1/92 |

Selected commutation functions and annuity values:

$D_{65} = 94{,}414 \quad N_{65} = 868{,}052$

$\ddot{a}_{35} = 13.8278 \quad \ddot{a}_{35:\overline{30}|} = 12.8570$

In what range is the normal cost for 1997 as of 1/1/97?

(A) Less than \$46,000
(B) \$46,000 but less than \$49,000
(C) \$49,000 but less than \$52,000
(D) \$52,000 but less than \$55,000
(E) \$55,000 or more

### 7.4 Solutions to Problems

**Problem 7 - 1**

Key Concept: To determine the normal cost, one needs to determine the unfunded accrued liability as of 1/1/83. Other required information is provided.

Step I: Calculate Normal cost as of 1/1/82.

$$\begin{aligned} PVFNC_{82} &= PVFB_{82} - AL_{82} - Assets_{82} \\ &= 1{,}000{,}000 - 400{,}000 - 0 = 600{,}000 \end{aligned}$$

$$\begin{aligned} NC_{82} &= PVFNC_{82} \div ATA_{82} \\ &= 600{,}000 \div 10 = 60{,}000 \end{aligned}$$

Step II: Calculate Unfunded Accrued Liability as of 1/1/83.

$$\begin{aligned} UAL_{83} &= (UAL_{82} + NC_{82})(1 + i) - C_{82} - I_c \\ &= (400{,}000 + 60{,}000)(1.06) - 100{,}000 - 0 = 387{,}600 \end{aligned}$$

Step III: Determine Normal Cost as of 1/1/83.

$$\begin{aligned} PVFNC_{83} &= PVFB_{83} - UAL_{83} - Assets_{83} \\ &= 1{,}200{,}000 - 387{,}600 - 100{,}000 = 712{,}400 \end{aligned}$$

$$\begin{aligned} NC_{83} &= PVFNC_{83} \div (PVFS_{83} \div Salary_{83}) \\ &= 712{,}400 \div 11 = 64{,}764 \end{aligned}$$

Answer is D.

**Problem 7 - 2**

Key Concept: It should be noted that the 35 year age group has no past service, hence, the accrued liability under either method is equal to zero. Therefore, only the 45 year old group need be considered.

The simplest approach is to compute the accrued liability for one participant under each funding method and then multiply the answer by the number of participants, or 60.

Step I: Entry Age Normal Method.

$$
\begin{aligned}
\text{(EAN) NC} &= (\text{Ben})(12\ddot{a}_{65}^{(12)})(D_{65}) \div (N_{35} - N_{65}) \\
&= (300)(120)(10) \div 1{,}190 \\
&= 360{,}000 \div 1{,}190 = 302.52
\end{aligned}
$$

Using the retrospective method:

$$
\begin{aligned}
\text{(EAN) AL} &= (\text{NC})(N_{35} - N_{45})/D_{45} \\
&= (302.52)(1{,}190 - 500) \div 46 = 4{,}537.80
\end{aligned}
$$

Step II: Attained Age Normal Method.

Accrued Liability is determined in the same manner as the Unit Credit Method. Accrued Liability is equal to the present value of Accrued Benefit.

For each participant:

$$
\begin{aligned}
\text{AL} &= (\text{Accrued Benefit})(12\ddot{a}_{65}^{(12)})(D_{65}/ D_{45}) \\
&= (100)(120)(10)/46 = 2{,}608.70
\end{aligned}
$$

Difference per participant, then, is 4,537.80 - 2,608.70 = 1,929.10

Total difference = 1,929.10 × 60 = 115,746

Answer is B.

**Problem 7 - 3**

Key Concept: Under the Attained Age Normal method, experience gains and losses do not affect the unfunded liability. In other words, the unfunded accrued liability is equal to the expected unfunded accrued liability.

Step I: Determine Unfunded Accrued Liability on 1/1/84.

$$
\begin{aligned}
UAL_{84} &= (UAL_{83} + NC_{83})(1 + i) - C_{83} - I_c \\
&= (495{,}000 + 40{,}000)(1.07) - 85{,}000 - 0 \\
&= 572{,}450 - 85{,}000 = 487{,}450
\end{aligned}
$$

Step II: Determine Unfunded Accrued Liability on 1/1/85.

$$\begin{aligned} UAL_{85} &= (UAL_{84} + NC_{84})(1 + i) - C_{84} - I_c \\ &= (487{,}450 + 45{,}000)(1.07) - 90{,}000 - 0 \\ &= 569{,}722 - 90{,}000 = 479{,}722 \end{aligned}$$

Answer is D.

**Problem 7 - 4**

Step I: Compute the Present Value of Future Benefits on basis of $15 per month formula.

| | Smith | Brown |
|---|---|---|
| Attained Age | 46 | 36 |
| Past years of service | 6 | 11 |
| Total years of service | 25 | 40 |
| Benefit at $15 per year of service | 375 | 600 |
| Present Value of Future Benefits (Benefit $\times v^{y-x} \times 12\ddot{a}_{65}^{(12)}$ ) | 10,369 | 8,434 |

Present Value of Future Benefits = 10,369 + 8,434 = 18,803

Step II: Compute increase in Unfunded Accrued Liability using the Unit Credit Cost Method.

Increase for Smith: $5 \times 6$ years $\times 100 \times v^{19} = 830$

Increase for Brown: $5 \times 11$ years $\times 100 \times v^{29} = 773$

Total = 830 + 773 = 1,603

Step III: Compute Present Value of Future Normal Costs.

$$\begin{aligned} PVFNC &= PVFB - \text{Plan Assets} - UAL - UAL\text{ increase} \\ &= 18{,}803 - 1{,}100 - 2{,}500 - 1{,}603 = 13{,}600 \end{aligned}$$

Answer is B.

**Problem 7 - 5**

Key Concept: Under the Attained Age Normal method, past service liability is determined using the Unit Credit approach.

| As of 1/1/88: | Smith | Brown |
|---|---|---|
| Attained Age | 45 | 60 |
| Entry Age | 30 | 40 |
| Projected Benefit | $(15)(10) + (20)(15) = 450$ | $(20)(10) + (5)(15) = 275$ |
| Accrued Benefit | $(15)(10) = 150$ | $(20)(10) = 200$ |
| PV Accrued Benefit | $(150)(9)(12)(1.06)^{-20} = 5{,}051$ | $(200)(9)(12)(1.06)^{-5} = 16{,}141$ |
| PV Future Benefits | $(450)(9)(12)(106)^{-20} = 15{,}153$ | $(275)(9)(12)(1.06)^{-5} = 22{,}194$ |
| PV Future Years | $\ddot{a}_{\overline{20}|} = 12.16$ | $\ddot{a}_{\overline{5}|} = 4.47$ |

Total PV Accrued Benefits = Initial Accrued Liability
= 5,051 + 16,141 = 21,192

| | |
|---|---|
| PV Future Benefits (15,153 + 22,194) | 37,347 |
| Less Accrued Liability (5,051 + 16,141) | - 21,192 |
| PV Future Normal Costs | 16,155 |
| Divided by PV Future Years (12.16 + 4.47) | ÷ 16.63 |
| Normal Cost per Participant | 971 |
| Number of Participants | × 2 |
| Total Normal Cost | 1,942 |

Answer is C.

Note: It is not necessary to calculate the initial accrued liability in this problem since the present value of future normal costs will simply be equal to the present value of future accrued benefits as follows:

Present Value of Future Accrued Benefits

Smith: $(20)(15)(9)(12)(1.06)^{-20} = 10{,}102$
Brown: $(5)(15)(9)(12)(1.06)^{-5} = 6{,}053$

Total Present Value of Accrued Benefits = 10,102 + 6,053 = 16,155

**Problem 7 - 6**

Key Concept: Under the Attained Age Normal method, the Initial Accrued Liability is equal to the Present Value of Accrued Benefits.

| On the effective date of 1/1/87: | Smith | Brown |
|---|---|---|
| (1) Attained Age | 50 | 60 |
| (2) Past Service | 25 | 15 |
| (3) Future Service | 15 | 5 |
| (4) Projected Benefit | 700 | 300 |
| (5) Accrued Benefit | 400 | 225 |
| (6) PV Accrued Benefit (5) × 112 × $1.06^{-3}$ | 18,693 | 18,831 |
| (7) PV Projected Benefit (4) × 112 × $1.06^{-3}$ | 32,714 | 25,108 |
| (8) PV of Future Years | 10.29 | 4.47 |

Step I: Total Accrued Liability as of 1/1/87 = 18,693 + 18,831 = 37,524

Step II: Calculate Normal Cost at 1/1/87.

| | |
|---|---|
| Present Value of Future Benefits = 32,714 + 25,108 | 57,822 |
| Less Accrued Liability | - 37,524 |
| Equals PV Future Normal Costs | 20,298 |
| Divided by PV Future Years | ÷ 14.76 |
| Equals NC per Participant | 1,375 |
| Multiplied by two participants equals total NC | 2,750 |

Step III: Determine Unfunded Accrued Liability at 1/1/88.

| | |
|---|---|
| Unfunded Accrued Liability as of 1/1/87 | 37,524 |
| Plus Normal Cost | + 2,750 |
| Plus interest on (1) and (2) | + 2,416 |
| Less Contribution for 1987 | - 5,750 |
| Less interest on Contribution | - 345 |
| Unfunded Liability at 1/1/88 | 36,595 |

Answer is B.

Note: It is not necessary to calculate normal cost for 1987 since the contribution was made 1/1/87.

$$UAL = (37{,}524 - 3000)(1.06) = 36{,}595$$

**Problem 7 - 7**

Key Concept: Under the Attained Age Normal method, the initial Accrued Liability is calculated using the Unit Credit method. Thereafter, the method is the same as Frozen Initial Liability.

| At 1/1/88 | Smith | Brown |
|---|---|---|
| Age | 45 | 35 |
| Past Years of Service | 15 | 0 |
| Future Years of Service | 20 | 30 |
| Total Years of Service | 35 | 30 |

Step I: Determine Unfunded Liability at effective date of 1/1/87.

Unfunded Liability = PV of Accrued Benefits.

$$UAL_{87} = PVAB_{87} = (10)(14 \text{ years})(10)(12)(1.06)^{-21} = 4{,}942$$

Step II: Determine Present Value of Future Benefits at 1/1/87.

$$\begin{aligned} PVFB_{87} &= (\text{Projected Benefit})(12)(\ddot{a}_{65}^{(12)})(1+i)^{-21} \\ &= (350)(10)(12)(1.06)^{-21} = 12{,}355 \end{aligned}$$

Step III: Determine Normal Cost at 1/1/87.

$$\begin{aligned} NC_{87} &= (PVFB_{87} - UAL_{87}) \,/\, \ddot{a}_{\overline{21|}} \\ &= (12{,}355 - 4{,}942) \,/\, 12.4699 = 594 \end{aligned}$$

Step IV: Determine Unfunded Liability at 1/1/88.

$$\begin{aligned} UAL_{88} &= (UAL_{87} + NC_{87})(1+i) - C_{87} - I_c \\ &= (4{,}942 + 594)(1.06) - (1{,}000 + 0) = 4{,}868 \end{aligned}$$

Notice that data for Brown does not enter into calculations at all.

Answer is B.

**Problem 7 - 8**

Key Concept: The unfunded liability in the Attained Age Normal funding method is based upon the unit credit accrued liability. Since the benefit formula increased by 50% as of 1/1/89, the unfunded liability increases by an amount equal to 50% of the present value of accrued benefits under the original plan.

Note that since Smith was hired 1/1/89, Smith has no PVAB. The $PVAB_{89}$ under the Original Plan is determined as follows:

Brown's accrued benefit = 10 × 10 years of service = 100
Green's accrued benefit = 10 × 20 years of service = 200

$$\text{Brown's } PVAB_{89} = 12 \times 100 \times \left(\frac{D_{65}}{D_{40}}\right) \ddot{a}_{65}^{(12)} = 12 \times 100 \times \left(\frac{1{,}738}{9{,}205}\right)(9.345) = 2{,}117$$

$$\text{Green's } PVAB_{89} = 12 \times 200 \times \left(\frac{D_{65}}{D_{50}}\right) \ddot{a}_{65}^{(12)} = 12 \times 200 \times \left(\frac{1{,}738}{4{,}968}\right)(9.345) = 7{,}846$$

$$\text{Total } PVAB_{89} = 2{,}117 + 7{,}846 = 9{,}963$$

$$\text{Increase in UAL} = .5 \times 9{,}963 = 4{,}982$$

Answer is A.

**Problem 7 - 9**

Key Concept: The increased benefit formula applies to all participants, increasing the present value of future benefits to all participants. In addition, the unfunded liability is to be increased by the increase in Unit Credit accrued liability due to the plan amendment.

Since the plan benefit formula provides a fixed benefit, the method spreads the Normal Cost as a level dollar amount.

$$NC_{88} = \frac{PVFB_{88} - (Assets_{88} + UL_{88})}{PV \text{ of future years}} \times \text{Number of Participants}$$

$$= \frac{(1{,}090{,}000 - (250{,}000 + 300{,}000))}{100} \times 10 = 54{,}000$$

As of 1/1/89,

$$PVFB_{Old} = 550{,}000 + 570{,}000 = 1{,}120{,}000$$
$$PVFB_{New} = 1{,}120{,}000 \times 54/50 = 1{,}209{,}600$$

Note that the Accrued Liability under Unit Credit for inactive participants is equal to their PVFB (PVAB).

$$\text{Unit Credit } AL_{Old} = 320{,}000 + 570{,}000 = 890{,}000$$
$$\text{Unit Credit } AL_{New} = 890{,}000 \times 54/50 = 961{,}200$$
$$\text{Increase in UAL} = 961{,}200 - 890{,}000 = 71{,}200$$

$$eUAL_{89} = (UAL_{88} + NC_{88}) \times (1 + i) - C_{88} - I_c$$
$$= (300{,}000 + 54{,}000) \times (1.06) - 88{,}000 = 287{,}240$$
$$UAL_{89} = 287{,}240 + 71{,}200 = 358{,}440$$

Calculate the Normal Cost for the new benefit:

$$NC_{89} = \frac{PVFB_{89} - (Assets_{89} + UAL_{89})}{PV \text{ of Future Years}} \times \text{Number of Participants}$$

$$= [1{,}209{,}600 - (345{,}000 + 358{,}440)] \times (8/96) = 42{,}180$$

Answer is D.

**Problem 7 - 10**

Key Concept: The initial Unfunded Accrued Liability under the Attained Age Normal Method is equal to the present value of accrued benefits.

As of 1/1/88, Accrued Benefit = 25 × 23 Years of Service = 575

$$UAL_{88} = PVAB_{88} = 575 \times 12\ddot{a}_{65}^{(12)} \times \frac{D_{65}}{D_{50}} = 18{,}393$$

Retirement Benefit = 25 × 38 Years of Service = 950

$$PVFB_{88} = 950 \times 12\ddot{a}_{65}^{(12)} \times D_{65}/D_{50} = 30{,}388$$

$$NC_{88} = \frac{PVFB - UL_{88} - Assets_{88}}{\ddot{a}_{50:\overline{15}|}}$$

$$= \frac{30{,}388 - 18{,}393 - 0}{\left(\dfrac{N_{50} - N_{65}}{D_{50}}\right)} = 1{,}288$$

Since there were no gains or losses in 1988, the normal cost as of 1/1/89 is the same as 1/1/88, or $1,288.

Answer is D.

**Problem 7 - 11**

Key Concept: Attained Age Normal is a frozen initial liability method with unfunded frozen liability based on the traditional unit credit funding method.

Step I: Calculate the retirement and accrued benefits.

Benefit at retirement = 10 × 12 × 30 years of service = 3,600

Accrued Benefit at 1/1/91 = 10 × 12 × 20 years of service = 2,400

Step II: Calculate unfunded liability under unit credit funding method.

$$AL_{91} = 2400 \times \left( \frac{D_{65}}{D_{50}} \right) \ddot{a}_{65}^{(12)}$$

$$= 2400 \times \left( \frac{N_{65}^{(12)}}{D_{50}} \right) = 2400 \times \left( \frac{849}{320} \right) = 6{,}368$$

Step III: Calculate the temporary annuity.

To convert $N_x^{(12)}$ to $N_x$, use the approximation

$$N_x = N_x^{(12)} + \tfrac{11}{24} D_x$$

$$N_{50} = 3716 + \frac{11}{24}(320) = 3863 \quad \text{and} \quad N_{65} = 849 + \frac{11}{24}(97) = 893$$

$$\text{Temporary annuity} = \frac{N_{50} - N_{65}}{D_{50}} = \frac{3863 - 893}{320} = 9.28125$$

Step IV: Calculate Present Value of Future Benefits.

$$PVFB_{91} = 3600 \times \left( \frac{D_{65}}{D_{50}} \right) \ddot{a}_{65}^{(12)} = 9551$$

Step V: Calculate the Normal Cost.

$$NC_{91} = \frac{PVFB_{91} - (UAL_{91} + Assets_{91})}{\ddot{a}_{50:\overline{15|}}}$$

$$= \frac{9551 - 6368}{9.28125} = 343$$

Answer is B.

**Problem 7 - 12**

Step I: Calculate retirement benefit

Benefit = 35 x 41 years of service = 1,435

Step II: Calculate the Present Value of Future Benefits.

$$PVFB_{91} = 1{,}435 \times 12\ddot{a}_{65}^{(12)} \times v^{19} = 31{,}921$$

Step III: Calculate the Unfunded Accrued Liability.

$$\begin{aligned} UAL_{91} &= (UAL_{90} + NC_{90})\,(1.08) - C_{90} - I_c \\ &= (12{,}818 + 1{,}296)(1.08) - 3{,}000 = 12{,}243 \end{aligned}$$

Step IV: Calculate the value of the Assets.

$$\begin{aligned} Assets_{91} &= Assets_{90}\,(1.08) + 500 + 3{,}000 \\ &= (3{,}000)\,(1.08) + 500 + 3{,}000 = 6{,}740 \end{aligned}$$

Step V: Calculate the Normal Cost.

$$NC_{91} = \frac{PVFB - (UAL + Assets)}{\ddot{a}_{\overline{19}|}} = \frac{31{,}921 - (12{,}243 + 6{,}740)}{10.3719} = 1{,}247$$

Answer is A.

Alternative Solution: If there had been no experience gain or loss, the Normal Cost would have remained $1,296. Since there was a $500 gain, the Normal Cost decreases by the gain, amortized over future years.

$$NC_{91} = 1{,}296 - \left[\frac{500}{\ddot{a}_{\overline{19}|}}\right] = 1{,}296 - 48 = 1{,}248$$

**Problem 7 - 13**

This problem requires us to use the identity:

$$PVFNC = PVFB - Assets - UL$$

$$PVFB_{93} = (900)(12\ddot{a}_{65}^{(12)})v^{20} = 27{,}909$$

$$PVFNC_{93} = (NC_{93})(\ddot{a}_{\overline{20}|}) = (1{,}231)(11.335595) = 13{,}954$$

Substituting into the identity,

$$13{,}954 = 27{,}909 - 10{,}000 - UAL_{93}$$

$$UAL_{93} = 3{,}955$$

Answer is B.

**Problem 7 - 14**

Key Concept: The initial unfunded liability determined using the Attained Age Normal cost method is equal to the Accrued Liability determined using the Unit Credit cost method.

Step I: Calculate the initial unfunded liability as of 1/1/93. Since the sole participant was hired on 1/1/92, there is only one year of past service, so the accrued benefit is \$20 as of 1/1/93.

$$\begin{aligned}\text{Unit Credit } AL_{93} &= (20)(12N_{65}^{(12)}/D_{49})\\ &= (20)(12)(N_{65} - \tfrac{11}{24}D_{65})/D_{49}\\ &= (240)(263{,}044 - (\tfrac{11}{24})(28{,}610))/101{,}241 = 592\end{aligned}$$

Step II: Calculate the Normal Cost as of 1/1/93.

Monthly retirement benefit = (20)(17 years of service) = 340

$$\begin{aligned}PVFB_{93} &= (\text{Benefit})(12N_{65}^{(12)})/D_{49}\\ &= (340)(12)(N_{65} - \tfrac{11}{24}D_{65})/D_{49}\\ &= (340)(12)(263{,}044 - (\tfrac{11}{24})(28{,}610))/101{,}241 = 10{,}072\end{aligned}$$

$$NC_{93} = (PVFB_{93} - \text{Unit Credit } AL_{93})/\ddot{a}_{49:\overline{16|}}$$
$$= (10{,}072 - 592)/[(N_{49} - N_{65})/D_{49}]$$
$$= (10{,}072 - 592)/[(1{,}238{,}268 - 263{,}044)/101{,}241 = 984$$

Step III: Calculate the Unfunded Accrued Liability as of 1/1/94.

$$UAL_{94} = (AL_{93} - NC_{93})(1.07) - \text{Contrib}_{93}$$
$$= (592 + 984)(1.07) - 1{,}125 = 561$$

Answer is B.

**Problem 7 - 15**

Step I: Calculate the Attained Age Normal Unfunded Liability.

Note that this is the Unit Credit Accrued Liability, which is just the present value of the accrued benefit.

$$\text{Unfunded Liability} = (10)(10 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(v^{20})$$
$$= 2{,}710$$

Step II: Calculate the Entry Age Normal Unfunded Liability.

$$\text{Unfunded Liability} = (10)(30 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})((\ddot{s}_{\overline{10|}}/\ddot{s}_{\overline{30|}})$$
$$= 4{,}602$$

Step III: Calculate the difference between the unfunded liabilities.

$$\text{Difference} = 4{,}602 - 2{,}710 = 1{,}892$$

Answer is D.

**Problem 7 - 16**

Key Concept: The Normal Cost for 1997 will be equal to the Normal Cost in 1996 (increased by the salary scale), reduced by an amortization of the investment gain.

Step I: Determine the average temporary annuity used in the 1/1/96 valuation.

$$\text{Final Salary} = (52{,}000)(1.04)^{18} = 105{,}342$$

Note that final salary is the same in both the 1996 and 1997 valuations since there was no experience gain or loss due to compensation. Since retirement is assumed to occur on 1/1/2016 (age 65), the final salary used will be in the 1/1/2015 valuation, 18 years from 1997.

$$\text{Benefit}_{96} = (105{,}342)(.02)(31 \text{ years of service}) = 65{,}312$$

$$\begin{aligned}\text{PVFB}_{96} &= (65{,}312)(\ddot{a}_{65}^{(12)})(v^{20})\\ &= (65{,}312)(8.74)(.2584)\\ &= 147{,}502\end{aligned}$$

$$\text{NC}_{96} = (\text{PVFB}_{96} - \text{Assets}_{96} - \text{UL}_{96})/\text{ATA}_{96}$$

$$6{,}300 = (147{,}502 - 5{,}000 - 45{,}000)/\text{ATA}_{96}$$

$$\text{ATA}_{96} = 15.4765$$

Step II: Determine the average temporary annuity to be used in the 1/1/97 valuation.

The formula for determining the current annuity from the prior year annuity (in the situation where there are no preretirement decrements) is:

$$\text{ATA}_{N+1} = (\text{ATA}_N - 1)[(1+i)/(1+s)]$$

So,

$$\text{ATA}_{97} = (15.4765 - 1)(1.07/1.04) = 14.8941$$

Step III: Determine the decrease in Normal Cost as of 1/1/97 due to investment gain.

$$\begin{aligned}\text{NC}_{\text{DECREASE}} &= \text{Investment gain}/\text{ATA}_{97}\\ &= 900/14.8941\\ &= 60\end{aligned}$$

Step IV: Determine the Normal Cost as of 1/1/97.

$$\begin{aligned}\text{NC}_{97} &= (\text{NC}_{96})(1.04) - \text{NC}_{\text{DECREASE}}\\ &= (6{,}300)(1.04) - 60\\ &= 6{,}492\end{aligned}$$

Answer is A.

**Problem 7-16 (Alternative Solution)**

Key Concept: The normal cost rate is constant each year provided there are no gains or losses. The normal cost rate for 1997 will decrease from 1996 due to the investment gain.

Step I: Determine the 1996 normal cost rate.

The normal cost rate is equal to the normal cost as a percentage of salary. Since the salary increase assumption was met for 1996, the 1996 salary was:

$$Salary_{1996} = Salary_{1997}/1.04 = 52{,}000/1.04 = 50{,}000$$

The normal cost rate for 1996 was:

$$\begin{aligned} NC\%_{1/1/96} &= NC_{1/1/96} \ / \ Salary_{1996} \\ &= 6{,}300/50{,}000 \\ &= .126 \end{aligned}$$

Step II: Determine the 1997 normal cost rate.

The normal cost rate decreased in 1997 by the ratio of the investment gain to the 1997 present value of future salary.

$$\begin{aligned} PVFS_{1/1/97} &= Salary_{1997} \times \ddot{a}_{\overline{19}|j} \qquad \text{where } j = 1.07/1.04 - 1 = .028846 \\ &= 52{,}000 \times 14.8887 \\ &= 774{,}212 \end{aligned}$$

And,

$$\begin{aligned} NC\%_{1/1/97} &= NC\%_{1/1/96} - (900/PVFS_{1/1/97}) \\ &= .126 - (900/774{,}212) \\ &= .1248 \end{aligned}$$

Step III: Determine the 1997 normal cost.

$$\begin{aligned} NC_{1/1/97} &= NC\%_{1/1/97} \times Salary_{1997} \\ &= .1248 \times 52{,}000 \\ &= 6{,}490 \end{aligned}$$

Answer is A.

**Problem 7 - 17**

Key Concept: The Unfunded Liability in the first year of the Attained Age Normal method is equal to the Unit Credit Accrued Liability.

Step I: Calculate the Unfunded Liability as of 1/1/97.

In order to calculate the unfunded liability, it is necessary to find the values of $\ddot{a}_{65}^{(12)}$ and $D_{35}$. These can be determined from the given data.

$$\begin{aligned}\ddot{a}_{65}^{(12)} &= [N_{65} - (11/24)D_{65}]/D_{65} \\ &= [868{,}052 - (11/24)(94{,}414)]/94{,}414 \\ &= 8.74\end{aligned}$$

$$\ddot{a}_{35} - \ddot{a}_{35:\overline{30|}} = (\ddot{a}_{65})(D_{65}/D_{35}) \quad \Rightarrow \quad 13.8278 - 12.8570 = (868{,}052/94{,}414)(94{,}414/D_{35})$$

$$\Rightarrow \quad D_{35} = 894{,}162$$

$$\begin{aligned}UL_{97} &= (100 \text{ participants})(20)(5 \text{ years of service})(12\ddot{a}_{65}^{(12)})(D_{65}/D_{35}) \\ &= 110{,}742\end{aligned}$$

Note: As an alternative the student can solve for $D_{65}/D_{35}$ and substitute into the final equation.

Step II: Calculate the Present Value of Future Normal Costs.

Each participant will have 35 years of service at retirement and currently has 5 years of service. Therefore, each participant will have 30 years of future service. Since the Unfunded Liability is based upon the 5 years of past service and zero assets, the Present Value of Future Normal Costs should be equal to 6 times the Unfunded Liability.

$$PVFNC_{97} = UL_{97} \times 6 = 110{,}742 \times 6 = 664{,}452$$

Step III: Calculate the Normal Cost.

$$NC_{97} = PVFNC_{97}/\ddot{a}_{35:\overline{30|}} = 664{,}452/12.8570 = 51{,}680$$

Answer is C.

# Chapter 8

# Miscellaneous

## 8.1 Ancillary Benefits and Other Actuarial Topics

In this, the final section, are those problems given in Joint Board Actuaries examinations that are not directly related to a specific Funding Method. Material could be extracted from such subjects as ancillary benefits, investment analysis, actuarial equivalence of alternate forms of distributions, employee contributions, cost of vesting and other areas of actuarial practice.

## 8.2 Problems

### Problem 8 - 1

Assumed interest rate: 7%

Data for retired participant:

Age of retiree: x
Age of spouse: y
Annual pension benefit: $10,000 per year payable at the beginning of each year.
Form of payment: Life annuity for participant with 50% continued for life of surviving spouse of participant.

Selected annuity values:

$\ddot{a}_x = 8.157$ $\ddot{a}_y = 10.301$ $\ddot{a}_{xy} = 7.281$

$\ddot{a}_{x+1} = 7.915$ $\ddot{a}_{y+1} = 10.059$

In what range is the loss from mortality if both the retiree and spouse are alive at the end of the first year?

(A) Less than $1,350
(B) $1,350 but less than $1,425
(C) $1,425 but less than $1,500
(D) $1,500 but less than $1,575
(E) $1,575 or more.

### Problem 8 - 2

Early retirement eligibility: Age 55

Early retirement benefit: Accrued benefit reduced 5/12% per month for the first 60 months, and 3/12% per month for each additional month, by which benefit commencement precedes age 65.

A level-income option is available at early retirement under which the pension reduces at age 62 by the amount of the estimated social security benefit.

The level income pension is "actuarially equivalent" to the normal pension. "Actuarial equivalence" is based on the plan's early retirement factors.

Smith retires at age 58 with an accrued benefit of $450 per month, and elects the level-income option. Smith's Social Security benefit commencing at age 62 is estimated at $300 per month.

In what range is Smith's monthly benefit at age 58?

(A) Less than $540
(B) $540 but less than $550
(C) $550 but less than $560
(D) $560 but less than $570
(E) $570 or more.

**Problem 8 - 3**

Employee contributions: Previously required, but discontinued as of 12/31/84
Death or termination benefit: Employee contributions plus interest at 5% are returned at the end of the year of death or termination.
Assumed interest rate: 7%
Assumed retirement age: 65

Decrements other than deaths are assumed to occur at the end of the year.

Data for the sole participant as of 1/1/85:

Attained age: 40.
Accumulated employee contributions plus interest as of 1/1/85 : $1,000

Selected functions:

| Age x | $l_x'^{(m)}$ (mortality only) | $l_x^{(T)}$ (all decrements) |
|---|---|---|
| 39 | 1,093 | 855 |
| 40 | 1,091 | 842 |
| 41 | 1,088 | 830 |

$K is the present value, as of 1/1/85, of the return of employee contributions with interest for all decrements other than death which occur during 1985. In what range is $K?

(A) Less than $11.00
(B) $11.00 but less than $11.25
(C) $11.25 but less than $11.50
(D) $11.50 but less than $11.75
(E) $11.75 or more.

**Problem 8 - 4**

The actuarial value of assets is determined as the average of book value and market value.

You are given the following values:

| | Book Value | Market Value |
|---|---|---|
| Value as of 1/1/84 | $1,000,000 | $ 800,000 |
| Contribution paid at 7/1/84 | 100,000 | 100,000 |
| Interest and dividends | 100,000 | 100,000 |
| Realized gains (losses) | (50,000) | (50,000) |
| Unrealized gains (losses) | N/A | 100,000 |
| Value as of 12/31/84 | $1,150,000 | $1,050,000 |

In what range is the dollar-weighted rate of return on the actuarial value of assets during 1984?

(A) Less than 10.5%
(B) 10.5% but less than 11.0%
(C) 11.0% but less than 11.5%
(D) 11.5% but less than 12.0%
(E) 12.0% or more.

**Problem 8 - 5**

Data for sole participant as of 1/1/86:

Date of birth: 1/1/16.
Spouse's date of birth: 1/1/16.
Status: Retired.

Retirement Benefit: $6,000 per year, payable annually on 1/1 for life, with 100% continued to surviving spouse.

Selected values:

$q_{70} = .050$

$\ddot{a}_{70} = 8.80 \qquad \ddot{a}_{70:70} = 6.11$

$\ddot{a}_{71} = 8.46 \qquad \ddot{a}_{71:71} = 5.83$

The participant and his spouse are both alive on 1/1/87.

In what range is the mortality loss for 1986?

(A) Less than $1,600
(B) $1,600 but less than $3,100
(C) $3,100 but less than $4,600
(D) $4,600 but less than $6,100
(E) $6,100 or more

**Problem 8 - 6**

Early retirement benefit: Actuarial equivalent of pension payable at normal retirement.
Pre-retirement death benefit: None.

Optional form of pension: Life annuity with 120 months certain, on an actuarially equivalent basis.

F(x) is the factor by which the amount of pension in the normal form commencing at age 65 must be multiplied in order to obtain the amount of pension in the optional form commencing at age x.

Which of the following correctly expresses F(55)?

(A) $\dfrac{D_{65}}{D_{55}} \ddot{a}^{(12)}_{\overline{10|}} + \dfrac{N^{(12)}_{65}}{N^{(12)}_{55}}$

(B) $\dfrac{D_{65}\,\ddot{a}^{(12)}_{\overline{10|}} + N^{(12)}_{75}}{D_{65}} \times \dfrac{N^{(12)}_{65}}{N^{(12)}_{55}}$

(C) $\dfrac{D_{55}\,\ddot{a}^{(12)}_{\overline{10|}} + N^{(12)}_{65}}{N^{(12)}_{65}}$

(D) $\dfrac{D_{65}\,\ddot{a}^{(12)}_{\overline{10|}} + N^{(12)}_{75}}{D_{55}\,\ddot{a}^{(12)}_{\overline{10|}} + N^{(12)}_{65}} \times \dfrac{N^{(12)}_{65}}{N^{(12)}_{55}}$

(E) $\dfrac{N^{(12)}_{65}}{D_{55}\,\ddot{a}^{(12)}_{\overline{10|}} + N^{(12)}_{65}}$

**Problem 8 - 7**

Normal retirement benefit: $1,000 per year, payable annually for life at the beginning of the year.

Assumed interest rate: 7%

Assumed retirement age: 65

It is assumed that there are no terminations prior to retirement, other than by death.

The sole participant is age 55 as of 1/1/86.

Selected values and commutation functions:

| $i$ | $\ddot{a}_{65}$ | $D_{55}$ | $D_{65}$ |
|---|---|---|---|
| 3% | 11.70 | 1,778 | 1,149 |
| 4% | 10.82 | 1,045 | 613 |
| 5% | 10.04 | 617 | 329 |
| 7% | 8.74 | 219 | 96 |

Effective 1/1/86 the plan is amended to provide an automatic post-retirement adjustment which will increase each annual payment by 3% of the preceding payment.

In what range is the increase in the present value of future benefits as of 1/1/86 due to the amendment?

(A) Less than $925
(B) $925 but less than $1,425
(C) $1,425 but less than $1,925
(D) $1,925 but less than $2,425
(E) $2,425 or more

**Problem 8 - 8**

| | Date | Amount |
|---|---|---|
| Market values of fund: | 1/1/87 | $ 50,000 |
| | 3/31/87 | 50,515 |
| | 6/30/87 | 46,705 |
| | 9/30/87 | 48,136 |
| | 1/1/88 | 63,651 |
| Benefits paid: | 4/1/87 | 5,200 |
| | 12/31/87 | 10,100 |
| Contributions received: | 10/1/87 | 20,000 |
| | 12/31/87 | 2,700 |

In what range is the dollar-weighted rate of return for 1987?

(A) Less than 11.55%
(B) 11.55% but less than 11.85%
(C) 11.85% but less than 12.15%
(D) 12.15% but less than 12.45%
(E) 12.45% or more

**Problem 8 - 9**

Accrued Benefit: \$15 per month for each year of service.

Vested Benefit: 25% of the accrued benefit for each year of service in excess of 6 years, to a maximum of 100% of the accrued benefit.

Assumed Retirement Age: 65.

Actuarial Cost Method: Aggregate.

It is assumed that all terminations occur at the beginning of the year.

Data for sole participant as of 1/1/87:

Attained Age: 40 Age at Hire: 35

$q_x^w$ is the probability of an employee aged x terminating before age (x + 1) according to the service table for active employees. $l_x$ is from the service table for active employees.

All commutation functions are based solely on mortality and interest.

Which of the following is the expression for the present value as of 1/1/87 of the participant's benefits for vested termination before age 65?

(A) $(180N_{65}^{(12)} \div D_{40})\,[.25 \sum_{t=42}^{44} q_t^w\,(t-35)(t-40) + \sum_{t=45}^{64} q_t^w(t-35)]$

(B) $(180N_{65}^{(12)} \div D_{40})\,[.25 \sum_{t=42}^{44} q_t^w\,(t-35)(t-41) + \sum_{t=45}^{64} q_t^w(t-35)]$

(C) $(180N_{65}^{(12)} \div D_{40})\,[.25 \sum_{t=42}^{44} q_t^w\,(t-35)(t-40)v^{t-40} + \sum_{t=45}^{64} q_t^w(t-35)\,v^{t-40}]$

(D) $(180N_{65}^{(12)}/l_{40})\,[.25 \sum_{t=42}^{44} q_t^w(t-35)(t-40)v^{t-40}l_t/D_t + \sum_{t=45}^{64} q_t^w\,(t-35)v^{t-40}l_t/D_t]$

(E) $(180N_{65}^{(12)}/l_{40})\,[.25 \sum_{t=42}^{44} q_t^w(t-35)(t-41)v^{t-40}l_t/D_t + \sum_{t=45}^{64} q_t^w(t-35)\,v^{t-40}\,l_t/D_t\,]$

**Problem 8 - 10**

Early retirement age: 55
Basis for conversion between annuity forms: Actuarial equivalence.

The following changes in the assumptions for actuarial equivalence are being considered:

Interest: Increase from 6% to 8%.

Mortality: Change from the 1971 Group Annuity Mortality Table (Males) to the 1971 Group Annuity Mortality Table (Males) set back three years.

Consider the following statements regarding a participant aged 40.

I. If only the mortality assumption is changed, early retirement benefits in the normal form will increase.

II. If only the interest assumption is changed, early retirement benefits in the normal form will increase.

III. If only the mortality assumption is changed, normal retirement benefits in the life with ten year certain optional form will increase.

Which, if any, of these statements is (are) true?

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D) above.

**Problem 8 - 11**

Normal retirement benefit: $1,000 each 1/1 until the retiree dies, then $500 to the surviving spouse (if any) each 1/1 thereafter for life.

Participant data as of 1/1/86: 100 married retirees, all aged 65 with spouses aged 62.

Deaths during 1986: 3 retirees die, leaving their spouses alive and 2 spouses die leaving the retirees alive.

There were no new retirees during 1986.

| | | |
|---|---|---|
| $\ddot{a}_{62} = 9.230$ | $\ddot{a}_{62:65} = 7.440$ | $p_{62} = .985$ |
| $\ddot{a}_{63} = 9.024$ | $\ddot{a}_{63:66} = 7.205$ | $p_{65} = .980$ |
| $\ddot{a}_{65} = 8.630$ | | |
| $\ddot{a}_{66} = 8.409$ | | |

In what range is the mortality gain for 1986?

(A) Less than $1,250
(B) $1,250 but less than $2,500
(C) $2,500 but less than $3,750
(D) $3,750 but less than $5,000
(E) $5,000 or more

**Problem 8 - 12**

Normal retirement benefit: $12,000 per year payable annually on 1/1.

Spouse's death benefit: $6,000 per year payable annually commencing on the 1/1 following the participant's death.

Assumed interest rate: 6%

Data for sole participant Smith:

| | |
|---|---|
| Date of birth | 1/1/22 |
| Date of retirement | 1/1/85 |
| Spouse's date of birth | 1/1/27 |

Smith's spouse died on 7/1/87.

Selected unisex values:

| | | |
|---|---|---|
| $\ddot{a}_{60} = 10.15$ | $\ddot{a}_{65} = 8.95$ | $\ddot{a}_{60:65} = 8.05$ |
| $\ddot{a}_{61} = 9.80$ | $\ddot{a}_{66} = 8.60$ | $\ddot{a}_{61:66} = 7.70$ |
| $p_{60} = .99$ | $p_{65} = .98$ | |

In what range is the mortality gain for 1987 recognized as of 1/1/88?

(A) Less than \$10,000
(B) \$10,000 but less than \$11,500
(C) \$11,500 but less than \$13,000
(D) \$13,000 but less than \$14,500
(E) \$14,500 or more

**Problem 8 - 13**

Normal retirement benefit: \$600 per month for life, with \$300 per month continuing to the participant's surviving spouse for life.
Pre-retirement death benefit: None.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/29 |
| Spouse's date of birth | 1/1/24 |

Selected probability and annuity values (based on a unisex mortality table and 6% interest):

$_{10}p_{60} = 0.80$ $\quad 12\ddot{a}_{60}^{(12)} = 127$ $\quad 12\ddot{a}_{60:65}^{(12)} = 94$

$_{5}p_{65} = 0.87$ $\quad 12\ddot{a}_{65}^{(12)} = 112$ $\quad 12\ddot{a}_{65:70}^{(12)} = 79$

$12\ddot{a}_{70}^{(12)} = 97$

In what range is the present value of the normal retirement benefit as of 1/1/89?

(A) Less than \$49,500
(B) \$49,500 but less than \$52,000
(C) \$52,000 but less than \$54,500
(D) \$54,500 but less than \$57,000
(E) \$57,000 or more

**Problem 8 - 14**

Normal retirement benefit: $15 per month for each year of service.

Early retirement benefit: Accrued benefit reduced by 6% for each year by which commencement of payments precedes age 65.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement deaths and terminations: None.
Retirement age:

| Age x | Probability at Age x of Retiring at Age x |
|---|---|
| 63 | 50% |
| 64 | 20% |
| 65 | 100% |

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/26 |
| Date of hire | 1/1/73 |

Selected annuity values:

$\ddot{a}_{63}^{(12)} = 9.80$ $\ddot{a}_{64}^{(12)} = 9.60$ $\ddot{a}_{65}^{(12)} = 9.35$

In what range is the present value of future benefits as of 1/1/89?

(A) Less than $24,800
(B) $24,800 but less than $25,800
(C) $25,800 but less than $26,800
(D) $26,800 but less than $27,800
(E) $27,800 or more

**Problem 8 - 15**

Normal retirement benefit: $20 per month for each year of service.

Early retirement benefit: Accrued benefit reduced by 5% for each year by which commencement of payments precedes age 65.

Pre-retirement death benefit: 50% of early retirement benefit payable monthly to the participant's spouse for life.

Actuarial assumptions:

Interest rate: 6%
Pre-retirement death decrement: 2% at the beginning of each year.
Pre-retirement terminations other than deaths: None.
Retirement age: 65

Marital characteristics: 90% of participants are married at the time of death; spouses are the same age as participants.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/26 |
| Date of hire | 1/1/59 |

Selected annuity values:

$\ddot{a}_{63}^{(12)} = 9.85$ $\ddot{a}_{64}^{(12)} = 9.60$ $\ddot{a}_{65}^{(12)} = 9.35$

In what range is the present value of pre-retirement death benefits as of 1/1/89?

(A) Less than $1,135
(B) $1,135 but less than $1,145
(C) $1,145 but less than $1,155
(D) $1,155 but less than $1,165
(E) $1,165 or more

**Problem 8 - 16**

Smith, age 60, is entitled to a pension payable under either Option A or Option B, which are actuarially equivalent.

| | |
|---|---|
| Option A: | A life annuity commencing immediately which pays (P + \$500) per month for two years, and P per month thereafter. |
| Option B: | A life annuity commencing immediately which pays \$500 per month as long as Smith and his spouse (also age 60) are both alive. Upon the death of Smith or his spouse, the monthly payment is reduced to (\$500 - .50 × P). |

Selected commutation functions and annuity values:

$D_{60} = 147.804$ $\qquad\qquad$ $\ddot{a}_{60}^{(12)} = 9.815$

$D_{62} = 125.296$ $\qquad\qquad$ $\ddot{a}_{62}^{(12)} = 9.394$

$\ddot{a}_{60:60}^{(12)} = 8.094$

In what range is P?

(A) Less than \$440
(B) \$440 but less than \$540
(C) \$540 but less than \$640
(D) \$640 but less than \$740
(E) \$740 or more

**Problem 8 - 17**

Normal retirement benefit: \$1,000 per month.

Basis for conversion for early retirement and between optional forms of payment: Actuarial equivalence.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of retirement | 1/1/90 |
| Spouse's date of birth | 1/1/38 |
| Form of payment elected | Monthly benefit for life, with 50% continuing to the participant's surviving spouse for life. |

Selected commutation functions:

| Age x | $N_x$ | $N_{x:x-3}$ |
|---|---|---|
| 51 | 377 | 3,234 |
| 52 | 345 | 2,935 |
| 53 | 316 | 2,659 |
| 54 | 289 | 2,403 |
| 55 | 263 | 2,168 |
| 56 | 240 | 1,951 |
| | | |
| 61 | 147 | 1,106 |
| 62 | 132 | 978 |
| 63 | 119 | 862 |
| 64 | 106 | 757 |
| 65 | 95 | 661 |
| 66 | 85 | 575 |

In what range is the spouse's monthly benefit if the participant dies first?

(A) Less than \$164
(B) \$164 but less than \$165
(C) \$165 but less than \$166
(D) \$166 but less than \$167
(E) \$167 or more

**Problem 8 - 18**

A master trust contains the assets for Plans A, B, and C. Total investment income each year is allocated to each plan in proportion to expected interest, using simple interest.

Trust values and transactions for 1989:

| | Plan A | Plan B | Plan C |
|---|---|---|---|
| Value of assets as of 1/1/89 | $100,000 | $60,000 | $0 |
| Contributions for 1989: | | | |
| 3/31/89 | 20,000 | 40,000 | 100,000 |
| 6/30/89 | 20,000 | 0 | 0 |
| 9/30/89 | 20,000 | 40,000 | 0 |
| 12/31/89 | 20,000 | 0 | 200,000 |
| Benefit payments for 1989: | | | |
| 6/30/89 | 10,000 | 15,000 | 0 |
| 12/31/89 | 10,000 | 0 | 5,000 |

Total investment income for 1989: $100,000

In what range is the value of assets for Plan C as of 1/1/90?

(A) Less than $320,750
(B) $320,750 but less than $333,500
(C) $333,500 but less than $346,250
(D) $346,250 but less than $359,000
(E) $359,000 or more

**Problem 8 - 19**

Data for all retirees as of 1/1/90:

| | Smith | Brown |
|---|---|---|
| Age | 60 | 70 |
| Annual benefit | $54,000 | $24,000 |
| Status as of 12/31/90 | Alive | Deceased |

Benefits are payable as of 1/1 of each year.
All deaths within a year are assumed to occur at the end of the year.

Selected probabilities of death and annuity values:

| Age x | $q_x$ | $\ddot{a}_x$ |
|---|---|---|
| 60 | .020 | 9.52 |
| 61 | .022 | 9.30 |
| 70 | .040 | 7.28 |
| 71 | .044 | 7.00 |

In what range is the mortality experience gain for 1990 due to the two retirees?

(A) Less than $140,000
(B) $140,000 but less than $145,000
(C) $145,000 but less than $150,000
(D) $150,000 but less than $155,000
(E) $155,000 or more

**Problem 8 - 20**

Normal retirement benefit: $2,500 per year, payable as of 1/1 of each year.
Normal form of payment: Annuity payable for the life of the retiree.

For married participants, the plan provides for the following two optional forms of payment which are actuarially equivalent to the normal form of payment.

Option A: Annuity payable for the life of the retiree. After the retiree's death, the surviving spouse will receive an annual payment each 1/1 equal to 50% of the amount payable to the retiree.

Option B: Annuity payable during the joint life of the retiree and the retiree's spouse. After either's death, the survivor will receive an annual payment each 1/1 equal to 50% of the amount payable during their joint lifetime.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/26 |
| Date of retirement | 1/1/91 |
| Spouse's date of birth | 1/1/29 |

Present value of future benefits as of 12/31/90: \$26,000
Initial annual benefit under Option B: \$2,376

Selected annuity value:

$\ddot{a}_{65:62} = 9.42$

In what range is the initial annual benefit under option A?

(A) Less than \$2,000
(B) \$2,000 but less than \$2,100
(C) \$2,100 but less than \$2,200
(D) \$2,200 but less than \$2,300
(E) \$2,300 or more

**Problem 8 - 21**

Assumed interest rate: 7% per year.
Age of sole retiree as of 1/1/90: 70

Retirement benefit: \$10,000 per year, payable as of 1/1 of each year for the life of the retiree.

Status of retiree as of 1/1/91: Alive.

Selected annuity value and life expectancies:

$\ddot{a}_{70} = 7.326 \qquad e_{70} = 13.80 \qquad e_{71} = 13.25$

In what range is the mortality experience loss for 1990 due to the retiree?

(A) Less than \$2,000
(B) \$2,000 but less than \$2,300
(C) \$2,300 but less than \$2,600
(D) \$2,600 but less than \$2,900
(E) \$2,900 or more

**Problem 8 - 22**

| | |
|---|---|
| Preretirement death benefit: | A lump sum payment of \$5,000 for each year of service, payable at the end of the year of death. A year of service is granted for the year of death regardless of when a participant dies in such year. |

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Date for sole participant:

| | |
|---|---|
| Date of birth | 1/1/31 |
| Date of hire | 1/1/71 |
| Status as of 1/1/91 | Active |

Selected commutation functions:

| Age x | $D_x$ |
|---|---|
| 60 | 14,863 |
| 61 | 13,694 |
| 62 | 12,600 |
| 63 | 11,575 |
| 64 | 10,616 |
| 65 | 9,718 |
| 66 | 7,926 |

In what range is the present value of the future preretirement death benefits as of 1/1/91?

(A) Less than \$7,600
(B) \$7,600 but less than \$7,700
(C) \$7,700 but less than \$7,800
(D) \$7,800 but less than \$7,900
(E) \$7,900 or more

**Problem 8 - 23**

Normal retirement benefit: $15 per month for each year of service up to 30 years.
Early retirement eligibility: Age 55.

Early retirement benefit:

| | |
|---|---|
| Less than 30 years of service: | Accrued benefit, reduced by 6% for each year by which the benefit commencement date precedes the normal retirement date. |
| 30 or more years of service: | Accrued benefit, unreduced for early commencement of payments. |

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement:

Less than 30 years service: 10% at beginning of each year.
30 or more years service: 40% at beginning of each year.
Age 65: 100%

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/29 |
| Date of hire | 1/1/63 |
| Status as of 1/1/91 | Active |

Selected commutation functions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 62 | 3,704 | 34,796 |
| 63 | 3,403 | 31,230 |
| 64 | 3,121 | 27,956 |
| 65 | 2,857 | 24,956 |

In what range is the present value of future benefits as of 1/1/91?

(A) Less than $39,100
(B) $39,100 but less than $40,100
(C) $40,100 but less than $41,100
(D) $41,100 but less than $42,100
(E) $42,100 or more

**Problem 8 - 24**

Normal retirement benefit: $20 per month for each year of service.
Early retirement eligibility: Age 60.

Early retirement benefit: Accrued benefit, reduced by 1/15 for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement:

| | |
|---|---|
| Age 62: | 25% |
| Age 63: | 50% |
| Age 64: | 75% |
| Age 65: | 100% |

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/30 |
| Date of hire | 1/1/80 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.18$ $\quad \ddot{a}_{63}^{(12)} = 8.96$

$\ddot{a}_{64}^{(12)} = 8.74$ $\quad \ddot{a}_{65}^{(12)} = 8.51$

In what range is the present value of future benefits as of 1/1/92?

(A) Less than $22,000
(B) $22,000 but less than $23,000
(C) $23,000 but less than $24,000
(D) $24,000 but less than $25,000
(E) $25,000 or more

**Problem 8 - 25**

Annual annuity: $10,000 payable each 1/1 for the life of retiree. Upon the death of the retiree, $5,000 is payable each 1/1 to the surviving spouse.

Assumed interest rate: 7% per year.

On 1/1/92, there are 100 retirees, all age 70. All are married with spouses age 67.

Mortality experience in 1992:

5 retirees die in 1992, and 3 of their spouses survive to 1/1/93.
95 retirees survive to 1/1/93, and 93 of their spouses survive to 1/1/93.

Selected annuity values:

$\ddot{a}_{67} = 8.74$ $\quad\quad$ $\ddot{a}_{70} = 8.06$ $\quad\quad$ $\ddot{a}_{67:70} = 6.51$

$\ddot{a}_{68} = 8.52$ $\quad\quad$ $\ddot{a}_{71} = 7.83$

In what range is the experience gain or loss from mortality for 1992 as of 12/31/92?

(A) Loss of $400,000 or more
(B) Loss of $200,000 but less than $400,000
(C) $0 or loss of less than $200,000
(D) Gain of more than $0 but less than $200,000
(E) Gain of $200,000 or more

**Problem 8 - 26**

Normal retirement benefit: $1,000 per month.
Normal form of payment: Life annuity with 5 years certain.

Actuarially equivalent optional forms of payment:

Option A: The initial monthly benefit is payable for 5 years certain and the remaining lifetime of the participant, with 50% of the initial monthly benefit payable for the surviving beneficiary's lifetime following the participant's death. However, there is no reduction in the benefit paid to the surviving beneficiary until the end of the 5-year certain period.

Option B: Same as Option A, except the percentage continuing to the surviving beneficiary is 75%.

Initial monthly benefit under Option A: $840

In what range is the initial monthly benefit under Option B?

(A) Less than $750
(B) $750 but less than $760
(C) $760 but less than $770
(D) $770 but less than $780
(E) $780 or more

**Problem 8 - 27**

Normal retirement benefit: $15 per month for each year of service.

Preretirement death benefit: Lump sum equal to 100 times the monthly accrued benefit, payable at the time of death.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65
Deaths are assumed to occur at the beginning of each year.

Data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/30 |
| Date of hire | 1/1/60 |

Selected annuity value and probabilities of mortality:

$\ddot{a}_{65}^{(12)} = 8.70 \qquad q_{62} = .017 \qquad q_{63} = .019 \qquad q_{64} = .021$

In what range is the present value of future preretirement death benefits as of 1/1/92?

(A) Less than $2,500
(B) $2,500 but less than $2,600
(C) $2,600 but less than $2,700
(D) $2,700 but less than $2,800
(E) $2,800 or more

**Problem 8 - 28**

Normal retirement benefit: $10,000 per year, payable on 1/1.

Form of payment: Upon the death of the retiree, a reduced payment of $P will be payable on 1/1 of each subsequent year to the surviving spouse.

Assumed interest rate: 7% per year.

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/23 |
| Date of retirement | 1/1/88 |
| Spouse's date of birth | 1/1/26 |

If Smith survives to 1/1/94, but Smith's spouse dies in 1993, the experience gain for 1993 due to mortality will be $11,300.

If Smith dies in 1993, but Smith's spouse survives to 1/1/94, the experience gain for 1993 due to mortality will be $K.

Selected annuity values:

$\ddot{a}_{67} = 8.287 \quad \ddot{a}_{68} = 8.061 \quad \ddot{a}_{70} = 7.603$

$\ddot{a}_{71} = 7.368 \quad \ddot{a}_{67:70} = 6.056$

In what range is $K?

(A) Less than $23,000
(B) $23,000 but less than $28,000
(C) $28,000 but less than $33,000
(D) $33,000 but less than $38,000
(E) $38,000 or more

**Problem 8 - 29**

Type of plan:

Before 1993: Contributory.
After 1992: Noncontributory.

Preretirement death or termination benefits: Employee contributions with interest accumulated at 6% per year are returned at the end of the year of death or termination.

Assumed interest rate: 6% per year.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/32 |
| Accumulated employee contributions with interest as of 12/31/92 | $15,000 |

Retirements are assumed to occur at the beginning of the year.

Selected commutation functions:

| Age x | $l_x^{(T)}$ | $q_x^{(r)}$ |
|---|---|---|
| 61 | 1,000 | 0.0 |
| 62 | 950 | 0.5 |
| 63 | 465 | 0.2 |
| 64 | 360 | 0.2 |
| 65 | 278 | 1.0 |

In what range is the present value of future preretirement death and termination benefits as of 1/1/93?

(A) Less than \$1,200
(B) \$1,200 but less than \$1,250
(C) \$1,250 but less than \$1,300
(D) \$1,300 but less than \$1,350
(E) \$1,350 or more

**Problem 8 - 30**

Consider the following actuarially equivalent benefit options available to a 58-year old retiring employee:

I. A monthly benefit of \$4,000 for life.

II. A monthly benefit of \$3,720 for life. Upon death of the retiree, the surviving spouse will receive a monthly benefit of \$1,860 for life.

III. A monthly benefit for life of \$4,000 to age 62 and \$3,500 thereafter. Upon death of the retiree at any time, the surviving spouse will receive a monthly benefit of \$K for life.

Selected factor:

$N_{62}^{(12)}/N_{58}^{(12)} = .6867$

In what range is \$K?

(A) Less than \$2,000
(B) \$2,000 but less than \$2,400
(C) \$2,400 but less than \$2,800
(D) \$2,800 but less than \$3,200
(E) \$3,200 or more

**Problem 8 - 31**

Retirement benefit: \$20,000 payable each 1/1 during the lifetime of the participant, with \$10,000 payable each 1/1 after the participant's death to the surviving spouse.

Date of birth for sole participant Smith (retired): 1/1/23

Date of birth of Smith's spouse: 1/1/28

Smith's spouse dies during 1993.

Selected annuity values and probabilities of survival:

$\ddot{a}_{65} = 10.00$ $\ddot{a}_{66} = 9.80$ $p_{65} = .9735$

$\ddot{a}_{70} = 9.00$ $\ddot{a}_{65:70} = 8.00$ $p_{70} = .9636$

$\ddot{a}_{71} = 8.80$

In what range is the net mortality gain during 1993 as of 1/1/94?

(A) Less than \$14,000
(B) \$14,000 but less than \$15,000
(C) \$15,000 but less than \$16,000
(D) \$16,000 but less than \$17,000
(E) \$17,000 or more

**Problem 8 - 32**

Normal retirement benefit: \$10 per month for each year of service.

Early retirement eligibility: Age 60.

| | |
|---|---|
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

| Age x | |
|---|---|
| 60 | 20% |
| 61 | 20% |
| 62 | 50% |
| 63 | 0% |
| 64 | 0% |
| 65 | 100% |

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/84 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 9.815$ $\quad$ $\ddot{a}_{62}^{(12)} = 9.394$ $\quad$ $\ddot{a}_{64}^{(12)} = 8.958$

$\ddot{a}_{61}^{(12)} = 9.607$ $\quad$ $\ddot{a}_{63}^{(12)} = 9.178$ $\quad$ $\ddot{a}_{65}^{(12)} = 8.736$

In what range is the present value of future benefits as of 1/1/94?

(A) Less than $9,000
(B) $9,000 but less than $10,000
(C) $10,000 but less than $11,000
(D) $11,000 but less than $12,000
(E) $12,000 or more

**Problem 8 - 33**

| | |
|---|---|
| Normal retirement benefit: | 2% of final year's compensation for each year of service. |
| Early retirement eligibility: | Age 55. |
| Early retirement benefit: | Accrued benefit, reduced by 5% for each year by which the benefit commencement date precedes the normal retirement date. |

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Probabilities of retirement (assumed to occur at beginning of year):

| | |
|---|---|
| At age 62 | 25% |
| At age 63 | 50% |
| At age 64 | 75% |
| At age 65 | 100% |

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/33 |
| Date of hire | 1/1/75 |
| 1994 compensation | $50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.18$ $\qquad$ $\ddot{a}_{64}^{(12)} = 8.74$

$\ddot{a}_{63}^{(12)} = 8.96$ $\qquad$ $\ddot{a}_{65}^{(12)} = 8.51$

In what range is the present value of future benefits as of 1/1/95?

(A) Less than $155,000
(B) $155,000 but less than $165,000
(C) $165,000 but less than $175,000
(D) $175,000 but less than $185,000
(E) $185,000 or more

**Problem 8 - 34**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service. |
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |

All optional forms of payment are actuarially equivalent to the normal form of payment.

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/36 |
| Date of hire | 1/1/72 |
| Date of retirement | 12/31/95 |
| Date of benefit commencement | 1/1/96 |

Smith originally elected an optional form of payment under which, for each $100 of monthly early retirement single life annuity benefit, he will receive a monthly benefit of $80 for his lifetime with one-half of this amount continuing after his death to his surviving spouse for her remaining lifetime.

Under an early retirement program, Smith's retirement benefits are enhanced by determining his normal and early retirement benefits with his service increased by one year, and his early retirement benefits reduced now by 3 % for each year by which commencement of payments precedes age 63.

Due to the early retirement program, Smith revises his election to an optional form of payment under which he will receive a monthly benefit of $80 for his lifetime with X% of this amount continuing after his death to his surviving spouse for her remaining lifetime.

In what range is the value of X%?

(A) Less than 75%
(B) 75% but less than 81%
(C) 81% but less than 87%
(D) 87% but less than 93%
(E) 93% or more

**Problem 8 - 35**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.

Postponed retirement benefit: Greater of (a) the normal retirement benefit determined as of the date of postponed retirement based on compensation and service up to the date of postponed retirement, or (b) the actuarial equivalent of the normal retirement benefit determined at age 65.

Normal form of payment: Life annuity.

Optional form of payment for married participants: Joint and 100% survivor annuity which is actuarially equivalent to the normal form of payment.

Preretirement death benefit: None.

Data for participant Smith (active as of 12/31/94):

| | |
|---|---|
| Date of birth | 1/1/29 |
| Date of hire | 1/1/64 |
| Date of retirement | 1/1/95 |
| Spouse's date of birth | 1/1/29 |

Annual compensation:

| | |
|---|---|
| 1991 | $30,000 |
| 1992 | 42,000 |
| 1993 | 45,000 |
| 1994 | 32,000 |

Selected commutation functions and annuity values:

| Age x | $D_x$ | $N^{(12)}$ |
|---|---|---|
| 65 | 94,414 | 824,779 |
| 66 | 86,246 | 734,109 |

$\ddot{a}^{(12)}_{65:65} = 6.5$ $\ddot{a}^{(12)}_{66:66} = 6.3$

In what range is Smith's annual benefit under the joint and 100% survivor annuity option?

(A) Less than \$20,000
(B) \$20,000 but less than \$21,000
(C) \$21,000 but less than \$22,000
(D) \$22,000 but less than \$23,000
(E) \$23,000 or more

**Problem 8 - 36**

Age of retiree and spouse at date of retirement: 60.

Actuarially-equivalent annuity options available to retiree and spouse:

Option A: Monthly benefit of (\$1,000 + \$X) for first 5 years of lifetime of retiree, and monthly benefit of \$X for remaining lifetime of retiree.

Option B: Monthly benefit of \$1,000 for joint lifetime of retiree and spouse, and monthly benefit of (\$1,000 - \$X) for remaining lifetime of survivor after the first death.

Selected commutation functions and annuity values:

$D_{60} = 144{,}405$ $\qquad \ddot{a}_{60}^{(12)} = 9.815$ $\qquad \ddot{a}_{60:60}^{(12)} = 8.094$

$D_{65} = 94{,}414$ $\qquad \ddot{a}_{65}^{(12)} = 8.736$

In what range is \$X?

(A) Less than \$500
(B) \$500 but less than \$600
(C) \$600 but less than \$700
(D) \$700 but less than \$800
(E) \$800 or more

**Problem 8 - 37**

Assumed interest rate: 7% per year.

Valuation data for all retired participants as of 1/1/95:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/35 | 1/1/30 | 1/1/25 |
| Monthly benefit (life annuity) | \$4,000 | \$5,000 | \$6,000 |

Brown died on 12/31/95. There were no other deaths or new retired participants during 1995.

Selected annuity values:

| $x$ | $\ddot{a}_x^{(12)}$ |
|---|---|
| 60 | 9.81 |
| 61 | 9.60 |
| 65 | 8.74 |
| 66 | 8.51 |
| 70 | 7.60 |
| 71 | 7.37 |

In what range is the experience gain during 1995 due to mortality for retired participants?

(A) Less than \$470,000
(B) \$470,000 but less than \$475,000
(C) \$475,000 but less than \$480,000
(D) \$480,000 but less than \$485,000
(E) \$485,000 or more

**Problem 8 - 38**

Assumed interest rate: 7% per year.

Data as of 1/1/96 for a retiree:

| | |
|---|---|
| Age of retiree: | x. |
| Age of spouse: | y. |
| Annual benefit: | \$10,000 payable each 1/1. |

| | |
|---|---|
| Form of payment: | Life annuity for the retiree, with 50% continuing for the life of the spouse if the retiree dies first. |

Selected annuity values:

$\ddot{a}_x = 8.157$ $\quad\quad$ $\ddot{a}_y = 10.301$ $\quad\quad$ $\ddot{a}_{xy} = 7.281$

$\ddot{a}_{x+1} = 7.915$ $\quad\quad$ $\ddot{a}_{y+1} = 10.059$

In what range is the experience loss during 1996 due to mortality if both the retiree and the spouse are still alive as of 12/31/96?

(A) Less than $1,350
(B) $1,350 but less than $1,425
(C) $1,425 but less than $1,500
(D) $1,500 but less than $1,575
(E) $1,575 or more

**Problem 8 - 39**

| | |
|---|---|
| Normal retirement benefit: | $1,000 per month. |
| Normal form of payment: | Life annuity. |
| Optional form of payment: | Joint and 100% survivor benefit equal to 88% of life annuity benefit. |
| Assumed form of payment: | Life annuity. |

Data for Smith:

| | |
|---|---|
| Date of birth | 1/1/32 |
| Spouse's date of birth | 1/1/37 |
| Date of retirement | 1/1/97 |

Selected annuity values:

| $x$ | $\ddot{a}_x^{(12)}$ |
|---|---|
| 60 | 9.8150 |
| 65 | 8.7358 |

| $x$ | $y$ | $\ddot{a}_x^{(12)} + \ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}$ |
|---|---|---|
| 65 | 60 | 11.1165 |
| 65 | 65 | 10.5755 |

In what range is the experience loss as of 1/1/97 due to Smith's election of the optional form of payment?

- (A) Less than \$12,700
- (B) \$12,700 but less than \$12,900
- (C) \$12,900 but less than \$13,100
- (D) \$13,100 but less than \$13,300
- (E) \$13,300 or more

**Problem 8 - 40**

Normal retirement benefit: \$1,500 per month.

Normal form of payment: Life annuity.

Actuarially-equivalent optional form of payment available to married participants:

> Annuity certain of \$X per month for the first 120 months, plus, after 120 months, an annuity of \$X per month for life of participant and \$X/2 per month for life of spouse if participant dies first.

Actuarial assumptions:

> Interest rate: 7% per year.
> Preretirement terminations other than deaths: None.
> Retirement age: 65.

Date of birth of Smith: 1/1/32

Date of birth of Smith's spouse: 1/1/32

Selected commutation functions and annuity values on the valuation basis:

| $x$ | $D_x$ | $N_x$ |
|---|---|---|
| 65 | 965 | 8,872 |
| 75 | 346 | 2,379 |

$\ddot{a}^{(12)}_{\overline{10|}} = 7.29$ $\quad\ddot{a}^{(12)}_{65} = 8.74$ $\quad\ddot{a}^{(12)}_{65:65} = 6.90$ $\quad\ddot{a}^{(12)}_{75} = 6.42$ $\quad\ddot{a}^{(12)}_{75:75} = 4.60$

Smith retires on 1/1/97 and elects the optional form of payment for married participants.

In what range is \$X for Smith?

(A) Less than \$1,225
(B) \$1,225 but less than \$1,250
(C) \$1,250 but less than \$1,275
(D) \$1,275 but less than \$1,300
(E) \$1,300 or more

**Problem 8 - 41**

Retirement benefit:

\$500 per month, reduced by 1/15 for each of the first 5 years and by 1/30 for each of the next 5 years by which benefits commence before age 65.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Preretirement decrements: | None. |
| Retirement age: | 65. |

Effective 1/1/97, two participants aged 56 and 63 retire and elect to commence receiving benefits immediately.

Selected annuity values:

| $x$ | $\ddot{a}_x^{(12)}$ |
|---|---|
| 56 | 10.5993 |
| 63 | 9.1775 |
| 65 | 8.7358 |

In what range is the increase in the present value of benefits as of 1/1/97 due to the retirements?

- (A) Less than \$6,000
- (B) \$6,000 but less than \$7,000
- (C) \$7,000 but less than \$8,000
- (D) \$8,000 but less than \$9,000
- (E) \$9,000 or more

**Problem 8 - 42**

Normal retirement benefit: \$500 per month.

Normal form of payment: Life annuity.

Optional form of payment:

- (A) Lump sum of \$60,000.
- (B) \$400 per month for life of spouse, and \$500 per month for life of retiree if spouse dies first.
- (C) \$K/12 per month for life of retiree, and \$K/24 per month for life of spouse if retiree dies first.

Each optional form of payment is actuarially equivalent to the normal form of payment.

Actuarial equivalence is based on unisex mortality.

The retiree and the spouse are the same age.

In what range is \$K?

(A) Less than \$4,200
(B) \$4,200 but less than \$4,600
(C) \$4,600 but less than \$5,000
(D) \$5,000 but less than \$5,400
(E) \$5,400 or more

## 8.3 Solutions to Problems

### Problem 8-1

Key Concept: The mortality loss (it must be a loss because both individuals survived) is the difference between the actual present value of future benefits and the expected present value of future benefits.

The following formulae are used:

$$PVFB_0 = (Ben)[\ddot{a}_x + 1/2(\ddot{a}_y - \ddot{a}_{xy})] \text{ and}$$
$$ePVFB_1 = (PVFB_0 + NC)(1 + i) - (Payments) - I$$

Step I: Determine the expected Accrued Liability at the end of the first year.

$$\begin{aligned} PVFB_0 &= (Ben)[\ddot{a}_x + (1/2)(\ddot{a}_y - \ddot{a}_{xy})] \\ &= 10{,}000[8.157 + (1/2)(10.301 - 7.281)] \\ &= 10{,}000(8.157 + 1.510) = 96{,}670 \end{aligned}$$

$$\begin{aligned} ePVFB_1 &= (PVFB_0 + NC)(1 + i) - Payments - I \\ &= (96{,}670 + 0)(1.07) - 10{,}000 - 700 \\ &= 103{,}437 - 10{,}700 = 92{,}737 \end{aligned}$$

Step II: Determine the actual Present Value of Future Benefits at the end of the first year. However, since we are not given a value for $\ddot{a}_{x+1:y+1}$, it must be calculated as shown below.

In general:

$$vp_x \times \ddot{a}_{x+1} = \ddot{a}_x - 1$$

or $$p_x = (\ddot{a}_x - 1)(1 + i) \div \ddot{a}_{x+1}$$

hence $$\begin{aligned} p_x &= (\ddot{a}_x - 1)(1.07) \div \ddot{a}_{x+1} \\ &= (8.157 - 1)(1.07) \div 7.915 = .9675 \end{aligned}$$

and $$p_y = (10.301 - 1)(1.07) \div 10.059 = .9894$$

also $$\begin{aligned} \ddot{a}_{x+1:y+1} &= (\ddot{a}_{xy} - 1)(1.07) \div (p_x p_y) \\ &= (7.281 - 1)(1.07) \div (.9675)(.9894) = 7.0208 \end{aligned}$$

The Present Value of Future Benefits may now be calculated.

$$\begin{aligned} PVFB &= (\text{Benefit})[\ddot{a}_{x+1} + 1/2(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})] \\ &= 10{,}000[7.915 + 1/2(10.059 - 7.0208)] \\ &= 10{,}000(7.915 + 1.5191) = 94{,}341 \end{aligned}$$

$$\begin{aligned} \text{Loss} &= PVFB - ePVFB \\ &= 94{,}341 - 92{,}737 = 1{,}604 \end{aligned}$$

Answer is E.

**Problem 8 - 2**

Key Concept: The key to the solution of a problem involving the "level-income option" lies in the recognition that Actuarial Equivalence means that Present Value of the Early Retirement Benefit is equal to (i) Present value of the "level-income" payment less (ii) the Present Value of the Social Security Benefit.

In this problem:

$$(ERB_{58})(\ddot{a}_{58}^{(12)}) = (LIB_{58})(\ddot{a}_{58}^{(12)}) - (\text{Soc. Sec. Benefit})(\ddot{a}_{62}^{(12)})(D_{62} / D_{58})$$

where ERB = Early Retirement Benefit at age 58
LIB = Level Income Benefit equal to the total pension and Social Security.

Step I: Calculate the Early Retirement Benefit.

| | |
|---|---|
| Accrued Benefit | \$450 |
| Reduction for years 1 through 5 | 25% |
| Reduction for years 6 and 7 | 6% |
| Total reduction | 31% |
| Early Retirement Benefit | 450 - (31%)(450) = 310.50 |

Step II: Determine the Level income payment.

$$(ERB_{58})(\ddot{a}_{58}^{(12)}) = (LIB_{58})(\ddot{a}_{58}^{(12)}) - (\text{Social Security Benefit})(\ddot{a}_{62}^{(12)})(D_{62}/D_{58})$$

and $$(LIB_{58})(\ddot{a}_{58}^{(12)}) = (ERB_{58})(\ddot{a}_{58}^{(12)}) + (\text{Social Security Benefit})(\ddot{a}_{62}^{(12)})(D_{62}/D_{58})$$

$$(LIB_{58}) = ERB_{58} + (\text{Social Security Benefit})[(\ddot{a}_{62}^{(12)} D_{62}) \div (\ddot{a}_{58}^{(12)} D_{58})]$$

$$= 310.50 + 300[(\ddot{a}_{62}^{(12)} D_{62}) \div (\ddot{a}_{58}^{(12)} D_{58})]$$

$$= 310.50 + 300(N_{62}^{(12)} / N_{58}^{(12)})$$

Key Concept: The problem becomes one of determining the factors to be applied to the Social Security Benefit. If the values for $N_{58}^{(12)}$ and $N_{62}^{(12)}$ were given, this would be a simple matter. Since they are not given, we must adopt a different approach. The problem states that "actuarial equivalence" is based on the plan's early retirement factors. With this information, we can compute the missing factors, $N_{58}^{(12)}$ and $N_{62}^{(12)}$.

If $\ddot{a}_{65}^{(12)} = 100\%$, then $N_{65}^{(12)} / N_{62}^{(12)} = 100 - 3(.05) = 85\%$

and $N_{65}^{(12)} / N_{58}^{(12)} = 100 - 25 - 6 = 69\%$

$$\text{Factor} = N_{62}^{(12)} / N_{58}^{(12)} = 69 \div 85$$

Our equation can now be stated:

$$LIB_{58} = 310.50 + 300(69/85)$$
$$= 310.50 + 243.53 = 554.03$$

Answer is C.

**Problem 8 - 3**

Key Concepts: (1) The present value of the return of employee contributions is based upon the probability of a "returning event" times the amount to be returned, all discounted to its present value.

(2) The value for all decrements other than death is equal to the value for all decrements, minus the value for occurrence of death.

(3) Note that the refund is made at the end of the year of death or termination.

Step I: Determine the Present Value of the return for all causes.

(a) Probability of decrement $= (l_{40}^{(T)} - l_{41}^{(T)}) / l_{40}^{(T)}$
$= (842 - 830) \div 842$
$= 12 \div 842$

(b) Amount to be returned $= 1{,}000 \times 1.05 = 1{,}050$
(Interest credited to date of payment)

(c) Present value $= (1{,}050)(12 \div 842)(1.07)^{-1} = 13.99$

Step II: Determine Present Value of return from death only.

(a) Probability of death $= (l'^{(m)}_{40} - l'^{(m)}_{41}) / l'^{(m)}_{40}$
$= (1{,}091 - 1{,}088) \div 1{,}091 = 3 \div 1{,}091$

(b) Present Value $= (1{,}050)(3 \div 1{,}091)(1.07)^{-1} = 2.70$

Step III: Calculate the difference.

Present Value of return of Employee contributions plus interest for decrements other than death:

PV(Refunds) $= 13.99 - 2.70 = 11.29$

Answer is C.

**Problem 8 - 4**

Key Concepts:

(1) The dollar-weighted rate of return for a fund, if one assumes all principal cash flows occur uniformly during the year (i.e. on the average at mid-year), can be written as:

$$i = 2I \div (A_0 + A_1 - I)$$

For this case, the only principal cash flow (the contribution) was made at mid-year.

(2) This problem calls for the rate of return on the actuarial value of assets which is the average of book and market values.

Step I: Determine the actuarial value.

At the beginning of the year:

$$A_0 = (1{,}000{,}000 + 800{,}000) \div 2 = 900{,}000$$

At the end of the year:

$$A_1 = (1{,}150{,}000 + 1{,}050{,}000) \div 2 = 1{,}100{,}000$$

Step II: Determine investment earnings on actuarial value.

$$\begin{aligned} I &= A_1 - A_0 - \text{Contribution} + \text{Benefits} \\ &= 1{,}100{,}000 - 900{,}000 - 100{,}000 + 0 = 100{,}000 \end{aligned}$$

Step III: Determine the rate of return.

$$\begin{aligned} i &= 2I \div (A_0 + A_1 - I) \\ &= 200{,}000 \div (900{,}000 + 1{,}100{,}000 - 100{,}000) \\ &= 200{,}000 \div 1{,}900{,}000 = .1053 = 10.53\% \end{aligned}$$

Answer is B.

**Problem 8 - 4 (Alternative solution)**

The more general method of determining a dollar-weighted rate of return is to divide the investment return by the time-weighted principal balance and cash flows.

In this case, A and I are determined as above. The time-weighted principal (which can be thought of as an "average balance" during the year) is calculated.

| | |
|---|---|
| $A_0 \times 1$ year = $900{,}000 \times 1$ | 900,000 |
| $C \times 1/2$ year = $100{,}000 \times ½$ | <u>+50,000</u> |
| Total = Average principal balance | 950,000 |

$$i = \text{I} \div \text{average principal balance} = 100{,}000 \div 950{,}000 = 10.53\%$$

**Problem 8 - 5**

Key Concept: As in all problems involving experience gains or losses, the comparison of actual Accrued Liability to the expected Accrued Liability must be made. (Refer to problem 8-1 which is similar.)

Step I: Compute Accrued Liability as of 1/1/86 and 1/1/87.

$$\begin{aligned} AL_{86} &= (\text{Benefit})[\ddot{a}_{70} + (\ddot{a}_{70} - \ddot{a}_{70:70})] \\ &= (6{,}000)[8.80 + (8.80 - 6.11)] = 68{,}940 \end{aligned}$$

$$AL_{87} = (6{,}000) \times [8.46 + (8.46 - 5.83)] = 66{,}540$$

Step II: Determine the expected Accrued Liability as of 1/1/87.

$$eAL_{87} = (AL_{86} + NC_{86})(1 + i) - (BP - I_{BP})$$

where BP represents Benefit Payments and $I_{BP}$ represents interest on benefit payments.

All of the terms are known except the assumed interest return which we can calculate from the formula:

$$vp_x\ddot{a}_{x+1} = \ddot{a}_x - 1$$

$$v(1 - .05)(8.46) = 8.80 - 1$$

$$\begin{aligned} v &= (7.80) \div [(.95)(8.46)] \\ &= 7.80 \div 8.037 \end{aligned}$$

and $(1 + i) = 8.037 \div 7.80 = 1.03038$

$$
\begin{aligned}
eAL_{87} &= (68{,}940 + 0)(1.03038) - (6{,}000 + 182) \\
&= 71{,}034 - 6{,}182 = 64{,}852
\end{aligned}
$$

Step III: Determine mortality loss.

$$
\begin{aligned}
\text{Loss} &= AL_{87} - eAL_{87} \\
&= 66{,}540 - 64{,}852 = 1{,}688
\end{aligned}
$$

Answer is B.

**Problem 8 - 5 (Alternative Solution)**

Key Concept: The Expected Accrued Liability is calculated as the probability-weightedsum of the Accrued Liabilities which would result under each of the possible combinations of survival or death of the participant and spouse. These probability combinations are:

(A) Probability of both participant and spouse living;

(B) Probability of participant living and spouse dying;

(C) Probability of participant dying and spouse living:

(D) Probability of both participant and spouse dying.

Each combination may be calculated as follows:

(A) $(\ddot{a}_{\overline{71:71}}) = \ddot{a}_{71} + \ddot{a}_{71} - \ddot{a}_{71:71} = 8.46 + 8.46 - 5.83 = 11.09$

$(p_1)(p_2)(\ddot{a}_{\overline{71:71}}) = (.95)(.95)(11.09) = 10.0087$

(B) $(p_1)(q_2)(\ddot{a}_{71}) = (.95)(.05)(8.46) = 0.4019$

(C) $(q_1)(p_2)(\ddot{a}_{71}) = (.05)(.95)(8.46) = 0.4019$

(D) $(q_1)(q_2)(0) = 0.0$

Total probability of all events A through D = 10.8125.
Therefore, the Accrued Liability = 6000 × 11.09 = 66,540
The expected Accrued Liability = 10.8125 × 6,000 = 64,875
Actuarial Loss = 66,540 - 64,875 = 1,665

## Problem 8 - 6

Key Concept: Expressions can be written for present value of benefit under the normal form as well as the optional form. Since actuarial equivalence is required, the expressions can be equated and reduced in form.

Using the principles of actuarial equivalence:

$$P_1(55) \times N_{65}^{(12)}/D_{55} = P_2(55) \times \left[\ddot{a}_{\overline{10|}}^{(12)} + N_{65}^{(12)}/D_{55}\right]$$

$$\text{Therefore, } P_2(55))/P_1(55) = N_{65}^{(12)}/D_{55} \div \left[\ddot{a}_{\overline{10|}}^{(12)} + N_{65}^{(12)}/D_{55}\right]$$

$$= (N_{65}^{(12)}/D_{55}) \div \left[\frac{\ddot{a}_{\overline{10|}}^{(12)} D_{55} + N_{65}^{(12)}}{D_{55}}\right]$$

$$= \frac{N_{65}^{(12)}}{\ddot{a}_{\overline{10|}}^{(12)} D_{55} + N_{65}^{(12)}}$$

Answer is E.

## Problem 8 - 7

Key Concept: The only variable affected by the plan amendment is $\ddot{a}_{65}$. By adopting the postretirement adjustment of 3%, the adjusted $\ddot{a}_{65}$ is equal to $\ddot{a}_{65}$ calculated using an interest return of 1.07 / 1.03.

Step I: Calculate the Present Value of Future Benefits prior to the plan amendment.

$$\text{PVFB} = (\text{Ben})(\ddot{a}_{65})(D_{65} \div D_{55})$$

$$= (1{,}000)(8.74)(96 \div 219) = 3{,}831$$

Step II: Calculate the Present Value of Future Benefits after the plan amendment.

For interest only at 7%:

$$\ddot{a}_{65} = \sum_{n=0}^{\infty} (1.07)^{-n}$$

With interest at 7% and annual adjustment of 3%:

$$\ddot{a}_{65} = \sum_{n=0}^{\infty} (1.07)^{-n} (1.03)^{n} = \sum_{n=0}^{\infty} (1.0388)^{-n}$$

By interpolation, we can calculate a value for $\ddot{a}_{65:3.88\%}$:

$$\ddot{a}_{65:3\%} = 11.70$$

$$\ddot{a}_{65:3.88\%} = ?$$

$$\ddot{a}_{4\%} = 10.82$$

From which $\ddot{a}_{65:3.88\%} = 10.92$

Step III: Present Value of Future Benefits after amendment can now be calculated.

$$\text{PVFB} = (1{,}000)(10.92)(96 \div 219) = 4{,}787$$

and the difference is 4,787 - 3,831 = 956.

Answer is B.

**Problem 8 - 8**

Step I: Determine the amount of interest earned during 1987.

$$I = (\text{Value at } 1/1/88) - \text{Contribution} + \text{Distributions} - (\text{Value at } 1/1/87)$$

$$= 63{,}651 - 22{,}700 + 15{,}300 - 50{,}000 = 6{,}251$$

Step II: Let $i$ equal the rate of return. Set up the equation of principal multiplied by interest prorated over each quarter.

$$6{,}251 = 50{,}000i - (5{,}200i)(3/4) + (20{,}000i)(1/4)$$
$$6{,}251 = 50{,}000i - 3{,}900i + 5{,}000i$$
$$6{,}251 = 51{,}100i$$
$$i = 12.23\%$$

Answer is D.

**Problem 8 - 9**

Key Concept: This problem is really an exercise in manipulation of actuarial symbols. The Present Value of Accrued Benefit at age 42 must be calculated, then discounted to age 40; then, follow the same procedure for age 43, 44, and so on.

Consider age 42 only:

$$\text{Accrued Benefit} = (15 \times 12 \text{ months} \times 7 \text{ years}) = 180 \times 7$$

$$\text{PV Accrued Benefit at age } 42 = (180 \times 7) \times (N_{65}^{(12)} \ / \ D_{42})$$

$$\text{Vested Accrued Ben at age } 42 = .25(180 \times 7) \times N_{65}^{(12)} \ / \ D_{42})$$

$$= (180N_{65}^{(12)}/D_{42})(.25)(7)(1)$$

This expression can be discounted to age 40 as follows:

$$\text{PVAB}_{42} \text{ (age 40)} = q_{42}^{w} \ (l_{42} \div l_{40})(v^2)(180N_{65}^{(12)} /D_{42})(.25)(7)(1)$$

$$=(180N_{65}^{(12)} \ / \ l_{40})(.25)(q_{42}^{w})(7)(1)(v^2)(l_{42}/D_{42})$$

It's beginning to look a lot like answer E.

In a similar way, we can discount the Present Value of Accrued Benefit at age 43 back to age 40:

$$\text{PVAB}_{43} \text{ (age 40)} = (180N_{65}^{(12)} \ / \ l_{40})(.25)(q_{43}^{w})(8)(2)(v^3)(l_{43}/D_{43})$$

$$\text{and} \quad \text{PVAB}_{44} \text{ (age 40)} = (180N_{65}^{(12)} \ / \ l_{40})(.25)(q_{44}^{w})(9)(3)(v^4)(l_{44}/D_{44})$$

By summing all three and using summation symbols, the expression becomes:

$$\text{PV Vested Accrued Benefit} = (180N_{65}^{(12)} \ / \ l_{40})[.25 \sum_{t=42}^{44} q_t^{w}(t\text{-}35)(t\text{-}41)(v^{t-40})(l_t/D_t)]$$

This is the first term of E.

It is a relatively easy problem to show that the second term of E is the Present Value at age 40 for the vested termination benefits for those years when the sole participant is fully vested.

Answer is E.

## Problem 8 - 9 (Alternative solution)

Not many students in a test situation would work out a detailed proof as presented above. When faced with a problem of this kind, it would be wiser to seek methods to eliminate answers that are patently false. Some, though by no means all, of the possibilities to accomplish this are as follows:

$(180N_{65}^{(12)}/D_{40})$ implies that benefits are valued at age 40 via mortality and interest only. Hence, formulas A, B, C are probably incorrect.

It should be apparent after some thought that the first term of each answer pertains to those years of partial vesting and the second term to those years of full vesting. The second term of A and B is discounting the full vesting years on mortality only. Hence, A and B can be eliminated on this count.

It should also be apparent that (t - 40) and/or (t - 41) pertain to the years of partial vesting. Since vesting does not begin until age 42, those answers involving (t - 40) can be eliminated. These are A, C and D.

Finally, since we have eliminated all answers except E, review that answer and verify that all components match the problem's conditions.

## Problem 8 - 10

Key Concept: Actuarial equivalence for early retirement may be expressed as $(N_{nr}/N_{er})$ where er represents early retirement age and nr represents normal age. This can be demonstrated as follows:

$$\begin{aligned}\text{PV of Accrued Benefit at age er} &= AB(N_{nr}/D_{nr})(D_{nr}/D_{er})\\ &= AB(N_{nr}/D_{er})\end{aligned}$$

If actuarially equivalent:

$$\begin{aligned}\text{Ben} &= AB(N_{nr}/D_{er}) \,/\, (N_{er}/D_{er})\\ &= AB\,(N_{nr}/N_{er})\end{aligned}$$

Assume a Normal Retirement Age of 65 and early Retirement Age of 64. We can do this without any loss of generality since, if our argument holds for age 64, it will hold for age 63, 62 and so on.

$$\text{Actuarial equivalence} = N_{65}/N_{64}$$

$$N_{65}/N_{64} = N_{65}/(N_{65} + D_{64})$$

Divide numerator and denominator by $N_{65}$

$$= 1 \div [1 + D_{64} / N_{65}]$$
$$= 1 \div [1 + 1 / a_{65}]$$

Now consider statement I. If mortality is set back three years, $a_{65}$ will increase and the entire expression will increase. Therefore Statement I is true.

Consider Statement II. If the interest assumption is increased, $a_{65}$ will decrease and the entire expression will decrease. Therefore statement II is false.

Consider Statement III. For the purpose of our argument, the only difference between a life only benefit and a ten year certain and life optional form is the addition of the factor $\ddot{a}_{\overline{10}|}$. Therefore, if statement I is true, statement III is true.

Answer is B.

**Problem 8 - 10 (Alternative solution):**

The problem is more easily dealt with intuitively rather than by mathematical proof as done in the previous solution.

The change in mortality assumption of setting ages back three years operates to increase the number of lifetime payments commencing at any age. In other words, an age 65 retiree will expect fewer payments under 1971 GAM than under 1971 GAM set back three years. Statement I is true.

The change in interest rate from 6% to 8% operates to make the Early Retirement payments relatively more valuable. At 6%, the payment (ignoring mortality) ten years after retirement is 34% more valuable than the payment 15 years after retirement. At 8%, the ten year payment is worth 47% more than the 15 year payment. Statement II is false.

Following the logic to show that Statement I is true, a decrease in mortality rates tends to (relatively speaking) stretch the payment stream over a longer period of time. An increase in interest rates tends to compress the payment stream. Statement III is true.

Answer is B.

**Problem 8 - 11**

Key Concept: As in most problems involving gains or losses, it is necessary to compare the actual accrued liability with the expected accrued liability.

In this problem, the expected accrued liability is developed based upon the sum of the probabilities that:

1. Both employee and spouse live for one year;
2. Employee lives and spouse dies during year;
3: Employee dies during year and spouse lives;
4. Both die during year.

The actual accrued liability is the sum of accrued liabilities based upon (1) both living, (2) employee only lives, and (3) spouse only lives.

Step I: Expected Accrued Liability based upon condition that both live.

$$\begin{aligned} e\text{AL} &= (p_{62})(p_{65})[\ddot{a}_{66} + (1/2)(\ddot{a}_{63} - \ddot{a}_{63:66})](1{,}000) \\ &= (.985)(.980)[8.409 + (1/2)(9.024 - 7.205)](1{,}000) = 8{,}995.15 \end{aligned}$$

Step II: Expected Accrued Liability based upon condition that employee lives and spouse dies.

$$\begin{aligned} e\text{AL} &= (p_{65})(q_{62})(\ddot{a}_{66})(1{,}000) \\ &= (.980)(.015)(8.409)(1{,}000) = 123.61 \end{aligned}$$

Step III: Expected Accrued Liability based upon condition that employee dies and spouse lives.

$$\begin{aligned} e\text{AL} &= (q_{65})(p_{62})(\ddot{a}_{63})(500) \\ &= (.020)(.985)(9.024))(500) = 88.89 \end{aligned}$$

$$e\text{AL} = 8{,}995.15 + 123.61 + 88.89 = 9{,}207.65$$
(for one employee/spouse)

$$e\text{AL} = 9{,}207.65 \text{ x } 100 \text{ (for 100 employees and spouses)} = 920{,}765$$

Step IV: Actual Accrued Liability for 95 couples who both live.

$$\begin{aligned} \text{AL} &= (95 \text{ couples})[\ddot{a}_{66} + (1/2)(\ddot{a}_{63} - \ddot{a}_{63:66})](1{,}000) \\ &= (95)[8.409 + (1/2)(9.024 - 7.205)](1{,}000) = 885{,}258 \end{aligned}$$

Step V: Actual Accrued Liability for 2 employees who survive spouses.

$$AL = (2 \text{ retirees})(\ddot{a}_{66})(1{,}000)$$
$$= (2)(8.409)(1{,}000) = 16{,}818$$

Step VI: Actual Accrued Liability for three spouses who survive retirees.

$$AL = (3 \text{ spouses})(\ddot{a}_{63})(500)$$
$$= (3)(9.024)(500) = 13{,}536$$

Step VII: Calculate total Accrued Liability and gain/loss.

$$\text{Total Accrued Liability} = 885{,}258 + 16{,}818 + 13{,}536 = 915{,}612$$
$$\text{Gain} = 920{,}765 - 915{,}612 = 5{,}153$$

Answer is E.

**Problem 8 - 12**

Key Concepts: (1) Accrued Liability under a 50% Joint and Survivor Annuity is determined by the formula:

$$AL = (\text{Benefit})[\ddot{a}_x + (50\%)(\ddot{a}_y - \ddot{a}_{x:y})]$$

(2) The expected Accrued Liability in the following year is determined by applying a factor (explained below) made up of interest and probability of living.

Step I: Determine Accrued Liability at 1/1/87. From the above formula:

$$AL_{87} = (12{,}000)[(\ddot{a}_{65}) + (50\%)(\ddot{a}_{60} - \ddot{a}_{60:65})]$$
$$= (12{,}000)[(8.95) + (50\%)(10.15 - 8.05)]$$
$$= (12{,}000)(10.00) = 120{,}000$$

Step II: Determine expected Accrued Liability at 1/1/88. The expected Accrued Liability at 1/1/88 is calculated by applying a commutation factor of interest and probabilities of the occurrence of all possible events.

Case 1: Assume both live $= 6 \times p_{60} \times p_{65}$
$= 6 \text{ x } .99 \text{ x } .98 = 5.8212$

Case 2: Assume Smith lives, spouse dies

$$= 6 \times p_{65} \times (1 - p_{60})$$
$$= 6 \times .98 \times (1 - .99) = .0588$$

Case 3: Assume Smith dies and spouse lives. Since spouse receives only half the benefit, the factor is only one half:

$$= (.5)[6 \times p_{60} \times (1 - p_{65})]$$
$$= (.5)[6 \times .99 \times (1 - .98)] = .0594$$

Factor to be used is $5.8212 + .0588 + .0594 = 5.9394$

$$eAL_{88} = (AL_{87} - \text{payments})(1 + .059394)$$
$$= (120{,}000 - 12{,}000)(1.059394) = 114{,}415$$

Step III: Determine actual Accrued Liability at 1/1/88.

$$AL_{88} = (\text{payment})(\ddot{a}_{66})$$
$$= (12{,}000)(8.60) = 103{,}200$$

Step IV: Determine Mortality gain.

$$\text{Gain} = eAL - AL$$
$$= 114{,}415 - 103{,}200 = 11{,}215$$

Answer is B.

**Problem 8 - 12 (Alternative solution)**

Step II: Prospective method. If both Smith and his spouse live, the accrued liability on 1/1/88 would equal:

$$12{,}000\ddot{a}_{66} + 6{,}000\ddot{a}_{61} - 6{,}000\ddot{a}_{61:66}$$

To determine the expected Accrued Liability, simply multiply each annuity due by the probability that the individuals live from 1/1/87 to 1/1/88. Therefore, the expected accrued liability is:

$$eAL = (12{,}000)(\ddot{a}_{66})(p_{65}) + (6{,}000)(\ddot{a}_{61})(p_{60}) - (6{,}000)(\ddot{a}_{61:66})(p_{60}p_{65})$$
$$= (12{,}000)(8.60)(.98) + (6{,}000)(9.80)(.99) - (6{,}000)(7.70)(.99)(.98)$$
$$= 101{,}136 + 58{,}212 - 44{,}823 = 114{,}525$$

Mortality gain = 114,525 - 103,200 = 11,325

Answer is still in range of B. Difference due to rounding.

**Problem 8 - 13**

Key Concept: The participant will reach retirement age in 5 years. In determining the present value of the retirement benefit, the life annuity portion of the benefit is only payable if the participant survives to retirement, and the survivor portion of the benefit is only payable if both the participant and the spouse survive to retirement.

$PVFB_{89}$ = Present value of life annuity + present value of survivor annuity.

$$= [{}_5p_{60}v^5 \times 12 \times 600\ddot{a}_{65}^{(12)}] + [{}_5p_{60:65}v^5 \times 12 \times 300(\ddot{a}_{70}^{(12)} - \ddot{a}_{65:70}^{(12)})]$$

$$= [{}_5p_{60}v^5 \times 12\ddot{a}_{65}^{(12)} \times 600] + [{}_{10}p_{60}v^5 \times 300(12\ddot{a}_{70}^{(12)} - 12\ddot{a}_{65:70}^{(12)})]$$

$$= 46{,}176 + 3{,}228 = 49{,}404$$

Note that ${}_5p_{60:65} = {}_5p_{60} \times {}_5p_{65} = {}_{10}p_{60}$

and ${}_5p_{60} = {}_{10}p_{60} \div {}_5p_{65} = .8/.87 = .91954$

Answer is A.

**Problem 8 - 14**

Key Concept: The total Present Value of Future Benefits is equal to the sum of the Present Value of Future Benefits at each possible retirement age multiplied by the probability of retirement at that age.

Step I: Calculate the Present Value of Future Benefits if retirement occurs at age 63.

Accrued Benefit at age 63 = 15 × 16 Years of Service × (1 - (.06)(2)) = 211.20

$$PVFB_{63} = (211.2)(12)(\ddot{a}_{63}^{(12)}) = 24{,}837$$

Step II: Calculate PVFB if retirement occurs at age 64.

Accrued Benefit at age 64 = 15 × 17 Years of Service × (1 - .06) = 239.70

$$PVFB_{64} = (239.70)(12)(\ddot{a}_{64}^{(12)})(v) = 26{,}050$$

Step III: Calculate PVFB if retirement occurs at age 65.

Accrued Benefit at 65 = 15 × 18 Years of Service = 270.00

$$PVFB_{65} = (270)(12)(\ddot{a}_{65}^{(12)})(v^2) = 26{,}962$$

Step IV: Calculate the probability of retirement at each age.

$$q_{63}^{(r)} = .5$$

$$q_{64}^{(r)} = (1 - q_{63}^{(r)})(q_{64}^{(r)}) = (.5)(.2) = .1$$

$$q_{65}^{(r)} = (1 - q_{63}^{(r)})(1 - q_{64}^{(r)})(q_{65}^{(r)}) = (.5)(.8)(1) = .4$$

Step V: Calculate the total PVFB.

$$PVFB_{Total} = (24{,}837)(.5) + (26{,}050)(.1) + (26{,}962)(.4) = 25{,}808$$

Answer is C.

**Problem 8 - 15**

Since death occurs on the first day of the year, we must look at benefits if death occurs on 1/1/89 or 1/1/90.

| Valuation Year | Att. Age | Accrued Benefit | Early Ret. Reduction | Reduced Acc. Ben. | Death Benefit |
|---|---|---|---|---|---|
| 1989 | 63 | 600 | .90 | 540 | 270.00 |
| 1990 | 64 | 620 | .95 | 589 | 294.50 |

Present Value of Death Benefit if death occurs 1/1/89:

$$= (270)(12\ddot{a}_{63}^{(12)})(q_{63}^{(d)})(.9)$$

$$= (270)(12)(9.85)(.02)(.9) = 574$$

Note that the present value is multiplied by .9 since only 90% of the participants are assumed to be married. Unmarried participants receive no death benefit.

Present Value of Death Benefit if death occurs 1/1/90:

$$= 294.50 \times 12\ddot{a}_{64}^{(12)} \times (v)(p_{63}^{(d)}) \times q_{64}^{(d)} \times (.9)$$

$$= (294.50)(12)(9.6)(\frac{1}{1.06})(.98)(.02)(.9) = 565$$

Total PV = 574 + 565 = 1,139

Answer is B.

**Problem 8 - 16**

Option A can be described as an life annuity of \$P plus a temporary life annuity of \$500 payable from age 60 to age 62:

$$(P)(12\ddot{a}_{60}^{(12)}) + (12)(500)\left[\ddot{a}_{60}^{(12)} - \ddot{a}_{62}^{(12)}\frac{D_{62}}{D_{60}}\right]$$

$$= (12P)(9.815) + (6{,}000)\left[9.815 - (9.394) \times \frac{125.296}{147.804}\right]$$

$$= 117.78P + 11{,}109$$

Option B can be described as a life annuity of \$500 payable as long as both Smith and his spouse are both alive plus a reduced pension payable to Smith if his spouse dies, and a reduced pension payable to his spouse if Smith dies:

$$(500)(12\ddot{a}^{(12)}_{60:60}) + (2)(500 - .5P)(12\ddot{a}^{(12)}_{60} - 12\ddot{a}^{(12)}_{60:60})$$

$$= (6{,}000)(8.094) + (24)(500 - .5P)(9.815 - 8.094)$$

$$= 69{,}216 - 20.652P$$

Since the two options are actuarially equivalent,

117.78P + 11,109 = 69,216 - 20.652P

P = 420

Answer is A.

**Problem 8 - 17**

Since the early retirement benefit at age 55 is actuarially equivalent to the benefit at age 65, we can determine $B_{55}$ as follows:

$$B_{55} \times \ddot{a}^{(12)}_{55} = 1{,}000 \times \ddot{a}^{(12)}_{65} \frac{D_{65}}{D_{55}}$$

$$B_{55} = 1{,}000 \frac{N^{(12)}_{65}}{N^{(12)}_{55}}$$

We are given $N_{65}$ but we need $N_{65}^{(12)}$? The approximation for $N_{65}^{(12)}$ derived from $N_{65}$ is

$$N_x^{(12)} = N_x - \frac{11}{24} D_x \quad and \quad D_x = N_x - N_{x+1}$$

$$N_{65}^{(12)} = 95 - \frac{11}{24}(10) = 90.417 \; and \; N_{55}^{(12)} = 263 - \frac{11}{24}(23) = 252.458$$

$$B_{55} = 1{,}000 \times (\frac{90.417}{252.458}) = 358.15$$

(Note that $D_{65} = N_{65} - N_{66}$ and $D_{55} = N_{55} - N_{56}$.)

The joint and survivor benefit ($B_{J\&S}$) can be determined as follows:

$$B_{55} \times \ddot{a}_{55}^{(12)} = B_{J+S} \times \left[ \ddot{a}_{55}^{(12)} + \frac{1}{2}\left(\ddot{a}_{52}^{(12)} - \ddot{a}_{55:52}^{(12)}\right)\right]$$

Developing the Commutation Functions:

$$D_{55} = N_{55} - N_{56} = 263 - 240 = 23$$

$$D_{52} = N_{52} - N_{53} = 345 - 316 = 29$$

$$D_{55:52} = N_{55:52} - N_{56:53} = 2{,}168 - 1{,}951 = 217$$

$$N_{52}^{(12)} = N_{52} - \frac{11}{24} D_{52} = 345 - \frac{11}{24}(29) = 331.708$$

$$N_{55:52}^{(12)} = N_{55:52} - \frac{11}{24} D_{55:52} = 2{,}168 - \frac{11}{24}(217) = 2{,}068.54$$

$$B_{J\&S} = \frac{358.15 \times (252.458/23)}{(252.458/23) + (.5)[(331.708/29) - (2{,}068.54/217)]}$$

$$= \frac{358.15 \times 10.976}{10.976 + (.5)(1.906)} = 329.54$$

Since one-half is payable to the spouse, the spousal benefit is

$$\text{Survivor Benefit} = 329.54/2 = 164.77$$

Answer is B.

**Problem 8 - 18**

Key Concept: This trust uses a Dollar Weighted Rate of Return. The relative weight of interest for each plan must be determined.

We must write an expression describing the expected interest for each plan. Note that transactions occurring on 12/31/89 do not receive interest credit.

*Plan A:*

$$100{,}000 \times i + 20{,}000 \times [\frac{3}{4}i + \frac{1}{2}i + \frac{1}{4}i] - 10{,}000\left(\frac{i}{2}\right) = 125{,}000i$$

*Plan B:*

$$60{,}000 \times i + 40{,}000\left(\frac{i}{4} + \frac{3}{4}i\right) - 15{,}000\left(\frac{i}{2}\right) = 92{,}500i$$

*Plan C:*

$$100{,}000\left(\frac{3}{4}i\right) = 75{,}000i$$

In total,

$$100{,}000 = 125{,}000i + 92{,}500i + 75{,}000i = 100{,}000$$
$$i = .34188$$

$$\text{Income for fund C} = 75{,}000i$$
$$= (75{,}000)(.34188) = 25{,}641$$
$$\text{Total value of fund C as of 1/1/90} = 100{,}000 + 200{,}000 - 5{,}000 + 25{,}641 = 320{,}641$$

Answer is A.

**Problem 8 - 19**

Key Concept: The experience gain is equal to the difference between the expected Accrued Liability and the actual Accrued Liability.

Step I: Calculate the expected liability.

Note that the expected liability as of 12/31/90 is the Present Value of Future Benefits as of 12/31/90 (assuming the participant has survived) times the probability the participant survived in 1990.

$$\begin{aligned} e\text{AL}_{\text{Smith}} &= 54{,}000\ \ddot{a}_{61} \times p_{60} \\ &= 54{,}000 \times 9.30 \times (.98) = 492{,}156 \end{aligned}$$

$$\begin{aligned} e\text{AL}_{\text{Brown}} &= 24{,}000 \times \ddot{a}_{71} \times p_{70} \\ &= 24{,}000 \times 7.00 \times (.96) = 161{,}280 \end{aligned}$$

$$e\text{AL}_{\text{Total}} = 492{,}156 + 161{,}280 = 653{,}436$$

Step II: Calculate the actual liability.

$$\begin{aligned} \text{AL}_{\text{Smith}} &= 54{,}000 \times \ddot{a}_{61} \\ &= 54{,}000 \times 9.30 = 502{,}200 \end{aligned}$$

$\text{AL}_{\text{Brown}} = 0$, since Brown is deceased.

$$\text{AL}_{\text{Total}} = 502{,}200$$

Step III: Calculate Gain.

$$\begin{aligned} \text{Gain} &= e\text{AL}_{\text{Total}} - \text{AL}_{\text{Total}} \\ &= 653{,}436 - 502{,}200 = 151{,}236 \end{aligned}$$

Answer is D.

**Problem 8 - 20**

The formulas for each of the benefit options are as follows:

Option A: Joint and 50% Survivor.

$$2500 \times \ddot{a}_x = A\left(\ddot{a}_x + \frac{1}{2}\left(\ddot{a}_y - \ddot{a}_{xy}\right)\right)$$

$$A = \frac{2500\ddot{a}_x}{\ddot{a}_x + \frac{1}{2}\left(\ddot{a}_y - \ddot{a}_{xy}\right)}$$

Option B: Joint and Survivor on a <u>joint</u> payment basis.

Note that this annuity is represented by $\ddot{a}_{xy} + \frac{1}{2}(\ddot{a}_x - \ddot{a}_{xy}) + \frac{1}{2}(\ddot{a}_y - \ddot{a}_{xy}) = \frac{1}{2}(\ddot{a}_x + \ddot{a}_y)$

$$2500 \times \ddot{a}_x = B\left(\frac{1}{2}\left(\ddot{a}_x + \ddot{a}_y\right)\right)$$

$$B = \frac{2500\ddot{a}_x}{\frac{1}{2}\left(\ddot{a}_x + \ddot{a}_y\right)}$$

The Normal form of benefit is Life only.

$$2{,}500\ \ddot{a}_x = 26{,}000 \text{ (Retiree only)}$$

$$\ddot{a}_x = 10.4$$

Now solve for $\ddot{a}_y$ using the formula for Option B.

$$2{,}376 = \frac{2{,}500(10.4)}{\frac{1}{2}(10.4 + \ddot{a}_y)}$$

$$\ddot{a}_y = 11.4855$$

Since x = 65 and y = 62 from the data, $\ddot{a}_{xy} = \ddot{a}_{65:62} = 9.42$

$$A = \frac{2500(10.4)}{\quad} = 2{,}274$$

$10.4+½(11.4855-9.42)$

Answer is D.

**Problem 8 - 21**

Key Concept: The experience loss is equal to the difference between the actual Accrued Liability and the expected Accrued Liability.

Actual Accrued Liability = $10{,}000 \times \ddot{a}_{71}$

Expected Accrued Liability = $10{,}000 \times \ddot{a}_{71} \times p_{70}$

We must now solve for $p_{70}$ and $\ddot{a}_{71}$.

$$e_x = \Sigma\, {}_t p_x$$

$$e_{70} = p_{70} + {}_2p_{70} + {}_3p_{70} + \ldots$$

$$= p_{70} + p_{70}\left(p_{71} + {}_2p_{71} + \ldots\right)$$

$$= p_{70} + p_{70}\left(e_{71}\right)$$

$$13.8 = p_{70} + p_{70}\ (13.25)$$

$$p_{70} = .9684$$

Also,

$$\ddot{a}_x = 1 + \left(v \times p_x\ \ddot{a}_{x+1}\right)$$

$$\ddot{a}_{70} = 1 + \left(\frac{.9684}{1.07}\right)\ \ddot{a}_{71}$$

$$\ddot{a}_{71} = 6.9897$$

Mortality Gain/Loss = Actual Accrued Liability less Expected Accrued Liability

$= 10{,}000 \times \ddot{a}_{71} - 10{,}000 \times \ddot{a}_{71} \times p_{70}$

$$= (10{,}000)(6.9897) - (10{,}000)(6.9897)(.9684) = 2{,}208$$

Answer is B.

**Problem 8 - 22**

The sole participant will have 21 Years of Service at the end of 1991.

The Death Benefit in 1991 is 21 Years of Service × 5,000 = 105,000.

For years 1992-1995, add 5,000 of additional lump sum payment for each year.

Recall, from Life Contingencies,

$$C_x = v^{x+1} \times d_x = vD_x - D_{x+1}$$

$$C_{60} = vD_{60} - D_{61} = 196.65$$

$$C_{61} = vD_{61} - D_{62} = 198.13$$

$$C_{62} = vD_{62} - D_{63} = 200.70$$

$$C_{63} = vD_{63} - D_{64} = 201.76$$

$$C_{64} = vD_{64} - D_{65} = 203.50$$

Present Value of Future Death Benefits as of 1/1/91:

$$= (105{,}000\ C_{60} + 110{,}000\ C_{61} + 115{,}000\ C_{62} + 120{,}000\ C_{63} + 125{,}000\ C_{64}) \div D_{60}$$

$$= \frac{115{,}171{,}750}{14{,}863} = 7{,}749$$

Answer is C.

**Problem 8 - 23**

Key Concept: PVFB is equal to the sum of the present value of benefits if retirement occurs in each year between now and Normal Retirement Age times the probability of retirement at that age.

Since the participant is active as of 1/1/91, the first possible date of retirement is 1/1/92.

The following summarizes the benefits at each possible retirement age:

| Age | Benefit Years of Service | Accrued Benefit | Reduced Acc. Ben. | $q^{(r)}$ |
|---|---|---|---|---|
| 63 | 29 | 435 | 382.80 | .10 |
| 64 | 30 | 450 | 450.00 | .40 |
| 65 | 30 | 450 | 450.00 | 1.00 |

$$PVFB_{91} = \sum v^t \times {}_t p_x^{(r)} \; q_{x+t}^{(r)} \; B_{x+t} \; 12\ddot{a}_{x+t}$$

$$= 382.80 \times (.10) \times v \times 12\ddot{a}_{63}^{(12)}$$

$$+ 450 \times (1 - .10)(.40) \times v^2 \times 12\ddot{a}_{64}^{(12)}$$

$$+ 450 \times (1 - .10)(1 - .40) \times v^3 \times 12\ddot{a}_{65}^{(12)}$$

$$= \frac{459.36}{1.07} \times \left(\frac{31230}{3403}\right) + \frac{1{,}944}{(1.07)^2} \times \left(\frac{27956}{3121}\right) + \frac{2{,}916}{(1.07)^3} \times \left(\frac{24956}{2857}\right)$$

$$= 3{,}940 + 15{,}209 + 20{,}792 = 39{,}941$$

Answer is B.

**Problem 8 - 24**

Key Concept: PVFB is equal to the sum of the present value of benefits if retirement occurs in each year between now and NRA times the probability of retirement at that age.

| NRA | $q^{(r)}_{NRA}$ | Accrued Benefit | Early Retirement Reduction | Benefit Payable |
|---|---|---|---|---|
| 62 | .25 | 240 | 3/15 | 192 |
| 63 | .50 | 260 | 2/15 | 225 |
| 64 | .75 | 280 | 1/15 | 261 |
| 65 | 1.00 | 300 | 0/15 | 300 |

$$PVFB_{92} = 192 \times 12\ddot{a}^{(12)}_{62} \times (.25) + (1\text{-}.25)(.5)(225 \times 12\ddot{a}^{(12)}_{63}) \times v$$

$$+ (1 - .25)(1 - .5)(.75)(261 \times 12\ddot{a}^{(12)}_{64}) \times v^2$$

$$+ (1 - .25)(1 - .5)(1 - .75)(300 \times 12\ddot{a}^{(12)}_{65}) \times v^3$$

$$= 5{,}288 + 8{,}478 + 6{,}724 + 2{,}345 = 22{,}835$$

Answer is B.

**Problem 8 - 25**

Key Concept: The experience gain or loss is the difference between the expected and actual liabilities.

Step I: Determine the value of $\ddot{a}_{68:71}$

Recall from life contingencies:

$$\ddot{a}_x = 1 + p_x v \ddot{a}_{x+1}$$

and $\ddot{a}_{xy} = 1 + p_{xy} v \ddot{a}_{x+1:y+1}$

so, $\ddot{a}_{67} = 1 + p_{67} v \ddot{a}_{68}$ $\qquad p_{67} = .972042$

$\ddot{a}_{70} = 1 + p_{70} v \ddot{a}_{71}$ $\qquad p_{70} = .964777$

$\ddot{a}_{67:70} = 1 + p_{67:70} v \ddot{a}_{68:71}$ $\qquad \ddot{a}_{68:71} = 6.2867$

Step II: Calculate the Actual Liability.

PVFB of 93 surviving retirees and spouses

$$= (10{,}000)(93)(\ddot{a}_{71}+\tfrac{1}{2}(\ddot{a}_{68}-\ddot{a}_{68:71})) = 8{,}320{,}385$$

PVFB of 2 surviving retirees without spouses

$$= (10{,}000)(2)(\ddot{a}_{71}) = 156{,}600$$

PVFB of 3 surviving spouses of deceased retires

$$= (5{,}000)(3)(\ddot{a}_{68}) = 127{,}800$$

Actual Liability

$$= 8{,}320{,}385 + 156{,}600 + 127{,}800 = 8{,}604{,}785$$

Step III: Calculate the Expected Liability.

Expected Liability = (1/1/92 Actual Liability - 1/1/92 Payments) × 1.07

1/1/92 Actual Liability = $[10{,}000\ddot{a}_{70} + 5{,}000(\ddot{a}_{67} - \ddot{a}_{67:70})] \times 100$ retirees = 9,175,000

1/1/92 Payments = 10,000 × 100 retirees = 1,000,000

Expected Liability = (9,175,000 - 1,000,000) × 1.07 = 8,747,250

Step IV: Calculate the Gain or Loss.

Since the expected liability is more than the actual liability, there is a gain.

$$\text{Gain} = 8{,}747{,}250 - 8{,}604{,}785 = 142{,}465$$

Answer is D.

**Problem 8 - 25 (Alternative Solution)**

We can also calculate the Expected Liability using a prospective method.

Expected Liability per dollar of benefit = $\ddot{a}_{71}p_{70} + \frac{1}{2}(\ddot{a}_{68}p_{67} - \ddot{a}_{68:71}p_{67:70}) = 8.747257$

Expected Liability = (10,000)(100 retirees)(8.747257) = 8,747,257

Gain = 8,747,257 - 8,604,785 = 142,472

**Problem 8 - 26**

Present value at retirement of Normal form:

$$= 1{,}000 \times 12 \times \left( \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} \right)$$

Option A: Life with 5 year certain on 50% survivor basis

Present value at retirement

$$= A \times 12 \times \left| \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} + \frac{1}{2}\left( {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right) \right|$$

Option B: Calculate Present Value at retirement of Life with 5 year certain on 75% survivor basis.

$$= B \times 12 \times \left[ \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} + \frac{3}{4}\left( {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right) \right]$$

Since Option A has the same value as the normal form,

$$1{,}000 \times 12 \times \left( \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} \right)$$

$$= 840 \times 12 \times \left[ \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} + \frac{1}{2}\left( {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right) \right]$$

$$160 \times \left( \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} \right) = 420 \times \left[ {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right]$$

$$2.625 = \frac{\left( \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} \right)}{\left[ {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right]}$$

Since Option B has the same value as the normal form,

$$1{,}000 \times 12 \times \left( \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} \right)$$

$$= B \times 12 \times \left[ \ddot{a}_{\overline{5}|}^{(12)} + \frac{N_{x+5}^{(12)}}{D_x} + \frac{3}{4}\left( {}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{x:y}^{(12)} \right) \right]$$

Dividing both sides of the equation by $12 \times ({}_{5|}\ddot{a}_y^{(12)} - {}_{5|}\ddot{a}_{xy}^{(12)})$ and substituting 2.625 from the above equation, we arrive at

$$1000 \times 2.625 = B \times (2.625 + 3/4)$$
$$B = 778$$

Answer is D.

**Problem 8 - 27**

The Present Value of Death Benefits (PVDB) are equal to

$$= \sum v^t \times \ {}_t p_x^{(T)} \times q_{x+t}^{(d)} \times AB_{x+t}$$

The following table represents the Death Benefits:

| Age x | Accrued Benefit | Lump Sum Benefit | $q_x^{(d)}$ |
|---|---|---|---|
| 62 | 480 | 48,000 | .017 |
| 63 | 495 | 49,500 | .019 |
| 64 | 510 | 51,000 | .021 |

$$\begin{aligned} PVDB_{92} &= 48{,}000 \times .017 + (1 - .017)(.019) \times v \times 49{,}500 \\ &\quad + (1 - .017)(1 - .019)(.021) \times v^2 \times 51{,}000 \\ &= 816 + 864 + 902 = 2{,}582 \end{aligned}$$

Answer is B.

**Problem 8 - 28**

Key Concept: Mortality gains or losses are computed by comparing the actual liability with the expected liability.

$$\begin{aligned} \text{Expected Liability} &= (\text{Prior liability - prior payment}) \times (1+i) \\ &= [(10{,}000\, \ddot{a}_{70} - P(\ddot{a}_{67} - \ddot{a}_{67:70})) - 10{,}000] \times (1.07) \\ &= 70{,}652 + 2.38717P \end{aligned}$$

Actual Liability (if spouse only dies) $= 10{,}000\, \ddot{a}_{71} = 73{,}680$

Since there is a gain (expected liability exceeds the actual liability),

$$\begin{aligned} 11{,}300 &= (70{,}652 + 2.38717P) - 73{,}680 \\ P &= 6{,}002 \end{aligned}$$

Therefore,

$$\text{Expected Liability} = 70{,}652 + 2.38717P$$
$$= 70{,}651 + (2.38717)(6{,}002) = 84{,}980$$

Actual Liability (if Smith only dies) = 6,002 $\ddot{a}_{68}$ = 48,382

The gain is 84,980 - 48,382 = 36,598

Answer is D.

**Problem 8 - 29**

Key Concept: The Present value of preretirement death and termination benefits is simply the present value of the benefit to be paid upon death or termination in each year before retirement multiplied by the probability that the participant dies or terminates while still alive.

Since there are no future employee contributions and the preretirement death or termination benefit is equal to the return of employee contributions with 6% interest while the assumed interest rate for calculating present values is 6%, the present value of the death or termination benefit is $15,000 (for each year in which death or termination could occur).

It remains to determine the probability that death or termination occurs each year.

Since no retirements occur at age 61, this probability is:

$$q_{61}^{(T)} = 1 - p_{61}^{(T)} = 1 - \frac{l_{62}^{(T)}}{l_{61}^{(T)}} = 1 - \frac{950}{1{,}000} = .05$$

In order to determine the probability of death or termination at age 62, the participant must first survive to age 62. This probability is $p_{61}^{(T)}$. Since retirements occur on the first day of the year and $q_{62}^{(r)}$=.5, exactly (.5) × (950) = 475 of the $l_{62}^{(T)}$ people retire at age 62. Therefore, since $l_{63}^{(T)}$= 465, there must be 10 deaths or terminations. This means the probability of a life age 62 dying or terminating ($q_{62}^{(-r)}$) is 10/950. To summarize, the probability of death or termination at age 62 for a participant currently age 61 is:

$$(p_{61}^{(T)})(q_{62}^{(-r)}) = (\frac{950}{1{,}000})(\frac{10}{950}) = .01$$

Using similar logic, the probability of death or termination at age 63, for a participant currently age 61 is:

$$({}_2p_{61}^{(T)})(q_{63}^{(-r)}) = (\frac{465}{1{,}000})(\frac{(.8)(465)-360}{465}) = .012$$

and the probability of death or termination at age 64, for a participant currently age 61 is:

$$({}_3p_{61}^{(T)})(q_{64}^{(-r)}) = (\frac{360}{1{,}000})(\frac{(.8)(360)-278}{360}) = .01$$

Since the total probability of death or termination before age 65 is

$$.082 = .05 + .01 + .012 + .01$$

the total liability can now be calculated as

$$\text{Total PV} = (15{,}000)(.082) = 1{,}230$$

Answer is B.

**Problem 8 - 30**

Since actuarial equivalence means equal present values, this problem is merely an exercise in algebra.

The present values can be described a follows:

Option I:

$$(4{,}000)(12\ddot{a}_{58}^{(12)})$$

Option II:

$$(3{,}720)(12\ddot{a}_{58}^{(12)}) + (1{,}860)(12\ddot{a}_{s}^{(12)} - 12\ddot{a}_{58:s}^{(12)})$$

Option III:

$$(4{,}000)(12\ddot{a}_{58}^{(12)}) - (500)(12_{4|}\ddot{a}_{58}^{(12)}) + (K)(12\ddot{a}_{s}^{(12)} - 12\ddot{a}_{58:s}^{(12)})$$

where s is the attained age of the spouse.

Setting options I and II equal to each other,

$$(4{,}000)(12\ddot{a}_{58}^{(12)}) = (3{,}720)(12\ddot{a}_{58}^{(12)}) + (1{,}860)\left(12\ddot{a}_{s}^{(12)} - 12\ddot{a}_{58:s}^{(12)}\right)$$

$$\ddot{a}_{s}^{(12)} - \ddot{a}_{58:s}^{(12)} = .150538\ddot{a}_{58}^{(12)}$$

Setting options I and III equal to each other and substituting the previous result,

$$(4{,}000)(12\ddot{a}_{58}^{(12)}) = (4{,}000)(12\ddot{a}_{58}^{(12)}) - (500)(12_{4|}\ddot{a}_{58}^{(12)}) + (K)(12\ddot{a}_{s}^{(12)} - 12\ddot{a}_{58:s}^{(12)})$$

$$= (4{,}000)(12\ddot{a}_{58}^{(12)}) - (500)(12_{4|}\ddot{a}_{58}^{(12)}) + (.150538)(K)(12\ddot{a}_{58}^{(12)})$$

$$(500)({}_{4|}\ddot{a}_{58}^{(12)}) = (.150538)(K)(\ddot{a}_{58}^{(12)})$$

$$(500)(N_{62}^{(12)}/D_{58}) = (.150538)(K)(N_{58}^{(12)}/D_{58})$$

$$K = (N_{62}^{(12)}/N_{58}^{(12)})(500/.150538)$$

$$= (.6867)(500/.150538) = 2{,}281$$

Answer is B.

**Problem 8 - 31**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since Smith's spouse died during the year, the benefit is now a life annuity of $20,000 payable on 1/1 of each year.

$$\begin{aligned}\text{Actual AL}_{94} &= (20{,}000)(\ddot{a}_{71})\\ &= (20{,}000)(8.8) = 176{,}000\end{aligned}$$

Step II: The expected Accrued Liability on 1/1/94 is equal to the Accrued Liability on 1/1/93 less the expected payment on 1/1/93, increased with interest. Since both Smith and Smith's spouse were alive on 1/1/93, $20,000 must have been paid on that date.

$$\begin{aligned}\text{AL}_{93} &= (20{,}000)(\ddot{a}_{70}) + 10{,}000(\ddot{a}_{65} - \ddot{a}_{65:70})\\ &= (20{,}000)(9) + 10{,}000(10 - 8) = 200{,}000\end{aligned}$$

$$e\text{AL}_{94} = (\text{AL}_{93} - \text{Benefit paid 1/1/93})(1 + i)$$

However, the interest assumption $i$ is not given, and must be calculated. Note that the formula for successive annuities due is

$$\ddot{a}_x = 1 + vp_x\, \ddot{a}_{x+1}$$

$$\begin{aligned}\text{Therefore,}\quad \ddot{a}_{65} &= 1 + vp_{65}\, \ddot{a}_{66}\\ 10 &= 1 + v(.9735)(9.8)\\ i &= .06\end{aligned}$$

$$e\text{AL}_{94} = (200{,}000 - 20{,}000)(1.06) = 190{,}800$$

Step III: Calculate the mortality gain, which is the difference between the actual and expected Accrued Liabilities.

$$\begin{aligned}\text{Gain} &= e\text{AL}_{94} - \text{actual AL}_{94}\\ &= 190{,}800 - 176{,}000 = 14{,}800\end{aligned}$$

Answer is B.

**Alternative Solution:**

The expected Accrued Liability can also be calculated using a prospective method. If both Smith and his spouse live, the accrued liability on 1/1/94 would be:

$$\text{AL (both live until 1/1/94)} = 20{,}000\ddot{a}_{71} + 10{,}000(\ddot{a}_{66} - \ddot{a}_{66:71})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individuals live from 1/1/93 to 1/1/94. Therefore, the expected Accrued Liability is:

$$e\text{AL}_{94} = 20{,}000\ddot{a}_{71}\, p_{70} + 10{,}000(\ddot{a}_{66}\, p_{65} - \ddot{a}_{66:71}\, p_{65}\, p_{70})$$

The value of $\ddot{a}_{71}$ and $\ddot{a}_{66:71}$ can be calculated using the formula for successive annuities due.

$$\begin{aligned} \ddot{a}_{70} &= 1 + vp_{70}\ddot{a}_{71} \\ 9 &= 1 + v(.9636)\ddot{a}_{71} \\ \ddot{a}_{71} &= 8.8 \end{aligned}$$

$$\begin{aligned} \ddot{a}_{65:70} &= 1 + vp_{65}\, p_{70}\, \ddot{a}_{66:71} \\ 8 &= 1 + v(.9735)(.9636)\ddot{a}_{66:71} \\ \ddot{a}_{66:71} &= 7.91 \end{aligned}$$

Therefore,

$$\begin{aligned} e\text{AL} &= (20{,}000)(8.8)(.9636) + (10{,}000)[(9.8)(.9735) - (7.91)(.9735)(.9636)] \\ &= 190{,}796 \end{aligned}$$

The difference between this result and the result from the retrospective method in Step II is due to rounding.

$$\text{Gain} = 190{,}796 - 176{,}000 = 14{,}796$$

Answer is B.

**Problem 8 - 32**

Step I: Calculate the probability of retirement at each age.

| Retirement Age | Probability of Retirement |
|---|---|
| 60 | $q_{60}^{(r)} = .2$ |
| 61 | $(p_{60}^{(r)})(q_{61}^{(r)}) = (.8)(.2) = .16$ |
| 62 | $(p_{60}^{(r)})(p_{61}^{(r)})(q_{62}^{(r)}) = (.8)(.8)(.5) = .32$ |
| 65 | $(p_{60}^{(r)})(p_{61}^{(r)})(p_{62}^{(r)})(q_{65}^{(r)}) = (.8)(.8)(.5)(1) = .32$ |

Step II: Calculate the retirement benefit at each retirement age.

Benefit at age 60: (10)(20 years of service)(1 - (.03)(5 years)) = \$170.00
Benefit at age 61: (10)(21 years of service)(1 - (.03)(4 years)) = \$184.80
Benefit at age 62: (10)(22 years of service)(1 - (.03)(3 years)) = \$200.20
Benefit at age 65: (10)(25 years of service) = \$250.000

Step III: Calculate the Present Value of Future Benefits.

$$\begin{aligned} PVFB_{94} &= (170)(12\ddot{a}_{60}^{(12)})(v^{10})(.2) + (184.80)(12\ddot{a}_{61}^{(12)})(v^{11})(.16) + \\ &\quad (200.20)(12\ddot{a}_{62}^{(12)})(v^{12})(.32) + (250)(12\ddot{a}_{65}^{(12)})(v^{15})(.32) \\ &= 2{,}036 + 1{,}619 + 3{,}207 + 3{,}040 = 9{,}902 \end{aligned}$$

Answer is B.

**Problem 8 - 33**

Key Concept: The total Present Value of Future Benefits is equal to the sum of the Present Value of Future Benefits at each possible retirement age multiplied by the probability of retirement at that age.

Step I: Calculate the Present Value of Future Benefits if retirement occurs at age 62.

Final Compensation = \$50,000

Accrued Benefit at age 62 = (.02)(\$50,000)(20 years of service)(1 - (.05)(3)) = 17,000

$$PVFB_{62} = (17{,}000)(\ddot{a}_{62}^{(12)}) = 156{,}060$$

Step II: Calculate the Present Value of Future Benefits if retirement occurs at age 63.

Final Compensation = \$50,000 × 1.05 = \$52,500

Accrued Benefit at age 63 = (.02)(\$52,500)(21 years of service)(1 - (.05)(2)) = 19,845

$$PVFB_{63} = (19{,}845)(\ddot{a}_{63}^{(12)})(v) = 166{,}179$$

Step III: Calculate the Present Value of Future Benefits if retirement occurs at age 64.

Final Compensation = \$50,000 × $(1.05)^2$ = 55,125

Accrued Benefit at age 64 = (.02)(\$55,125)(22 years of service)(1 - .05) = 23,042

$$PVFB_{64} = (23{,}042)(\ddot{a}_{64}^{(12)})(v^2) = 175{,}899$$

Step IV: Calculate the Present Value of Future Benefits if retirement occurs at age 65.

Final Compensation = \$50,000 × $(1.05)^3$ = 57,881

Accrued Benefit at age 65 = (.02)(\$57,881)(23 years of service) = 26,625

$$PVFB_{65} = (26{,}625)(\ddot{a}_{65}^{(12)})(v^3) = 184{,}956$$

Step V: Calculate the probability of retirement at each age.

$$q_{62}^{(r)} = .25$$
$$q_{63}^{(r)} = (1 - q_{62}^{(r)})(q_{63}^{(r)}) = (.75)(.5) = .375$$
$$q_{64}^{(r)} = (1 - q_{62}^{(r)})(1 - q_{63}^{(r)})(q_{64}^{(r)}) = (.75)(.5)(.75) = .28125$$
$$q_{65}^{(r)} = (1 - q_{62}^{(r)})(1 - q_{63}^{(r)})(1 - q_{64}^{(r)})(q_{65}^{(r)}) = (.75)(.5)(.25)(1) = .09375$$

Step VI: Calculate the total Present Value of Future Benefits.

$$PVFB_{Total} = (156{,}060)(.25) + (166{,}179)(.375) + (175{,}899)(.28125) + (184{,}956)(.09375) = 168{,}143$$

Answer is C.

**Problem 8 - 34**

Under the original option,

$$100\,(12\ddot{a}_{60}^{(12)}) = 80\,(12\ddot{a}_{60}^{(12)}) + 40\,(12\ddot{a}_{y}^{(12)} - 12\ddot{a}_{60:y}^{(12)})$$

where y is the age of the spouse.

$$20\ddot{a}_{60}^{(12)} = 40(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})$$
$$.5 = (\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})/(\ddot{a}_{60}^{(12)})$$

We must determine the amount of benefit Smith is due under the early retirement program.

Amount of benefit under the old program = (.02)(Salary)(24 years of service)(1- (.03)(5))
= .408(Salary)

Amount of benefit under the new program = (.02)(Salary)(25 years of service)(1- (.03)(3))
= .455(Salary)

So, the life annuity benefit under the early retirement program is

$$100 \times (.455/.408) = 111.52$$

And,

$$111.52\,(12\ddot{a}_{60}^{(12)}) = 80(12\ddot{a}_{60}^{(12)}) + 80(X)(12\ddot{a}_{y}^{(12)} - 12\ddot{a}_{60:y}^{(12)})$$
$$31.52\,\ddot{a}_{60}^{(12)} = 80(X)(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})$$
$$31.52 = 80(X)(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})/\ddot{a}_{60}^{(12)} = 40X$$
$$X = .788, \text{ or } 78.8\%$$

Answer is B.

**Problem 8 - 35**

Step I: Calculate the benefit on the postponed retirement date of 1/1/95.

Final 3-year average salary = (42,000 + 45,000 + 32,000)/3 = 39,667

Benefit = (39,667)(.02)(31 years of service) = 24,594

Step II: Calculate the actuarial equivalent of the normal retirement benefit.

Final 3-year average salary on the normal retirement
date of 1/1/94 = (42,000 + 45,000 + 30,000)/3 = 39,000

$$\text{Benefit} = (39{,}000)(.03)(30 \text{ years of service})(N_{65}^{(12)} / N_{66}^{(12)})$$
$$= (23{,}400)(824{,}779/734{,}109) = 26{,}290$$

Step III: Calculate the postponed retirement benefit. This is the greater of the benefits calculated in Steps I and II, which is \$26,290.

Step IV: Calculate the 100% joint and survivor annuity.

$$\ddot{a}_{66}^{(12)} = N_{66}^{(12)} / D_{66} = 734{,}109/86{,}246 = 8.5118$$

$$26{,}290 \times \ddot{a}_{66}^{(12)} = X\,(\ddot{a}_{66}^{(12)} + \ddot{a}_{66}^{(12)} - \ddot{a}_{66:66}^{(12)})$$
$$X = 20{,}868$$

Answer is B.

**Problem 8 - 36**

Option A can be described as a life annuity of \$X, plus a temporary life annuity of \$1,000 payable from age 60 to age 65:

$$(X)(12\ddot{a}_{60}^{(12)}) + (12)(1{,}000)[\ddot{a}_{60}^{(12)} - (\ddot{a}_{65}^{(12)})(D_{65} / D_{60})]$$

$$= (12X)(9.815) + (12{,}000)[9.815 - (8.716)(94{,}414/144{,}405)]$$

$$= 117.78X + 49{,}239$$

Option B can be described as a life annuity of \$1,000 payable as long as both the retiree and the spouse are alive, plus a reduced pension of \$1,000 - \$X payable to the retiree if the spouse dies or to the spouse if the retiree dies:

$$(1{,}000)(12\ddot{a}_{60:60}^{(12)}) + (2)(12)(1{,}000-X)[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}]$$

$$= (12{,}000)(8.094) + (24{,}000 - 24X)[9.815 - 8.094]$$

$$= 138{,}432 - 41.304X$$

Since the two options are actuarially equivalent,

$$117.78X + 49{,}239 = 138{,}432 - 41.304X$$

$$X = 561$$

Answer is B.

**Problem 8 - 37**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since Brown died at the end of the year, only Smith and Green will continue to receive benefits.

$$\begin{aligned}\text{Actual } AL_{96} &= (4{,}000)(12\ddot{a}_{61}^{(12)}) + (6{,}000)(12\ddot{a}_{71}^{(12)}) \\ &= (4{,}000)(12)(9.60) + (6{,}000)(12)(7.37) \\ &= 460{,}800 + 530{,}640 = 991{,}440\end{aligned}$$

Step II: Calculate the expected Accrued Liability. If each retired participant were still alive on 1/1/96, the Accrued Liability would be:

$$\text{AL (all live until 1/1/96)} = (4{,}000)(12\ddot{a}_{61}^{(12)}) + (5{,}000)(12\ddot{a}_{66}^{(12)}) + (6{,}000)(12\ddot{a}_{71}^{(12)})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individual lives from 1/1/95 to 1/1/96. Therefore, the expected Accrued Liability is:

$$\text{eAL96} = (4{,}000)(12\ddot{a}_{61}^{(12)})p_{60} + (5{,}000)(12\ddot{a}_{66}^{(12)})p_{65} + (6{,}000)(12\ddot{a}_{71}^{(12)})p_{70}$$

The values of $p_{60}$, $p_{65}$, and $p_{70}$ can be calculated using the formula for successive annuities due.

$$\begin{aligned}\ddot{a}_{60} &= 1 + v\, p_{60}\, \ddot{a}_{61} \\ 10.26833 &= 1 + (1/1.07)(p_{60})(10.05833) \\ p_{60} &= .98596\end{aligned}$$

$$\begin{aligned}\ddot{a}_{65} &= 1 + v\, p_{65}\, \ddot{a}_{66} \\ 9.19833 &= 1 + (1/1.07)(p_{65})(8.96833) \\ p_{65} &= .97813\end{aligned}$$

$$\ddot{a}_{70} = 1 + v\, p_{70}\, \ddot{a}_{70}$$
$$8.05833 = 1 + (1/1.07)(p_{70})(7.82833)$$
$$p_{70} = .96475$$

Note the use of the formula $\ddot{a}_x^{(12)} = \ddot{a}_x - 11/24$.

Therefore,

$$\begin{aligned} eAL_{96} &= (4{,}000)(12)(9.60)(.98596) + (5{,}000)(12)(8.51)(.97813) \\ &\quad + (6{,}000)(12)(7.37)(.96475) \\ &= 454{,}330 + 499{,}433 + 511{,}935 = 1{,}465{,}698 \end{aligned}$$

Step III: Calculate the mortality gain. This is the difference between the actual and expected Accrued Liabilities.

$$\begin{aligned} \text{Gain} &= eAL_{96} - \text{Actual } AL_{96} \\ &= 1{,}465{,}698 - 991{,}440 \\ &= 474{,}258 \end{aligned}$$

Answer is B.

**Problem 8 - 38**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since both the retiree and the spouse are alive as of 12/31/96, both are entitled to benefits.

$$\text{Actual } AL_{97} = (10{,}000)(\ddot{a}_{x+1}) + (5{,}000)(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$$

The value of $\ddot{a}_{xy}$ can be calculated using the formula for successive annuities due.

$$\ddot{a}_x = 1 + v\, p_x\, \ddot{a}_{x+1}$$
$$8.157 = 1 + (1/1.07)(p_x)(7.915)$$
$$p_x = .96753$$

$$\ddot{a}_y = 1 + v\, p_y\, \ddot{a}_{y+1}$$
$$10.301 = 1 + (1/1.07)(p_y)(10.059)$$
$$p_y = .98937$$

$$\ddot{a}_{xy} = 1 + v\, p_{xy}\, \ddot{a}_{x+1:y+1}$$
$$7.281 = 1 + (1/1.07)(.96753)(.98937)(\ddot{a}_{x+1:y+1})$$
$$\ddot{a}_{x+1:y+1} = 7.021$$

Therefore,

$$\text{Actual } AL_{97} = (10{,}000)(7.915) + (5{,}000)(10.059 - 7.021)$$
$$= 94{,}340$$

Step II: Calculate the expected Accrued Liability. If the retiree and the spouse were both still alive on 1/1/97, the Accrued Liability would be:

$$AL \text{ (both live until 1/1/97)} = (10{,}000)(\ddot{a}_{x+1}) + (5{,}000)(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individual survives from 1/1/96 to 1/1/97. Therefore, the expected Accrued Liability is:

$$eAL_{97} = (10{,}000)(\ddot{a}_{x+1})p_x + (5{,}000)[(\ddot{a}_{y+1})p_y - (\ddot{a}_{x+1:y+1})p_{xy}]$$
$$= (10{,}000)(7.915)(.96753)$$
$$+ (5{,}000)[(10.059)(.98937) - (7.021)(.96753)(.98937)]$$
$$= 92{,}736$$

Step III: Calculate the mortality loss. This is the difference between the actual and expected Accrued Liabilities.

$$\text{Loss} = \text{Actual } AL_{97} - eAL_{97}$$
$$= 94{,}340 - 92{,}736$$
$$= 1{,}604$$

Answer is E.

**Problem 8 - 38 (Alternative Solution)**

The expected Accrued Liability could have been calculated using a retrospective method.

$$eAL_{97} = (\text{Prior liability - prior payment}) \times (1+i)$$
$$= [10{,}000\ddot{a}_x + 5{,}000(\ddot{a}_y - \ddot{a}_{xy}) - 10{,}000](1.07)$$
$$= 92{,}736$$

$$\begin{aligned}\text{Loss} &= \text{Actual } AL_{97} - eAL_{97} \\ &= 94{,}340 - 92{,}736 \\ &= 1{,}604\end{aligned}$$

Answer is E.

**Problem 8 - 39**

Step I: Calculate the Accrued Liability using the normal form of payment.

$$AL_{NORMAL} = (1{,}000)(12\ddot{a}_{65}^{(12)}) = (1{,}000)(12)(8.7358) = 104{,}830$$

Step II: Calculate the Accrued Liability using the optional form of payment.

$$\begin{aligned}AL_{OPTIONAL} &= (.88)(1{,}000)(12)(\ddot{a}_{65}^{(12)} + \ddot{a}_{60}^{(12)} - \ddot{a}_{60:65}^{(12)}) \\ &= (880)(12)(11.1165) \\ &= 117{,}390\end{aligned}$$

Step III: Calculate the experience loss.

$$\text{Loss} = 117{,}390 - 104{,}830 = 12{,}560$$

Answer is A.

**Problem 8 - 40**

The present value of the normal form (life annuity) can be set equal to the present value of the optional form as follows:

$$\begin{aligned}(1500)(12\ddot{a}_{65}^{(12)}) = {} & (X)(12\ddot{a}_{\overline{10|}}^{(12)}) + (X)(12\ddot{a}_{75}^{(12)})(D_{75}/D_{65}) \\ & + (12)(X/2)[(\ddot{a}_{75}^{(12)})(D_{75}/D_{65}) - (\ddot{a}_{75:75}^{(12)})(D_{75}/D_{65})({}_{10}p_{65})]\end{aligned}$$

It is necessary to solve for ${}_{10}p_{65}$.

$$\begin{aligned}D_{75}/D_{65} = v^{10}{}_{10}p_{65} \quad &\Rightarrow \quad 346/965 = v^{10}{}_{10}p_{65} \\ &\Rightarrow \quad {}_{10}p_{65} = .7053\end{aligned}$$

We can now solve for X in the original equation.

$$(1500)(12\ddot{a}_{65}^{(12)}) = (X)(12\ddot{a}_{\overline{10}|}^{(12)}) + (X)(12\ddot{a}_{75}^{(12)})(D_{75}/D_{65})$$
$$+ (12)(X/2)[(\ddot{a}_{75}^{(12)})(D_{75}/D_{65}) - (\ddot{a}_{75:75}^{(12)})(D_{75}/D_{65})({}_{10}p_{65})]$$

$\Rightarrow$ $X = 1{,}290$

Answer is D.

**Problem 8 - 41**

Step I: Calculate the PVFB before the early retirement election.

The participants are currently 9 and 2 years from the assumed retirement age of 65, respectively. The Present Value of Future Benefits is:

$$\text{PVFB (before election)} = (500)(12\ \ddot{a}_{65}^{(12)})(v^9 + v^2)$$
$$= (500)(12)(8.7358)(.5439 + .8734)$$
$$= 74{,}287$$

Step II: Calculate the PVFB after the early retirement election.

There is no discount factor used, but the early retirement reductions must be applied. The reduction for the participant currently age 56 is:

$5/15 + 4/30 = 14/30$ $\Rightarrow$ Multiply benefit by a 16/30 factor.

The reduction for the participant currently age 63 is:

$2/15 + 0/30 = 2/15$ $\Rightarrow$ Multiply benefit by a 13/15 factor.

$$\text{PVFB (after election)} = (500)(16/30)(12\ \ddot{a}_{56}^{(12)}) + (500)(13/15)(12\ \ddot{a}_{63}^{(12)})$$
$$= (500)(16/30)(12)(10.5993) + (500)(13/15)(12)(9.1775)$$
$$= 33{,}918 + 47{,}723$$
$$= 81{,}641$$

Step III: Calculate the increase in the PVFB.

$$\begin{aligned} PVFB_{INCREASE} &= PVFB \text{ (after election)} - PVFB \text{ (before election)} \\ &= 81{,}641 - 74{,}287 \\ &= 7{,}354 \end{aligned}$$

Answer is C.

**Problem 8 - 42**

Step I: Set the present value of the normal form equal to the present value of option B.

Assume the participant and spouse are age *x*.

$$(500)(12\ddot{a}_x^{(12)}) = (400)(12\ddot{a}_x^{(12)}) + (500)(12)(\ddot{a}_x^{(12)} - \ddot{a}_{x:x}^{(12)})$$

$$\Rightarrow \quad (100)(12\ddot{a}_x^{(12)}) = (500)(12)(\ddot{a}_x^{(12)} - \ddot{a}_{x:x}^{(12)})$$

$$\Rightarrow \quad (\ddot{a}_x^{(12)} - \ddot{a}_{x:x}^{(12)})/\ddot{a}_x^{(12)} = .2$$

Step II: Set the present value of the normal form equal to the present value of option C.

$$(500)(12\ddot{a}_x^{(12)}) = (K/12)(12\ddot{a}_x^{(12)}) + (K/24)(12)(\ddot{a}_x^{(12)} - \ddot{a}_{x:x}^{(12)})$$

$$\Rightarrow \quad 500 = K/12 + (K/24)(\ddot{a}_x^{(12)} - \ddot{a}_{x:x}^{(12)})/\ddot{a}_x^{(12)}$$

$$\Rightarrow \quad 500 = K/12 + (K/24)(.2)$$

$$\Rightarrow \quad K = 5{,}455$$

Answer is E.

# 1995 Supplement to

# ACTUARIAL COST METHODS

# A REVIEW

Supplement to

# ACTUARIAL COST METHODS A REVIEW

By
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David Farber, M.S.P.A.

A S P A
Dedicated to the Private Pension System

American Society of Pension Actuaries
Actuaries, Consultants, Administrators and other Benefits Professionals

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**Chapter Eight - Miscellaneous**

## Guide to Problems

The problems herein were taken from the May, 1994 EA-1(B) Examination. For those students wishing to use this material in conjunction with this examination, the following cross reference list should be helpful.

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| 3 | Chap. 2 - 37 |
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## Glossary of Abbreviations

Since many actuarial terms are lengthy, we will use abbreviations in equations and in the text for the sake of brevity. Listed below are many of the abbreviations which will be seen here:

| | |
|---|---|
| $\ddot{a}_x$ | Present value of an annuity due at age x |
| AA, aa or x | Attained Age |
| Assets | Actuarial Value of Assets |
| AL | Accrued Liability |
| ATA | Average Temporary Annuity |
| B(x) | Benefit at age x |
| BP | Benefit Payments |
| $C_t$ | Contributions for year t |
| CV | Cash Value |
| DB | Death Benefit |
| *e*Assets | Expected Value of Assets |
| *e*AL | Expected Accrued Liability |
| *e*PVFB | Expected Present Value of Future Benefits |
| *e*PVFNC | Expected Present Value of Future Normal Cost |
| *e*UAL | Expected Unfunded Accrued Liability |
| *e*UL | Expected Unfunded Liability |
| EA or ea | Entry Age |
| EAN | Entry Age Normal |
| FIL | Frozen Initial Liability |
| $I_{BP}$ | Interest on Benefit Payments |
| $I_c$ | Interest on contributions |
| ILP | Individual Level Premium |
| J&S or J+S | Joint and Survivor |
| NC | Normal Cost |
| NRA | Normal Retirement Age |
| NRD | Normal Retirement Date |
| PVAB | Present Value of Accrued Benefits |
| PVFB | Present Value of Future Benefits |
| PVFNC | Present Value of Future Normal Costs |
| PVFS | Present Value of Future Salary |
| PVFY | Present Value of Future Years |
| RA, ra or y | Retirement Age |
| S, Salary | Salary or Compensation |
| TA | Temporary Annuity |
| UAL | Unfunded Accrued Liability |
| UC | Unit Credit |
| UL | Unfunded Liability |

# Introduction

This Supplement is designed to assist students who are preparing for the EA-1(B) Examination, Basic Pension Mathematics, which is offered jointly by the American Society of Pension Actuaries, the Society of Actuaries and the Joint Board for the Enrollment of Actuaries. The problems and solutions in this Supplement are from the May, 1994, EA-1(B) Examination. Material is arranged according to the recognized actuarial cost methods, with the last section dealing with those principles common to all cost methods.

**Actuarial Cost Methods, A Review**, in conjunction with this supplement, provides a complete, up to date review of the principles of the various pension cost methods and related actuarial topics.

# Chapter One

# Unit Credit Cost Method (Accrued Benefit)

# Chapter 1

# Unit Credit Cost Method (Accrued Benefit)

## 1.4 Problems

**Problem 1 - 42**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service up to 20 years, plus 1% of final 3-year average compensation for each additional year of service. |
| Preretirement death benefit: | None. |
| Actuarial cost method: | Projected unit credit (based on actual accrual percentages as of valuation date). |

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data for only participants (both active as of 1/1/94):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/44 | 1/1/44 |
| Date of hire | 1/1/69 | 1/1/84 |
| 1994 valuation compensation | $50,000 | $50,000 |

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 50 | 310,647 | 3,752,218 |
| 65 | 94,414 | 868,052 |

In what range is the accrued liability as of 1/1/94?

(A) Less than $140,000
(B) $140,000 but less than $150,000
(C) $150,000 but less than $160,000
(D) $160,000 but less than $170,000
(E) $170,000 or more

**Problem 1 - 43**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.

Early retirement eligibility: Age 55

Early retirement benefit: Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

Before 1994: 100% at age 65.
After 1993: 50% at age 62, 0% at ages 63 and 64, and 100% at age 65.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/74 |
| 1994 valuation compensation | $50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.394$ $\ddot{a}_{65}^{(12)} = 8.736$

In what range is the change in the accrued liability as of 1/1/94 due to the change in the retirement age assumption?

(A) Less than $2,150
(B) $2,150 but less than $2,200
(C) $2,200 but less than $2,250
(D) $2,250 but less than $2,300
(E) $2,300 or more

**Problem 1 - 44**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.
Early retirement eligibility: Age 60.
Early retirement benefit: Unreduced accrued benefit.
Actuarial cost method: Projected unit credit.

Optional form of payment: 95% of accrued benefit, payable until the last death of the participant and his surviving spouse.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65
Form of payment: Life annuity.

Valuation data for sole participant Smith:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/74 |
| 1993 valuation compensation | $40,000 |
| Actual annual compensation: | |
| 1991 | $36,281 |
| 1992 | 38,095 |
| 1993 | 40,000 |
| Spouse's date of birth | 1/1/34 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 9.815$ $\qquad$ $\ddot{a}_{60:60}^{(12)} = 8.094$

$\ddot{a}_{65}^{(12)} = 8.736$ $\qquad$ $\ddot{a}_{65:65}^{(12)} = 6.896$

On 12/31/93, Smith retires and elects to receive his annuity under the optional form of payment commencing 1/1/94.

In what range is the experience loss as of 1/1/94 due to Smith's retirement?

(A) Less than $30,000
(B) $30,000 but less than $40,000
(C) $40,000 but less than $50,000
(D) $50,000 but less than $60,000
(E) $60,000 or more

**Problem 1 - 45**

Normal retirement benefit: $20 per month for each year of service.

Postponed retirement benefit: Actuarial equivalent of normal retirement benefit, based on postretirement valuation assumptions.

Preretirement death benefit: None.

Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probabilities of retirement (retirements are assumed to occur at beginning of year):

| Age x | |
|---|---|
| 65 | 60% |
| 66 | 80% |
| 67 | 100% |

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/29 |
| Date of hire | 1/1/70 |

Selected commutation functions based on postretirement valuation assumptions:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 94 | 825 |
| 66 | 86 | 734 |
| 67 | 79 | 651 |

In what range is the accrued liability as of 1/1/94?

(A) Less than $50,000
(B) $50,000 but less than $51,000
(C) $51,000 but less than $52,000
(D) $52,000 but less than $53,000
(E) $53,000 or more

**Problem 1 - 46**

Normal retirement benefit: $20 per month for each year of service.
Termination benefit: Accrued benefit payable at normal retirement age.
Vesting: Full and immediate.
Preretirement death benefit: None.
Actuarial cost method: Unit credit.

Actuarial assumptions:

> Interest rate: 7% per year.
> Pretermination deaths: None.
> Post-termination deaths: Included in commutation functions below.
> Preretirement terminations: 10% at age 50 only (terminations are assumed to occur at beginning of year).
> Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/54 |
| Date of hire | 1/1/84 |

Selected commutation functions based on post-termination assumptions:

| Age x | $D_x$ |
|---|---|
| 30 | 1,262 |
| 40 | 632 |
| 50 | 311 |
| 65 | 94 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.7$$

In what range is the accrued liability as of 1/1/94?

(A) Less than $3,700
(B) $3,700 but less than $3,800
(C) $3,800 but less than $3,900
(D) $3,900 but less than $4,000
(E) $4,000 or more

## 1.5 Solutions to Problems

**Problem 1 - 42**

Key Concept: The Accrued Liability determined using the Projected Unit Credit method is equal to the present value of the accrued benefit using projected salary.

Step I: Calculate the Final Average Salary. Note that since Smith and Brown have the same salary and date of birth, they will have the same Final Average Salary.

$$\begin{aligned}\text{Final Average Salary} &= 50{,}000\left[(1.05)^{14} + (1.05)^{13} + (1.05)^{12}\right]/3 \\ &= 94{,}357\end{aligned}$$

Step II: Calculate the accrued benefits using the Final Average Salary.

Smith: $(94{,}357)\left[(.02)(20 \text{ years of service}) + (.01)(5 \text{ years of service})\right] = 42{,}461$

Brown: $(94{,}357)(.02)(10 \text{ years of service}) = 18{,}871$

Step III: Since $\ddot{a}_{65}^{(12)}$ was not provided, this factor must be calculated.

$$\begin{aligned}\ddot{a}_{65}^{(12)} &= (N_{65} - \tfrac{11}{24} D_{65})/D_{65} \\ &= 8.73577\end{aligned}$$

Step IV: Calculate the Present Value of the Accrued Benefits.

Smith: $42{,}461 \times \ddot{a}_{65}^{(12)} \times (D_{65}/D_{50}) = 112{,}735$

Brown: $18{,}871 \times \ddot{a}_{65}^{(12)} \times (D_{65}/D_{50}) = 50{,}103$

Total Accrued Liability = 112,735 + 50,103 = 162,838

Answer is D.

**Problem 1 - 43**

Step I: Calculate the Final Average Salary at each retirement age. See Problem 4 - 53 for a discussion of Final Average Salary.

$$\text{Final Average Salary RA} = 65: \quad (50{,}000)\left[(1.05)^{14} + (1.05)^{13} + (1.05)^{12}\right]/3 = 94{,}357$$

$$\text{Final Average Salary RA} = 62: \quad (50{,}000)\left[(1.05)^{11} + (1.05)^{10} + (1.05)^{9}\right]/3 = 81{,}509$$

Step II: Calculate the Accrued Liability under the old retirement age assumptions (100% at age 65).

$$\text{Projected Accrued Benefit} = (94{,}357)(.02)(20 \text{ years of service}) = 37{,}743$$

$$\begin{aligned} \text{AL (RA 65)} = \text{PVAB} &= (37{,}743)(\ddot{a}_{65}^{(12)})(v^{15}) \\ &= (37{,}743)(8.736)(.362446) = 119{,}507 \end{aligned}$$

Step III: Calculate the Accrued Liability at retirement age 62.

$$\begin{aligned} \text{Projected Accrued Benefit} &= (81{,}509)(.02)(20 \text{ years of service})(1 - (.03)(3 \text{ years})) \\ &= 29{,}669 \end{aligned}$$

$$\begin{aligned} \text{AL (RA 62)} = \text{PVAB} &= (29{,}669)(\ddot{a}_{62}^{(12)})(v^{12}) \\ &= (29{,}669)(9.394)(.444012) = 123{,}751 \end{aligned}$$

Step IV: Calculate the Accrued Liability under the new retirement age assumptions (50% at age 62, 100% at age 65).

$$\text{AL} = (123{,}751)(.5) + (119{,}507)(.5) = 121{,}629$$

Step V: Calculate the increase in the Accrued Liability due to the change in assumptions.

$$\text{Increase} = 121{,}629 - 119{,}507 = 2{,}122$$

Answer is A.

**Problem 1 - 44**

Key Concept: The experience loss due to early retirement is the difference between the actual liability and the Accrued Liability determined using the funding assumptions and cost method.

Step I: Calculate the Projected Unit Credit Accrued Liability as of 1/1/94.

$$\text{Final Average Salary} = (40{,}000)\left[(1.05)^5 + (1.05)^4 + (1.05)^3\right]/3 = 48{,}659$$

Note that 1993, not 1994, salary is given. See Problem 4 - 53 for a discussion of Final Average Salary.

$$\text{Projected Accrued Benefit} = (48{,}659)(.02)(20 \text{ years of service}) = 19{,}464$$

$$\begin{aligned} AL = PVAB &= (19{,}464)(\ddot{a}_{65}^{(12)})(v^5) \\ &= (19{,}464)(8.736)(.712986) = 121{,}234 \end{aligned}$$

Step II: Calculate the value of the early retirement benefit.

$$\text{Actual Final Average Salary} = (36{,}281 + 38{,}095 + 40{,}000)/3 = 38{,}125$$

$$\text{Actual Accrued Benefit} = (38{,}125)(.02)(20 \text{ years of service}) = 15{,}250$$

The actual liability is the present value of the actual accrued benefit, calculated at age 60 using the optional form of benefit, since that is the benefit which was elected.

$$\begin{aligned} PVAB &= (15{,}250)(.95)(\ddot{a}_{\overline{60:60}}^{(12)}) \\ &= (15{,}250)(.95)(\ddot{a}_{60}^{(12)} + \ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}) \\ &= (15{,}250)(.95)(9.815 + 9.815 - 8.094) = 167{,}128 \end{aligned}$$

Step IV: Calculate the difference between the Accrued Liability and the actual liability, which is the actuarial loss.

$$\text{Actuarial Loss} = 167{,}128 - 121{,}234 = 45{,}894$$

Answer is C.

**Problem 1 - 45**

Key Concept: The Accrued Liability is equal to the sum of the liabilities for each possible retirement age, each multiplied by the probability of retirement at that age.

Step I: Calculate the retirement benefit at each possible retirement age. Since there is no preretirement death benefit, actuarial equivalence includes both interest and mortality.

$$\text{Benefit at RA } 65 = (20)(24 \text{ years of service}) = 480$$

$$\begin{aligned}\text{Benefit at RA } 66 &= \text{Benefit at } 65 \times (N_{65}^{(12)}/N_{66}^{(12)}) \\ &= 480 \times (825/734) = 539.51\end{aligned}$$

$$\begin{aligned}\text{Benefit at RA } 67 &= \text{Benefit at } 65 \times (N_{65}^{(12)}/N_{67}^{(12)}) \\ &= 480 \times (825/651) = 608.29\end{aligned}$$

Step II: Calculate the liability assuming retirement at each age. Since there are no assumed preretirement decrements, the discount is done on an interest only basis.

Retirement at age 65: $= (\text{Benefit})(12)(N_{65}^{(12)}/D_{65})$
$= (480)(12)(825/94) = 50{,}553$

Retirement at age 66: $= (\text{Benefit})(12)(N_{66}^{(12)}/D_{66})(v)$
$= (539.51)(12)(734/86)(.934579) = 51{,}641$

Retirement at age 67: $= (\text{Benefit})(12)(N_{67}^{(12)}/D_{67})(v^2)$
$= (608.29)(12)(651/79)(.873439) = 52{,}539$

Step III: Calculate the Accrued Liability.

$$\begin{aligned}AL &= (q_{65}^{(r)})(50{,}553) + (p_{65}^{(r)})(q_{66}^{(r)})(51{,}641) + (p_{65}^{(r)})(p_{66}^{(r)})(q_{67}^{(r)})(52{,}539) \\ &= (.6)(50{,}553) + (.4)(.8)(51{,}641) + (.4)(.2)(1)(52{,}539) \\ &= 30{,}332 + 16{,}525 + 4{,}203 = 51{,}060\end{aligned}$$

Answer is C.

**Problem 1 - 46**

Key Concept: The Accrued Liability determined using the Unit Credit method is the sum of the present value of the accrued termination benefit plus the present value of the accrued retirement benefit.

Step I: Calculate the accrued monthly benefit.

$$\text{Accrued Benefit} = (20)(10 \text{ years of service}) = 200$$

Step II: Calculate the present value of the retirement benefit. Note that the only preretirement decrement assumed is the 10% termination assumption at age 50. There is no preretirement mortality assumed for retirement benefits.

$$\begin{aligned} \text{PVFB} &= (200)(12\ddot{a}_{65}^{(12)})(v^{25})({}_{25}p_{40}) \\ &= (200)(12)(8.7)(.18425)(.9) = 3{,}462 \end{aligned}$$

Step III: Calculate the present value of the termination benefit. Note that for the termination benefit, the post-termination mortality assumption is in effect from age 50 to 65.

$$\begin{aligned} \text{PVFB} &= (200)(12\ddot{a}_{65}^{(12)})(v^{10})(q_{50}^{(w)})(\text{D}_{65}/\text{D}_{50}) \\ &= (200)(12)(8.7)(.50835)(.1)(94/311) = 321 \end{aligned}$$

Step IV: Calculate the Accrued Liability as of 1/1/94.

$$\text{AL}_{94} = 3{,}462 + 321 = 3{,}783$$

Answer is B.

# Chapter Two

# Aggregate Cost Method

# Chapter 2

# Aggregate Cost Method

## 2.4 Problems

**Problem 2 - 37**

Normal retirement benefit: 50% of 5-year final average compensation.
Actuarial cost method: Aggregate (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases:

Before 1994: 5% per year.
After 1993: 6% per year.

Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/69 |
| 1994 valuation compensation | $100,000 |

Selected valuation results as of 1/1/94 before change in assumptions:

| | |
|---|---|
| Normal cost as of 1/1 | $ 38,120 |
| Value of assets | $200,000 |

In what range is the normal cost for 1994 as of 1/1/94 after the change in actuarial assumptions?

(A) Less than \$38,800
(B) \$38,800 but less than \$39,200
(C) \$39,200 but less than \$39,600
(D) \$39,600 but less than \$40,000
(E) \$40,000 or more

**Problem 2 - 38**

Normal retirement benefit: \$2,000 per month.

Actuarial cost method: Aggregate with side fund and life insurance (split-funded).

Actuarial assumptions:

Compensation increases: None.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data as of 1/1/94:

| | |
|---|---|
| Value of assets in side fund | \$10,000 |
| Annual premium for life insurance | 2,000 |
| Cash value of life insurance at age 65 | 50,000 |

Date of birth for sole participant (active as of 1/1/94): 1/1/49

Selected commutation functions based on preretirement assumptions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 45 | 445 | 5,691 |
| 55 | 214 | 2,405 |
| 65 | 94 | 868 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.78$$

In what range is the total normal cost for 1994 as of 1/1/94?

(A) Less than $4,000
(B) $4,000 but less than $4,300
(C) $4,300 but less than $4,600
(D) $4,600 but less than $4,900
(E) $4,900 or more

## 2.5 Solutions to Problems

**Problem 2 - 37**

Key Concept: As in Problem 6 - 28, we must determine the value of $\ddot{a}_{65}^{(12)}$ in order to solve this problem.

Step I: Calculate the Final Average Salary at each salary scale rate.

Using 5% salary scale,

$$\begin{aligned}\text{Final Average Salary} &= (100{,}000)\left[(1.05)^4 + (1.05)^3 + (1.05)^2 + (1.05) + 1\right]/5 \\ &= 110{,}513\end{aligned}$$

Using 6% salary scale,

$$\begin{aligned}\text{Final Average Salary} &= (100{,}000)\left[(1.06)^4 + (1.06)^3 + (1.06)^2 + (1.06) + 1\right]/5 \\ &= 112{,}742\end{aligned}$$

See the discussion in Problem 4 - 53 concerning the calculation of Final Average Salary.

Step II: To determine the single sum value of the retirement benefit at age 65, calculate the annuity purchase rate using the valuation results before the salary scale was increased to 6%.

$$\text{Retirement Benefit} = (.5)(110{,}513) = 55{,}256$$

$$\begin{aligned}\text{PVFNC} &= \text{NC} \times \ddot{a}_{\overline{5}|\,j} \qquad \text{where } j = 1.07/1.05 - 1 = .0190476 \\ &= 38{,}120 \times 4.8165 = 183{,}605\end{aligned}$$

$$\begin{aligned}\text{PVFB} &= \text{PVFNC} + \text{Assets} \\ &= 183{,}605 + 200{,}000 = 383{,}605\end{aligned}$$

$$\begin{aligned}\text{Also, PVFB} &= (\text{Benefit})(\ddot{a}_{65}^{(12)})v^5 \\ 383{,}605 &= (55{,}256)(\ddot{a}_{65}^{(12)})v^5\end{aligned}$$

$$\text{Therefore, } \ddot{a}_{65}^{(12)} = 9.737$$

Step III: Determine the new normal cost using the 6% salary scale.

$$\text{Retirement Benefit} = (.5)(112,742) = 56,371$$

$$\begin{aligned}\text{PVFB} &= (\text{Benefit})(\ddot{a}_{65}^{(12)})v^5 \\ &= (56,371)(9.737)(.712986) = 391,347\end{aligned}$$

$$\begin{aligned}\text{PVFNC} &= \text{PVFB - assets} \\ &= 391,347 - 200,000 = 191,347\end{aligned}$$

$$\begin{aligned}\text{NC} &= \text{PVFNC}/\ddot{a}_{\overline{5|}\,j}\,, \qquad \text{where } j = 1.07/1.06 - 1 = .009434 \\ &= 191,347/4.9074 = 38,992\end{aligned}$$

Answer is B.

**Problem 2 - 38**

Step I: Calculate the Present Value of Future Benefits for the side fund.

$$\begin{aligned}\text{PVFB}_{65} &= (\text{Benefit})(12\ddot{a}_{65}^{(12)}) \\ &= (2,000)(12)(8.78) = 210,720\end{aligned}$$

$$\begin{aligned}\text{Side Fund PVFB}_{45} &= (\text{PVFB}_{65} - \text{Cash Value})(D_{65}/D_{45}) \\ &= (210,720 - 50,000)(94/445) = 33,950\end{aligned}$$

Step II: Calculate the side fund Normal Cost.

$$\begin{aligned}\text{Side Fund NC} &= (\text{PVFB}_{45} - \text{assets})/\left[(N_{45} - N_{65})/D_{45}\right] \\ &= (33,950 - 10,000)/\left[(5,691 - 868)/445\right] = 2,210\end{aligned}$$

Step III: Calculate the total Normal Cost.

$$\begin{aligned}\text{Total NC} &= \text{Side Fund NC} + \text{Premium} \\ &= 2,210 + 2,000 = 4,210\end{aligned}$$

Answer is B.

# Chapter Three

# Individual Aggregate Cost Method

# Chapter 3

# Individual Aggregate Cost Method

## 3.3 Problems

**Problem 3 - 15**

Plan effective date: 1/1/94
Normal retirement benefit: $900 per month.
Early retirement eligibility: Age 60.

Early retirement benefit: Normal retirement benefit minus $50 per month for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

| Age x | |
|---|---|
| 63 | 33.33% |
| 64 | 66.67% |
| 65 | 100.00% |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/49 |
| Date of hire | 1/1/94 |

Selected annuity values:

$$\ddot{a}_{63}^{(12)} = 8.96 \qquad \ddot{a}_{64}^{(12)} = 8.74 \qquad \ddot{a}_{65}^{(12)} = 8.51$$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $2,200
(B) $2,200 but less than $2,300
(C) $2,300 but less than $2,400
(D) $2,400 but less than $2,500
(E) $2,500 or more

## 3.4 Solutions to Problems

**Problem 3 - 15**

Step I: Calculate the Present Value of Future Benefits for each retirement age.

For RA = 63, Retirement Benefit = 900 - (50)(2 years of service) = 800
For RA = 64, Retirement Benefit = 900 - 50 = 850
For RA = 65, Retirement Benefit = 900

$$\text{For RA} = 63,\ PVFB_{63} = (\text{Benefit})(12\ddot{a}_{63}^{(12)})(v^{18})$$
$$= (800)(12)(8.96)(.29586) = 25{,}449$$

$$\text{For RA} = 64,\ PVFB_{64} = (\text{Benefit})(12\ddot{a}_{64}^{(12)})(v^{19})$$
$$= (850)(12)(8.74)(.27651) = 24{,}650$$

$$\text{For RA} = 65,\ PVFB_{65} = (\text{Benefit})(12\ddot{a}_{65}^{(12)})(v^{20})$$
$$= (900)(12)(8.51)(.25842) = 23{,}751$$

Step II: Calculate the total Present Value of Future Benefits. The total Present Value of Future Benefits is equal to the sum of the individual Present Value of Future Benefits at each retirement age, each multiplied by the probability of retirement at that age.

$$PVFB = (PVFB_{63})(q_{63}^{(r)}) + (PVFB_{64})(p_{63}^{(r)})(q_{64}^{(r)}) + (PVFB_{65})(p_{64}^{(r)})(p_{63}^{(r)})(q_{65}^{(r)})$$
$$= (25{,}449)(1/3) + (24{,}650)(2/3)(2/3) + (23{,}751)(2/3)(1/3)(1)$$
$$= 24{,}717$$

Step III: Calculate the Normal Cost. The temporary annuity used to calculate the Normal Cost is equal to the sum of the individual temporary annuities at each retirement age, each multiplied by the probability of retirement at that age.

$$\text{Temporary Annuity} = \ddot{a}_{\overline{18|}}(q_{63}^{(r)}) + \ddot{a}_{\overline{19|}}(p_{63}^{(r)})(q_{64}^{(r)}) + \ddot{a}_{\overline{20|}}(p_{64}^{(r)})(p_{63}^{(r)})(q_{65}^{(r)})$$
$$= \ddot{a}_{\overline{18|}}(1/3) + \ddot{a}_{\overline{19|}}(2/3)(2/3) + \ddot{a}_{\overline{20|}}(2/3)(1/3)(1)$$
$$= 3.58774 + 4.91515 + 2.51902 = 11.02191$$

$$NC = PVFB/(\text{Temporary Annuity})$$
$$= 24{,}717/11.02191 = 2{,}243$$

Answer is B.

**Alternative Solution:**

The temporary annuity factor determined in Step III above could also be calculated as the present value of future annual payments of \$1, with each payment multiplied by the probability that the participant is still active.

$$\begin{aligned}\text{Temporary Annuity} &= \ddot{a}_{\overline{18}|} + v^{18}(p_{63}^{(\tau)}) + v^{19}(p_{63}^{(\tau)})(p_{64}^{(\tau)}) + v^{20}(p_{63}^{(\tau)})(p_{64}^{(\tau)})(p_{65}^{(\tau)}) \\ &= \ddot{a}_{\overline{18}|} + v^{18}(.6667) + v^{19}(.6667)(.3333) + v^{20}(.6667)(.3333)(0) \\ &= 10.7632 + .1973 + .0614 = 11.0219\end{aligned}$$

$$\begin{aligned}\text{NC} &= \text{PVFB}/(\text{Temporary Annuity}) \\ &= 24{,}717/11.0219 = 2{,}243\end{aligned}$$

Answer is B.

# Chapter Four

# Entry Age Normal Cost Method

# Chapter 4

# Entry Age Normal Cost Method

## 4.5 Problems

**Problem 4 - 53**

Normal retirement benefit:

Before 1994: 50% of final 5-year average compensation.
After 1993: 60% of final 5-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Compensation increases: 5% per year.
Preretirement terminations other than death: None.
Retirement age: 65

Valuation data for sole participant

| | |
|---|---|
| Date of birth | 1/1/60 |
| Date of hire | 1/1/90 |
| 1994 valuation compensation | $25,000 |

Selected commutation functions:

| Age x | $D_x$ | ${}^sD_x$ | $N_x$ | ${}^sN_x$ |
|---|---|---|---|---|
| 30 | 1,262 | 5,454 | 17,888 | 164,704 |
| 34 | 958 | 5,033 | 13,323 | 143,532 |
| 65 | 94 | 2,241 | 868 | 30,013 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.776$

In what range is the change in the accrued liability as of 1/1/94 due to the plan amendment?

(A) Less than $1,350
(B) $1,350 but less than $1,450
(C) $1,450 but less than $1,550
(D) $1,550 but less than $1,650
(E) $1,650 or more

**Problem 4 - 54**

Vesting: 100% after 5 years of service.

Actuarial cost method: Individual entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 4% per year.
Preretirement deaths: None.
Retirement age: 65

Valuation data for only participants (both active as of 1/1/94):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/59 | 1/1/56 |
| Date of hire | 1/1/94 | 1/1/91 |

Normal cost for 1994 as of 1/1/94:

For Smith: $17,000
For Brown: $16,000

Selected probabilities of termination:

| Age x | $q_x^{(w)}$ |
|---|---|
| 35 | 0.25 |
| 36 | 0.20 |
| 37 | 0.15 |
| 38 | 0.10 |
| 39 | 0.05 |

In what range is the accrued liability as of 1/1/94?

(A) Less than $65,000
(B) $65,000 but less than $80,000
(C) $80,000 but less than $95,000
(D) $95,000 but less than $110,000
(E) $110,000 or more

**Problem 4 - 55**

Normal retirement benefit:

Effective 1/1/93: \$15 per month for each year of service.
Effective 1/1/94: \$18 per month for each year of service.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate:

Before 1994: 7% per year.
After 1993: 6% per year.

Preretirement deaths and terminations: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/53 |
| Date of hire | 1/1/80 |

Selected annuity values:

| | <u>6%</u> | <u>7%</u> |
|---|---|---|
| $12\ddot{a}_{65}^{(12)}$ | 112.14 | 104.83 |

As of 1/1/94, the increase in the accrued liability due to the change in the assumed interest rate is determined before the increase in the accrued liability due to the plan amendment.

In what range is the absolute value of the difference between (a) the increase in the accrued liability as of 1/1/94 due to the change in the assumed interest rate, and (b) the increase in the accrued liability due to the plan amendment?

(A) Less than \$100
(B) \$100 but less than \$200
(C) \$200 but less than \$300
(D) \$400 but less than \$500
(E) \$500 or more

**Problem 4 - 56**

Normal retirement benefit: $50 per month for each year of service.

Termination benefit: Accrued benefit payable at normal retirement date.

Vesting: Full and immediate.
Preretirement death benefit: None.
Actuarial cost method: Entry age normal (level dollar amount).
Actuarial assumptions:

> Interest rate: 7% per year.
> Pretermination deaths: None.
> Post-termination deaths: Included in commutation functions below.
> Preretirement terminations: 30% at age 50 only (terminations are assumed to occur at beginning of year).
> Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/54 |
| Date of hire | 1/1/84 |

Selected commutation functions based on post-termination assumptions:

| Age x | $D_x$ |
|---|---|
| 30 | 1,262 |
| 40 | 632 |
| 50 | 311 |
| 65 | 94 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.7$$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $950
(B) $950 but less than $1,000
(C) $1,000 but less than $1,050
(D) $1,050 but less than $1,100
(E) $1,100 or more

**Problem 4 - 57**

Normal retirement benefit: 50% of final 5-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Compensation increases: 6% per year.
Preretirement terminations other than deaths: None.
Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/84 |
| 1994 valuation compensation | \$53,000 |

Selected commutation functions:

| Age x | $D_x$ | ${}^{s}D_x$ | $N_x$ | ${}^{s}N_x$ |
|---|---|---|---|---|
| 40 | 49,876 | 513,015 | 666,789 | 15,607,843 |
| 50 | 24,505 | 451,387 | 295,992 | 10,748,428 |
| 65 | 7,448 | 328,780 | 68,476 | 4,770,425 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability as of 1/1/94?

(A) Less than \$60,000
(B) \$60,000 but less than \$62,500
(C) \$62,500 but less than \$65,000
(D) \$65,000 but less than \$67,500
(E) \$67,500 or more

## 4.6 Solutions to Problems

**Problem 4 - 53**

<u>Key Concept</u>: In problems involving salary scales, it is necessary to count the number of salary changes. In this problem, the participant's attained age is 34. Since his 1994 valuation compensation is given, there will only be 30 salary changes before retirement, with the last change occurring on the valuation date as of 1/1/2024.

Step I: Calculate the Final Average Salary.

$$\begin{aligned}\text{Final Average Salary} &= (25{,}000)\left[(1.05)^{30} + (1.05)^{29} + (1.05)^{28} + (1.05)^{27} + (1.05)^{26}\right]/5 \\ &= 98{,}237\end{aligned}$$

Note that other formulas could be used to determine Final Average Salary. For example,

$$\text{Final Average Salary} = (25{,}000)(\ddot{s}_{\overline{30}|} - \ddot{s}_{\overline{25}|})/5 = 98{,}237$$

$$\begin{aligned}\text{Final Average Salary} &= (\text{Final Salary})(\ddot{a}_{\overline{5}|}/5) \\ &= (25{,}000)(1.05)^{30}(\ddot{a}_{\overline{5}|}/5) = 98{,}237\end{aligned}$$

Step II: Calculate the increase in the retirement benefit.

$$\text{Increase} = (.1)(98{,}237) = 9{,}823.70$$

Step III: Determine the increase in the Normal Cost at entry age.

$$\begin{aligned}PVFB_{30} &= (9{,}823.70)(\ddot{a}_{65}^{(12)})(D_{65}/D_{30}) \\ &= (9{,}823.70)(8.776)(94/1{,}262) = 6{,}422\end{aligned}$$

$$\begin{aligned}NC_{30} &= PVFB_{30} \div \left[({}^sN_{30} - {}^sN_{65})/{}^sD_{30}\right] \\ &= 6{,}422 \div \left[(164{,}704 - 30{,}013)/5{,}454\right] = 260\end{aligned}$$

Step IV: Determine the increase in the Accrued Liability.

$$\begin{aligned}AL_{34} &= NC_{30} \times (1.05)^4 \times \left[({}^sN_{30} - {}^sN_{34})/{}^sD_{34}\right] \\ &= 260 \times 1.2155 \times \left[(164{,}704 - 143{,}532)/5{,}033\right] = 1{,}329\end{aligned}$$

Answer is A.

**Problem 4 - 54**

<u>Key Concept</u>: The Accrued Liability is defined as either the accumulation of prior Normal Costs or the difference between the Present Value of Future Benefits and the Present Value of Future Normal Costs. Since Normal Cost is given in this problem, the Accrued Liability should be calculated by accumulating the prior Normal Costs. Note that Smith was just hired on 1/1/94 and therefore has no prior Normal Cost.

Step I: Calculate the Normal Cost for Brown for each prior year.

Since the Normal Costs increase by the 4% salary scale rate each year,

$$NC_{91} = 16{,}000 \div (1.04)^3 = 14{,}224$$
$$NC_{92} = 16{,}000 \div (1.04)^2 = 14{,}793$$
$$NC_{93} = 16{,}000 \div (1.04) = 15{,}385$$

Step II: Calculate the Accrued Liability.

$$\begin{aligned} AL &= NC_{91} \times (1.07)^3 \times (1/{}_3p_{35}^{(w)}) + \\ &\quad NC_{92} \times (1.07)^2 \times (1/{}_2p_{36}^{(w)}) + NC_{93} \times (1.07) \times (1/p_{37}^{(w)}) \\ &= (14{,}224)(1.225043)\big[1/[(.75)(.8)(.85)]\big] + \\ &\quad (14{,}793)(1.1449)\big[1/[(.8)(.85)]\big] + (15{,}385)(1.07)(1/.85) \\ &= 34{,}167 + 24{,}906 + 19{,}367 = 78{,}440 \end{aligned}$$

Answer is B.

**Problem 4 - 55**

Step I: Calculate the Accrued Liability based upon the original plan provisions and assumptions.

Monthly Retirement Benefit = (15)(38 years of service) = 570

$$\begin{aligned} NC &= (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38|}} \qquad (i = 7\%) \\ &= (570)(104.83)/184.64 = 323.62 \end{aligned}$$

$$AL = (\text{Normal Cost})(\ddot{s}_{\overline{14}|})$$
$$= (323.62)(24.129) = 7{,}809$$

Step II: Calculate the Accrued Liability with the new 6% interest rate assumption.

$$NC = (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38}|} \qquad (i = 6\%)$$
$$= (570)(112.14)/144.058 = 443.71$$

$$AL = (NC)(\ddot{s}_{\overline{14}|})$$
$$= (443.71)(22.276) = 9{,}884$$

Step III: Calculate the Accrued Liability with both the new benefit formula and the new interest rate.

Monthly Retirement Benefit = (18)(38 years of service) = 684

$$NC = (\text{Benefit})(12\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{38}|} \qquad (i = 6\%)$$
$$= (684)(112.14)/144.058 = 532.45$$

$$AL = (NC)(\ddot{s}_{\overline{14}|})$$
$$= (532.45)(22.276) = 11{,}861$$

Step IV: Calculate the difference between the increase in the Accrued Liability due to the change in the interest rate and the increase due to the plan amendment.

Increase due to interest rate = 9,884 - 7,809 = 2,075

Increase due to plan amendment = 11,861 - 9,884 = 1,977

Difference = 2,075 - 1,977 = 98

Answer is A.

**Problem 4 - 56**

Key Concept: In situations where decrements are assumed to occur preretirement, the Present Value of Future Benefits is equal to the present value of each type of benefit (retirement, termination, death, for example).

Step I: Calculate the present value of the termination benefit at entry age.

Termination Benefit = (50)(20 years of service) = 1,000

$$\begin{aligned} PVFB_{30} &= (1{,}000)(12\ddot{a}_{65}^{(12)})(q_{50}^{(w)})(D_{65}/D_{50})(v^{20}) \\ &= (1{,}000)(12)(8.7)(.3)(94/311)(.2584) = 2{,}446 \end{aligned}$$

Note that commutation functions are used to discount only from age 65 to age 50, since preretirement mortality is assumed to occur only <u>after</u> termination.

Step II: Calculate the present value of the retirement benefit at entry age.

Retirement Benefit = (50)(35 years of service) = 1,750

$$\begin{aligned} PVFB_{30} &= (1{,}750)(12\ddot{a}_{65}^{(12)})(p_{50}^{(w)})(v^{35}) \\ &= (1{,}750)(12)(8.7)(.7)(.0937) = 11{,}983 \end{aligned}$$

Step III: Calculate the value of the temporary annuity from entry age to retirement age.

$$\begin{aligned} \ddot{a}_{30:\overline{35}|} &= (1 + v + v^2 + \ldots + v^{19}) + (p_{50}^{(w)}v^{20} + \ldots + p_{50}^{(w)}v^{34}) \\ &= (1 + v + \ldots + v^{19}) + p_{50}^{(w)}v^{20}(1 + v + \ldots + v^{14}) \\ &= \ddot{a}_{\overline{20}|} + (.7)v^{20}\,\ddot{a}_{\overline{15}|} \\ &= 11.3356 + (.7)(.2584)(9.7455) = 13.0984 \end{aligned}$$

Step IV: Calculate the Normal Cost.

$$\begin{aligned} NC &= PVFB_{30} \div \ddot{a}_{30:\overline{35}|} \\ &= (2{,}446 + 11{,}983) \div 13.0984 = 1{,}101 \end{aligned}$$

Answer is E.

**Problem 4 - 57**

Step I: Calculate the final five year average salary. See the discussion in Problem 4 - 53 concerning the calculation of Final Average Salary.

$$\begin{aligned} \text{Final Average Salary} &= (53{,}000)\left[(1.06)^{14} + (1.06)^{13} + (1.06)^{12} + (1.06)^{11} + (1.06)^{10}\right]/5 \\ &= 107{,}009 \end{aligned}$$

Step II: Calculate the Accrued Liability.

$$\begin{aligned}
PVFB_{40} &= (107{,}009)(.5)(\ddot{a}_{65}^{(12)})(D_{65}/D_{40}) \\
&= (107{,}009)(.5)(8.74)(7{,}448/49{,}876) = 69{,}831
\end{aligned}$$

$$\begin{aligned}
NC_{40} &= (PVFB_{40})/[({}^{s}N_{40} - {}^{s}N_{65})/{}^{s}D_{40}] \\
&= (69{,}831)/[(15{,}607{,}843 - 4{,}770{,}425)/513{,}015] = 3{,}306
\end{aligned}$$

$$\begin{aligned}
AL_{50} &= (NC_{40})(1 + s)^{10}\left[({}^{s}N_{40} - {}^{s}N_{50})/{}^{s}D_{50}\right] \\
&= (3{,}306)(1.06)^{10}\left[(15{,}607{,}843 - 10{,}748{,}428)/451{,}387\right] \\
&= 63{,}738
\end{aligned}$$

Answer is C.

**Alternative Solution**:

An alternative way to calculate the Accrued Liability is to determine the Normal Cost as a rate of salary at entry age. The Present Value of Future Benefits at entry age is calculated as shown in Step II above. Then the Normal Cost rate at entry age is determined as follows:

$$\begin{aligned}
\text{NC Rate}_{40} &= (PVFB_{40}/[({}^{s}N_{40} - {}^{s}N_{65})/{}^{s}D_{40}] \div \left[(53{,}000)/(1 + s)^{10}\right] \\
&= (69{,}831/21.12495) \div (53{,}000/1.79085) = .1117
\end{aligned}$$

The Normal Cost rate can now be multiplied by the current salary, and accumulated from entry age to attained age to determine the Accrued Liability.

$$\begin{aligned}
AL_{50} &= \text{NC Rate}_{40} \times 53{,}000 \times ({}^{s}N_{40} - {}^{s}N_{50})/{}^{s}D_{50} \\
&= (.1117)(53{,}000)\left[(15{,}607{,}843 - 10{,}748{,}428)/451{,}387\right] \\
&= 63{,}733
\end{aligned}$$

Answer is C.

# Chapter Five

# Individual Level Premium Cost Method

# Chapter 5

# Individual Level Premium Cost Method

## 5.3 Problems

**Problem 5 - 25**

Plan effective date: 1/1/90
Normal retirement benefit: 50% of highest 3-year average compensation.
Compensation: Base rate of pay as of 1/1.

Actuarial cost method: Individual level premium.

Actuarial assumptions:

> Interest rate: 7% per year.
> Compensation increases: None.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/30 |
| Date of hire | 1/1/90 |
| Base rate of pay as of 1/1/90 | $60,000 |
| Base rate of pay as of 1/1/91 | 60,000 |
| Base rate of pay as of 1/1/92 | 80,000 |
| Base rate of pay as of 1/1/93 | 80,000 |
| Base rate of pay as of 1/1/94 | 65,000 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.786$$

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $45,000
(B) $45,000 but less than $47,500
(C) $47,500 but less than $50,000
(D) $50,000 but less than $52,500
(E) $52,500 or more

## 5.4 Solutions to Problems

**Problem 5 - 25**

Key Concept: The Normal Cost under the Individual Level Premium method is the sum of the Normal Cost determined to fund the initial benefit at the time the plan is established, plus the Normal Costs determined to fund the increases (decreases) in the projected retirement benefit each year. If the projected retirement benefit decreases in a given year, the Normal Cost determined to "fund" the decrease will be negative, and the total Normal Cost for that participant will decrease for the year.

Step I: Calculate the Normal Cost as of 1/1/90.

$$\text{Benefit} = (60{,}000)(.5) = 30{,}000$$

$$\begin{aligned} NC_{90} &= (\text{Benefit})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{5}|} \\ &= (30{,}000)(8.786)/6.15329 = 42{,}836 \end{aligned}$$

Step II: Calculate the increase in the Normal Cost due to the increase in benefits as of 1/1/92.

$$\text{Benefit Increase} = (80{,}000 - 60{,}000)(.5) = 10{,}000$$

$$\begin{aligned} \Delta NC_{92} &= (\text{Benefit Increase})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{3}|} \\ &= (10{,}000)(8.786)/3.43994 = 25{,}541 \end{aligned}$$

Step III: Calculate the decrease in Normal Cost due to the benefit decrease in benefits as of 1/1/94.

$$\text{3 year average salary} = (80{,}000 + 80{,}000 + 65{,}000)/3 = 75{,}000$$

$$\text{Benefit Decrease} = (75{,}000 - 80{,}000)(.5) = (2{,}500)$$

$$\begin{aligned} \Delta NC_{94} &= (\text{Benefit Decrease})(\ddot{a}_{65}^{(12)})/\ddot{s}_{\overline{1}|} \\ &= (2{,}500)(8.786)/1.07 = (20{,}528) \end{aligned}$$

Step IV: Calculate the total Normal Cost as of 1/1/94.

$$\begin{aligned} NC_{94} &= NC_{90} + \Delta NC_{92} + \Delta NC_{94} \\ &= 42{,}836 + 25{,}541 - 20{,}528 = 47{,}849 \end{aligned}$$

Answer is C.

# Chapter Six

# Frozen Initial Liability Cost Method

# Chapter 6

# Frozen Initial Liability Cost Method

## 6.3 Problems

**Problem 6 - 28**

Plan effective date: 1/1/94

Normal retirement benefit: $20 per month for each year of service.
Actuarial cost method: Frozen initial liability (entry age is age at hire).

Actuarial assumptions:

> Preretirement interest rate: 8% per year.
> Postretirement interest rate: 7% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/34 |
| Date of hire | 1/1/84 |

Accrued liability under unit credit method as of 1/1/94, based on a 7% rate for preretirement and postretirement interest: $14,900

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $1,100
(B) $1,100 but less than $1,200
(C) $1,200 but less than $1,300
(D) $1,300 but less than $1,400
(E) $1,400 or more

**Problem 6 - 29**

Normal retirement benefit: $15 per month for each year of service.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

> Interest rate: 7% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65

Date of birth for only participant ever covered by plan (active as of 1/1/94): 1/1/42

Selected valuation results:

| | 1/1/93 | 1/1/94 |
|---|---|---|
| Present value of future benefits | $122,000 | |
| Value of assets | | $27,500 |
| Unfunded liability | | 48,500 |
| Value of assets plus unfunded liability | 65,000 | 76,000 |

In what range is the investment experience gain or loss during 1993?

(A) Loss of $200 or more
(B) Loss of less than $200
(C) $0 or gain of less than $200
(D) Gain of $200 but less than $400
(E) Gain of $400 or more

**Problem 6 - 30**

Normal retirement benefit: 50% of final 5-year average compensation.
Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

> Interest rate: 7% per year.
> Compensation increases: 5% per year.
> Preretirement deaths and terminations: None.
> Retirement age: 65, but not before 1/1/98.

Selected valuation results as of 1/1/93:

| | |
|---|---|
| Present value of future benefits | $ 500,000 |
| Unfunded liability | 100,000 |
| Value of assets | 100,000 |
| Present value of future compensation | 3,000,000 |
| Valuation compensation | 200,000 |

Contribution for 1993: $30,000 paid on 1/1/93.

There were no experience gains or losses from any source during 1993.

There were no terminations, deaths, or retirements during 1993, and there were no new entrants on or before 1/1/94.

In what range is the normal cost for 1994 as of 1/1/94?

(A) Less than $20,940
(B) $20,940 but less than $20,980
(C) $20,980 but less than $21,020
(D) $21,020 but less than $21,060
(E) $21,060 or more

## 6.4 Solutions to Problems

**Problem 6 - 28**

Key concept: In a plan with just one participant, the first year Normal Cost calculated under the Frozen Initial Liability method is identical to the Normal Cost calculated under the Entry Age Normal method.

Step I: Calculate the annuity purchase rate at age 65, using the value of the Accrued Liability determined under the Unit Credit method. The monthly accrued benefit as of 1/1/94 is:

$$\text{Accrued Benefit}_{94} = (20)(10 \text{ years of service}) = 200$$

Since the Unit Credit Accrued Liability is equal to the present value of the accrued benefit,

$$14{,}900 = (200)(12\ddot{a}_{65}^{(12)})(v^5)$$

$$\text{and } 12\ddot{a}_{65}^{(12)} = 104.49$$

Step 2: Calculate the Normal Cost under the Entry Age Normal method.

$$\text{Monthly Normal Retirement Benefit} = (20)(15 \text{ years of service}) = 300$$

$$NC_{EAN} = 300 \times 12\ddot{a}_{65}^{(12)}/\ddot{s}_{\overline{15|}}$$

$$= 300 \times 104.49/29.3243 = 1{,}069$$

Answer is A.

**Problem 6 - 29**

Key Concepts: Since there is only one participant and no deaths or terminations were assumed, the only experience gains or losses could have come from investments. Under the Frozen Initial Liability method, gains and losses are spread over future Normal Costs. Therefore, the difference between the expected and actual Present Value of Future Normal Costs must be the experience gain or loss.

Step I: Calculate the expected Present Value of Future Normal Costs.

$$\begin{aligned} PVFNC_{93} &= PVFB_{93} - Assets_{93} - UL_{93} \\ &= 122{,}000 - 65{,}000 = 57{,}000 \end{aligned}$$

$$\begin{aligned} NC_{93} &= 57{,}000 \div \ddot{a}_{\overline{14}|} \\ &= 57{,}000 \div 9.35765 = 6{,}091 \end{aligned}$$

$$\begin{aligned} ePVFNC_{94} &= (PVFNC_{93} - NC_{93}) \times 1.07 \\ &= (57{,}000 - 6{,}091) \times 1.07 = 54{,}473 \end{aligned}$$

Step II: Calculate the actual Present Value of Future Normal Costs.

$$\begin{aligned} PVFB_{94} &= PVFB_{93} \times 1.07 \\ &= 122{,}000 \times 1.07 = 130{,}540 \end{aligned}$$

$$\begin{aligned} PVFNC_{94} &= PVFB_{94} - Assets_{94} - UL_{94} \\ &= 130{,}540 - 27{,}500 - 48{,}500 = 54{,}540 \end{aligned}$$

Step III: Calculate the experience gain or loss by subtracting the actual Present Value of Future Normal Costs from the expected Present Value of Future Normal Costs.

$$\begin{aligned} \text{Experience Gain/Loss} &= ePVFNC_{94} - PVFNC_{94} \\ &= 54{,}473 - 54{,}540 = (67) \quad \text{Loss} \end{aligned}$$

Answer is B.

**Problem 6 - 30**

Key Concept: Since there were no gains or losses and the plan population has remained the same, the Normal Cost as of 1/1/94 is equal to the Normal Cost as of 1/1/93, increased by the 5% salary scale (in order to keep the Normal Cost a level percent of salary).

Step I: Calculate the Normal Cost as of 1/1/93.

$$\begin{aligned} NC_{93} &= (PVFB_{93} - Assets_{93} - UL_{93})/(PVFS_{93}/Salary_{93}) \\ &= (500{,}000 - 100{,}000 - 100{,}000)/(3{,}000{,}000/200{,}000) = 20{,}000 \end{aligned}$$

Step II: Calculate the Normal Cost as of 1/1/94.

$$\begin{aligned} NC_{94} &= NC_{93} \times (1 + s) \\ &= 20{,}000 \times (1.05) = 21{,}000 \end{aligned}$$

Answer is C.

# Chapter Seven

# Attained Age Normal Cost Method

# Chapter 7

# Attained Age Normal Cost Method

## 7.3 Problems

**Problem 7 - 14**

Plan effective date: 1/1/93
Normal retirement benefit: $20 per month for each year of service.
Preretirement death benefit: None.

Actuarial cost method: Attained age normal.

Actuarial assumptions:

> Interest rate: 7% per year.
> Preretirement terminations other than deaths: None.
> Retirement age: 65

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/92 |

Contribution for 1993: $1,125 paid on 12/31/93.

Selected commutation functions:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 49 | 101,241 | 1,238,268 |
| 50 | 94,135 | 1,137,027 |
| 65 | 28,610 | 263,044 |

In what range is the unfunded liability as of 1/1/94?

(A) Less than \$550
(B) \$550 but less than \$565
(C) \$565 but less than \$580
(D) \$580 but less than \$595
(E) \$595 or more

## 7.4 Solutions to Problems

**Problem 7 - 14**

Key Concept: The initial unfunded liability determined using the Attained Age Normal cost method is equal to the Accrued Liability determined using the Unit Credit cost method.

Step I: Calculate the initial unfunded liability as of 1/1/93. Since the sole participant was hired on 1/1/92, there is only one year of past service, so the accrued benefit is \$20 as of 1/1/93.

$$\begin{aligned}
\text{Unit Credit } AL_{93} &= (20)(12N_{65}^{(12)}/D_{49}) \\
&= (20)(12)(N_{65} - \tfrac{11}{24}D_{65})/D_{49} \\
&= (240)(263{,}044 - (\tfrac{11}{24})(28{,}610))/101{,}241 = 592
\end{aligned}$$

Step II: Calculate the Normal Cost as of 1/1/93.

$$\text{Monthly retirement benefit} = (20)(17 \text{ years of service}) = 340$$

$$\begin{aligned}
PVFB_{93} &= (\text{Benefit})(12N_{65}^{(12)})/D_{49} \\
&= (340)(12)(N_{65} - \tfrac{11}{24}D_{65})/D_{49} \\
&= (340)(12)(263{,}044 - (\tfrac{11}{24})(28{,}610))/101{,}241 = 10{,}072
\end{aligned}$$

$$\begin{aligned}
NC_{93} &= (PVFB_{93} - \text{Unit Credit } AL_{93})/\ddot{a}_{49:\overline{16|}} \\
&= (10{,}072 - 592)/\left[(N_{49} - N_{65})/D_{49}\right] \\
&= (10{,}072 - 592)/\left[(1{,}238{,}268 - 263{,}044)/101{,}241\right] = 984
\end{aligned}$$

Step III: Calculate the Unfunded Accrued Liability as of 1/1/94.

$$\begin{aligned}
UAL_{94} &= (AL_{93} - NC_{93})(1.07) - \text{Contrib}_{93} \\
&= (592 + 984)(1.07) - 1{,}125 = 561
\end{aligned}$$

Answer is B.

# Chapter Eight

# Miscellaneous

# Chapter 8

# Miscellaneous

## 8.2 Problems

**Problem 8 - 31**

Retirement benefit: $20,000 payable each 1/1 during the lifetime of the participant, with $10,000 payable each 1/1 after the participant's death to the surviving spouse.

Date of birth for sole participant Smith (retired): 1/1/23

Date of birth of Smith's spouse: 1/1/28

Smith's spouse dies during 1993.

Selected annuity values and probabilities of survival:

| | | |
|---|---|---|
| $\ddot{a}_{65} = 10.00$ | $\ddot{a}_{66} = 9.80$ | $p_{65} = .9735$ |
| $\ddot{a}_{70} = 9.00$ | $\ddot{a}_{65:70} = 8.00$ | $p_{70} = .9636$ |
| $\ddot{a}_{71} = 8.80$ | | |

In what range is the net mortality gain during 1993 as of 1/1/94?

(A) Less than $14,000
(B) $14,000 but less than $15,000
(C) $15,000 but less than $16,000
(D) $16,000 but less than $17,000
(E) $17,000 or more

**Problem 8 - 32**

Normal retirement benefit: $10 per month for each year of service.

Early retirement eligibility: Age 60.

Early retirement benefit: Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Probability of retirement (retirements are assumed to occur at beginning of year):

| Age x | |
|---|---|
| 60 | 20% |
| 61 | 20% |
| 62 | 50% |
| 63 | 0% |
| 64 | 0% |
| 65 | 100% |

Valuation data for sole participant (active as of 1/1/94):

| | |
|---|---|
| Date of birth | 1/1/44 |
| Date of hire | 1/1/84 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 9.815$ $\ddot{a}_{62}^{(12)} = 9.394$ $\ddot{a}_{64}^{(12)} = 8.958$

$\ddot{a}_{61}^{(12)} = 9.607$ $\ddot{a}_{63}^{(12)} = 9.178$ $\ddot{a}_{65}^{(12)} = 8.736$

In what range is the present value of future benefits as of 1/1/94?

(A) Less than $9,000
(B) $9,000 but less than $10,000
(C) $10,000 but less than $11,000
(D) $11,000 but less than $12,000
(E) $12,000 or more

## 8.3 Solutions to Problems

**Problem 8 - 31**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since Smith's spouse died during the year, the benefit is now a life annuity of $20,000 payable on 1/1 of each year.

$$\text{Actual } AL_{94} = (20{,}000)(\ddot{a}_{71})$$
$$= (20{,}000)(8.8) = 176{,}000$$

Step II: The expected Accrued Liability on 1/1/94 is equal to the Accrued Liability on 1/1/93 less the expected payment on 1/1/93, increased with interest. Since both Smith and Smith's spouse were alive on 1/1/93, $20,000 must have been paid on that date.

$$AL_{93} = (20{,}000)(\ddot{a}_{70}) + 10{,}000(\ddot{a}_{65} - \ddot{a}_{65:70})$$
$$= (20{,}000)(9) + 10{,}000(10 - 8) = 200{,}000$$

$$e AL_{94} = (AL_{93} - \text{Benefit paid } 1/1/93)(1 + i)$$

However, the interest assumption $i$ is not given, and must be calculated. Note that the formula for successive annuities due is

$$\ddot{a}_x = 1 + v p_x \, \ddot{a}_{x+1}$$

Therefore,
$$\ddot{a}_{65} = 1 + v p_{65} \, \ddot{a}_{66}$$
$$10 = 1 + v(.9735)(9.8)$$
$$i = .06$$

$$e AL_{94} = (200{,}000 - 20{,}000)(1.06) = 190{,}800$$

Step III: Calculate the mortality gain, which is the difference between the actual and expected Accrued Liabilities.

$$\text{Gain} = e AL_{94} - \text{actual } AL_{94}$$
$$= 190{,}800 - 176{,}000 = 14{,}800$$

Answer is B.

**Alternative Solution:**

The expected Accrued Liability can also be calculated using a prospective method. If both Smith and his spouse live, the accrued liability on 1/1/94 would be:

$$\text{AL (both live until 1/1/94)} = 20{,}000\ddot{a}_{71} + 10{,}000(\ddot{a}_{66} - \ddot{a}_{66:71})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individuals live from 1/1/93 to 1/1/94. Therefore, the expected Accrued Liability is:

$$e\text{AL}_{94} = 20{,}000\ddot{a}_{71}\, p_{70} + 10{,}000(\ddot{a}_{66}\, p_{65} - \ddot{a}_{66:71}\, p_{65}\, p_{70})$$

The value of $\ddot{a}_{71}$ and $\ddot{a}_{66:71}$ can be calculated using the formula for successive annuities due.

$$\ddot{a}_{70} = 1 + vp_{70}\ddot{a}_{71}$$
$$9 = 1 + v(.9636)\ddot{a}_{71}$$
$$\ddot{a}_{71} = 8.8$$

$$\ddot{a}_{65:70} = 1 + vp_{65}\, p_{70}\, \ddot{a}_{66:71}$$
$$8 = 1 + v(.9735)(.9636)\ddot{a}_{66:71}$$
$$\ddot{a}_{66:71} = 7.91$$

Therefore,

$$e\text{AL} = (20{,}000)(8.8)(.9636) + (10{,}000)\left[(9.8)(.9735) - (7.91)(.9735)(.9636)\right]$$
$$= 190{,}796$$

The difference between this result and the result from the retrospective method in Step II is due to rounding.

$$\text{Gain} = 190{,}796 - 176{,}000 = 14{,}796$$

Answer is B.

**Problem 8 - 32**

Step I: Calculate the probability of retirement at each age.

| Retirement Age | Probability of Retirement |
|---|---|
| 60 | $q_{60}^{(r)} = .2$ |
| 61 | $(p_{60}^{(r)})(q_{61}^{(r)}) = (.8)(.2) = .16$ |
| 62 | $(p_{60}^{(r)})(p_{61}^{(r)})(q_{62}^{(r)}) = (.8)(.8)(.5) = .32$ |
| 65 | $(p_{60}^{(r)})(p_{61}^{(r)})(p_{62}^{(r)})(q_{65}^{(r)}) = (.8)(.8)(.5)(1) = .32$ |

Step II: Calculate the retirement benefit at each retirement age.

| | |
|---|---|
| Benefit at age 60: | (10)(20 years of service)(1 - (.03)(5 years)) = \$170.00 |
| Benefit at age 61: | (10)(21 years of service)(1 - (.03)(4 years)) = \$184.80 |
| Benefit at age 62: | (10)(22 years of service)(1 - (.03)(3 years)) = \$200.20 |
| Benefit at age 65: | (10)(25 years of service) = \$250.000 |

Step III: Calculate the Present Value of Future Benefits.

$$\begin{aligned}PVFB_{94} &= (170)(12\ddot{a}_{60}^{(12)})(v^{10})(.2) + (184.80)(12\ddot{a}_{61}^{(12)})(v^{11})(.16) + \\ &\quad (200.20)(12\ddot{a}_{62}^{(12)})(v^{12})(.32) + (250)(12\ddot{a}_{65}^{(12)})(v^{15})(.32) \\ &= 2{,}036 + 1{,}619 + 3{,}207 + 3{,}040 = 9{,}902\end{aligned}$$

Answer is B.

# 1996 Supplement to

# ACTUARIAL COST METHODS

# A REVIEW

## 1996 Supplement to

# ACTUARIAL COST METHODS A REVIEW

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## Guide to Problems

The problems herein were taken from the May, 1995 EA-1(B) Examination. For those students wishing to use this material in conjunction with this examination, the following cross reference list should be helpful. Problems and solutions for the May, 1994 EA-1(B) Examination are contained in the 1995 Supplement. The cross reference to those problems is provided here for your convenience.

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| 20 | 1 - 46 | 4 - 61 |

## Glossary of Abbreviations

Since many actuarial terms are lengthy, we will use abbreviations in equations and in the text for the sake of brevity. Listed below are many of the abbreviations which will be seen here:

| | |
|---|---|
| $\ddot{a}_x$ | Present value of an annuity due at age x |
| AA, aa or x | Attained Age |
| Assets | Actuarial Value of Assets |
| AL | Accrued Liability |
| ATA | Average Temporary Annuity |
| B(x) | Benefit at age x |
| BP | Benefit Payments |
| $C_t$ | Contributions for year t |
| CV | Cash Value |
| DB | Death Benefit |
| *e*Assets | Expected Value of Assets |
| *e*AL | Expected Accrued Liability |
| *e*PVFB | Expected Present Value of Future Benefits |
| *e*PVFNC | Expected Present Value of Future Normal Cost |
| *e*UAL | Expected Unfunded Accrued Liability |
| *e*UL | Expected Unfunded Liability |
| EA or ea | Entry Age |
| EAN | Entry Age Normal |
| ERB | Early Retirement Benefit |
| FIL | Frozen Initial Liability |
| $I_{BP}$ | Interest on Benefit Payments |
| $I_c$ | Interest on contributions |
| ILP | Individual Level Premium |
| J&S or J+S | Joint and Survivor |
| NC | Normal Cost |
| NRA | Normal Retirement Age |
| NRD | Normal Retirement Date |
| PVAB | Present Value of Accrued Benefits |
| PVFB | Present Value of Future Benefits |
| PVFNC | Present Value of Future Normal Costs |
| PVFS | Present Value of Future Salary |
| PVFY | Present Value of Future Years |
| RA, ra or y | Retirement Age |
| S, Salary | Salary or Compensation |
| TA | Temporary Annuity |
| UAL | Unfunded Accrued Liability |
| UC | Unit Credit |
| UL | Unfunded Liability |

# Introduction

This Supplement is designed to assist students who are preparing for the EA-1(B) Examination, Basic Pension Mathematics, which is offered jointly by the American Society of Pension Actuaries, the Society of Actuaries and the Joint Board for the Enrollment of Actuaries. The problems and solutions in this Supplement are from the May, 1995, EA-1(B) Examination. Material is arranged according to the recognized actuarial cost methods, with the last section dealing with those principles common to all cost methods.

**Actuarial Cost Methods, A Review**, in conjunction with the 1995 and 1996 supplements, provides a complete, up to date review of the principles of the various pension cost methods and related actuarial topics.

# Chapter One

# Unit Credit
# Cost Method
# (Accrued Benefit)

# Chapter 1

# Unit Credit Cost Method (Accrued Benefit)

## 1.4 Problems

**Problem 1 - 47**

Normal retirement benefit: $20 per month for each year of service.
Preretirement death benefit: None.
Termination benefit: Vested accrued benefit, payable at normal retirement date.
Vesting eligibility: 100% after 5 years.
Actuarial cost method: Unit credit.

Actuarial assumptions:

Interest rate: 6% per year.
Preretirement terminations other than deaths and withdrawals: None.
Retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/62 |
| Date of hire | 1/1/92 |

Selected values from preretirement single-decrement tables:

| Age x | $l_x'^{(d)}$ | $l_x'^{(w)}$ |
|---|---|---|
| 30 | 10,000 | 10,000 |
| 31 | 9,985 | 9,500 |
| 32 | 9,969 | 9,000 |
| 33 | 9,952 | 8,500 |
| 34 | 9,934 | 8,000 |
| 35 | 9,915 | 7,500 |
| 65 | 7,900 | 2,000 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 9.439$$

In what range is the accrued liability for preretirement vested termination benefits as of 1/1/95?

(A) Less than \$200
(B) \$200 but less than \$400
(C) \$400 but less than \$600
(D) \$600 but less than \$800
(E) \$800 or more.

**Problem 1 - 48**

Normal retirement benefit: 2% of final year's compensation for each year of service.

Early retirement eligibility: Age 55.

Early retirement benefit: Accrued benefit, reduced by 1/15 for each of the first 5 years and 1/30 for each of the next 5 years by which the benefit commencement date precedes the normal retirement date.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 9% per year.
Compensation increases: 3% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Date of birth for selected participants (all active as of 12/31/94):

| Smith | Brown | Green |
|---|---|---|
| 1/1/39 | 1/1/35 | 1/1/32 |

Smith, Brown, and Green all retire and elect to commence receiving benefits as of 1/1/95.

Selected annuity values:

$\ddot{a}_{56}^{(12)} = 9.84$ $\ddot{a}_{63}^{(12)} = 8.71$

$\ddot{a}_{60}^{(12)} = 9.25$ $\ddot{a}_{65}^{(12)} = 8.39$

Which, if any, of the participants generated an experience gain as of 1/1/95 due to his retirement?

(A) Smith only
(B) Brown only
(C) Smith and Brown only
(D) Brown and Green only
(E) The correct answer is not given by (A), (B), (C), or (D) above.

## 1.5 Solutions to Problems

**Problem 1 - 47**

The benefit to be used to determine the Accrued Liability in the Unit Credit method is the accrued benefit as of the beginning plan year.

$$\text{Accrued Benefit}_{1/1/95} = 20 \times 3 \text{ years of service} = 60$$

This benefit will not become vested until 1/1/97, when the participant will be 35 years old.

The Accrued Liability to be determined is the present value of the Accrued Benefit at 1/1/95, payable if the participant terminates employment between ages 35 and 65. Note that the participant must still survive to age 65 in order to receive a benefit.

$$\begin{aligned}
\text{AL(due to vested termination)} &= (60)(12\ddot{a}_{65}^{(12)})({}_{32}p_{33}'^{(d)})({}_{2}p_{33}'^{(w)})({}_{30}q_{35}'^{(w)})(v^{32}) \\
&= (60)(12\ddot{a}_{65}^{(12)})\frac{(l_{65}'^{(d)})(l_{35}'^{(w)})(l_{35}'^{(w)} - l_{65}'^{(w)})(v^{32})}{(l_{33}'^{(d)})(l_{33}'^{(w)})(l_{35}'^{(w)})} \\
&= (60)(12)(9.439)(7900/9952)(7500/8500) \times \\
&\qquad ((7500\text{-}2000)/7500)(.155) = 541
\end{aligned}$$

Answer is C.

**Problem 1 - 48**

Key Concept: An experience loss is created if the actual liability due to the early retirement election exceeds the Accrued Liability under the funding method. We must look at each participant individually.

Smith: Projected Unit Credit Accrued Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1.03)^9(\ddot{a}_{65}^{(12)})v^9$$
$$= (.02)(\text{Years of service})(\text{Salary})(5.04)$$

Early Retirement Liability

$$= (.02)(\text{Years of service})(\text{Salary})(1 - 5/15 - 4/30)(\ddot{a}_{56}^{(12)})$$
$$= (.02)(\text{Years of service})(\text{Salary})(5.248)$$

Experience Loss, since early retirement yields a larger liability.

Brown: Projected Unit Credit Accrued Liability

$= (.02)(\text{Years of service})(\text{Salary})(1.03)^5(\ddot{a}_{65}^{(12)})v^5$
$= (.02)(\text{Years of service})(\text{Salary})(6.32)$

Early Retirement Liability

$= (.02)(\text{Years of service})(\text{Salary})(1 - 5/15)(\ddot{a}_{60}^{(12)})$
$= (.02)(\text{Years of service})(\text{Salary})(6.17)$

Experience Gain, since early retirement yields a smaller liability.

Green: Projected Unit Credit Accrued Liability

$= (.02)(\text{Years of service})(\text{Salary})(1.03)^2(\ddot{a}_{65}^{(12)})v^2$
$= (.02)(\text{Years of service})(\text{Salary})(7.49)$

Early Retirement Liability

$= (.02)(\text{Years of service})(\text{Salary})(1 - 2/15)(\ddot{a}_{63}^{(12)})$
$= (.02)(\text{Years of service})(\text{Salary})(7.55)$

Experience Loss, since early retirement yields a larger liability.

Answer is B.

# Chapter Two

# Aggregate Cost Method

# Chapter 2

# Aggregate Cost Method

## 2.4 Problems

### Problem 2 - 39

Plan effective date: 1/1/95.
Normal retirement benefit: $10 per month for each year of service since date of hire.
Preretirement death benefit: None.
Actuarial cost method: Aggregate.

Actuarial assumptions:

> Interest rate: 7% per year.
> Preretirement terminations other than deaths: None.
> Retirement age: 65.

Selected valuation data for only participants (all active as of 1/1/95):

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/60 | 1/1/60 | 1/1/50 |
| Date of hire | 1/1/81 | 1/1/81 | 1/1/71 |

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 35 | 894,190 | 12,364,650 |
| 45 | 445,008 | 5,690,850 |
| 65 | 94,414 | 868,052 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than $1,600
(B) $1,600 but less than $1,700
(C) $1,700 but less than $1,800
(D) $1,800 but less than $1,900
(E) $1,900 or more

**Problem 2 - 40**

Mandatory employee contribution: 0.5% of compensation, paid on 1/1.
Actuarial cost method: Aggregate.

Present value of future benefits (excluding any benefits due to voluntary employee contributions) as of 1/1/95:

| | |
|---|---|
| Retirement benefits | $1,000,000 |
| Return of mandatory employee contributions at death or termination | 20,000 |
| Employer-provided termination benefits | 80,000 |

Present value of future compensation as of 1/1/95 $4,800,000

Total valuation compensation for 1995: $600,000
Mandatory employee contributions for 1995: $3,000 paid on 1/1/95.
Voluntary employee contributions for 1995: $2,000 paid on 1/1/95.

Value of assets as of 12/31/94:

| | |
|---|---|
| Mandatory employee contributions with interest | $30,000 |
| Voluntary employee contributions with interest | 20,000 |
| Other assets | 150,000 |
| Total assets | 200,000 |

In what range is the employer normal cost for 1995 as of 1/1/95?

(A) Less than $109,500
(B) $109,500 but less than $111,000
(C) $111,000 but less than $112,500
(D) $112,500 but less than $114,000
(E) $114,000 or more

**Problem 2 - 41**

Normal retirement benefit:

Before 1995: \$30 per month for each year of service.
After 1994: \$35 per month for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

As of 1/1/94, all participants were active and under age 63.

Selected valuation results as of 1/1/94:

| | |
|---|---|
| Present value of future benefits | \$900,000 |
| Value of assets | 500,000 |
| Average present value of future working lifetime of active participants | 11 |

The contribution for 1994 was paid on 1/1/94 in an amount equal to the normal cost for 1994 as of 1/1/94.

There were no deaths, terminations, or retirements during 1994, and there are no new participants on 1/1/95.

There were no investment experience gains or losses during 1994.

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than \$43,300
(B) \$43,300 but less than \$47,300
(C) \$47,300 but less than \$51,300
(D) \$51,300 but less than \$54,300
(E) \$54,300 or more

**Problem 2 - 42**

Normal retirement benefit: 50% of final year's compensation.
Actuarial cost method: Aggregate.
Assumed compensation increases: 3% per year.

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | $ 149,000 |
| Value of assets | 20,000 |
| Present value of future compensation | 1,200,000 |
| Annual compensation | 150,000 |

Investment gain during 1994: $1,600.
Actual compensation increases during 1994: 10%.
There were no other experience gains or losses during 1994.

There have never been any inactive participants.

In what range is the change in the normal cost for 1995 as of 1/1/95 due to experience gains and losses?

(A) Less than $1,000
(B) $1,000 but less than $1,100
(C) $1,100 but less than $1,200
(D) $1,200 but less than $1,300
(E) $1,300 or more

**Problem 2 - 43**

| | |
|---|---|
| Actuarial cost method: | Aggregate. |
| Assumed interest rate: | 7% per year. |

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | $1,200,000 |
| Present value of future compensation | 9,500,000 |
| Annual compensation | 750,000 |

The plan's assets earned a 4% return in 1994; the resulting investment experience loss increased the normal cost for 1995 as of 1/1/95 by $100.

There were no contributions or disbursements during 1994.

In what range is the value of assets as of 1/1/95?

(A) Less than $34,000
(B) $34,000 but less than $37,000
(C) $37,000 but less than $40,000
(D) $40,000 but less than $43,000
(E) $43,000 or more

## 2.5 Solutions to Problems

**Problem 2 - 39**

Step I: Calculate the retirement benefit. Each participant will have a total of 44 years of service at retirement.

$$\text{Benefit} = \$10 \times 44 \text{ years of service} = 440$$

Step II: Calculate the Present Value of Future Benefits. Note that the calculation is identical for Smith and Brown, since they each have the same date of birth.

$$\begin{aligned} \text{Smith/Brown PVFB} &= \$440 \times 12\ddot{a}_{65}^{(12)} \times D_{65}/D_{35} \\ &= \$440 \times 12 \times 8.74 \times 94{,}414/894{,}190 = 4{,}873 \end{aligned}$$

$$\begin{aligned} \text{Green PVFB} &= \$440 \times 12\ddot{a}_{65}^{(12)} \times D_{65}/D_{45} \\ &= \$440 \times 12 \times 8.74 \times 94{,}414/445{,}008 = 9{,}791 \end{aligned}$$

$$\text{Total PVFB} = 4{,}873 + 4{,}873 + 9{,}791 = 19{,}537$$

Step III: Calculate the Average Temporary Annuity. Again, the calculation is identical for Smith and Brown.

$$\begin{aligned} \text{Smith/Brown TA} &= (N_{35} - N_{65})/D_{35} \\ &= (12{,}364{,}650 - 868{,}052)/894{,}190 = 12.857 \end{aligned}$$

$$\begin{aligned} \text{Green TA} &= (N_{45} - N_{65})/D_{45} \\ &= (5{,}690{,}850 - 868{,}052)/445{,}008 = 10.8376 \end{aligned}$$

$$\text{Average TA} = (12.857 + 12.857 + 10.8376)/3 = 12.1839$$

Step IV: Calculate the Normal Cost.

$$\begin{aligned} \text{NC} &= \text{Total PVFB/ATA} \\ &= 19{,}537/12.1839 = 1{,}604 \end{aligned}$$

Answer is B.

**Problem 2 - 40**

Key Concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' mandatory contributions. The actual amount of employees' voluntary contributions and the assets associated with the employees' voluntary contributions are irrelevant as they are kept as a separate account.

$$PVFB = 1{,}000{,}000 + 20{,}000 + 80{,}000 = 1{,}100{,}000$$

$$\text{Assets} = 30{,}000 + 150{,}000 = 180{,}000$$

$$\begin{aligned} PVFNC &= PVFB - \text{Assets} \\ &= 1{,}100{,}000 - 180{,}000 = 920{,}000 \end{aligned}$$

$$\begin{aligned} NC\,\% &= PVFNC \div PVFS \\ &= 920{,}000 \div 4{,}800{,}000 = 19.1667\% \end{aligned}$$

$$\begin{aligned} NC &= NC\% \times \text{Salary} \\ &= 19.1667\% \times 600{,}000 = 115{,}000 \end{aligned}$$

$$\begin{aligned} \text{Employer } NC &= NC - \text{Mandatory employee contributions} \\ &= 115{,}000 - 3{,}000 = 112{,}000 \end{aligned}$$

Answer is C.

**Problem 2 - 40 (Alternative Solution)**

Key Concept: Present Value of Future Benefits equals Present Value of Future Employer Costs plus Present Value of Future Employee Costs plus Assets.

$$PVFB = PVFNC_{ER} + PVFNC_{EE} + \text{Assets}$$

$$\text{Where } PVFNC_{ER} = NC\%_{ER} \times PVFS$$

$$\text{And } PVFNC_{EE} = .5\% \times PVFS$$

We will use the above relationship to solve for $NC\%_{ER}$:

$$\begin{aligned} 1{,}100{,}000 &= (NC\%_{ER})(4{,}800{,}000) + (.005)(4{,}800{,}000) + 180{,}000 \\ 896{,}000 &= NC\%_{ER} \times 4{,}800{,}000 \end{aligned}$$

$$NC\%_{ER} = 18.6667\%$$
$$NC_{ER} = (.186667)(600{,}000) = 112{,}000$$

Answer is C.

**Problem 2 - 40 (Alternative Solution)**

The following equation of value can be written:

$$\text{Total PVFB} = PVFNC_{ER} + PVFNC_{EE} + \text{Assets}$$

The total PVFB includes retirement benefits, return of mandatory employee contributions, and employer-provided termination benefits.

The Assets include both employer and mandatory employee contributions with interest.

Also, $PVFNC_{EE} = (.005)(PVFS)$
$= (.005)(4{,}800{,}000) = 24{,}000$

So, $1{,}100{,}000 = PVFNC_{ER} + 24{,}000 + 180{,}000$
and $PVFNC_{ER} = 896{,}000$

$$\text{NC Rate}_{ER} = PVFNC_{ER}/PVFS$$
$$= 896{,}000/4{,}800{,}000 = .186667$$

$$NC_{ER} = (\text{NC Rate}_{ER})(\text{Salary})$$
$$= (.186667)(600{,}000) = 112{,}000$$

Answer is C.

**Problem 2 - 41**

Key Concept: The Normal Cost under the Aggregate method remains the same each year (since the benefit formula is a flat dollar amount), unless there are gains or losses. Since the benefit formula increased in 1995, the 1995 Normal Cost is equal to the 1994 Normal Cost plus an amount to fund the increased benefit.

Step I: Calculate the increase in the PVFB due to the plan amendment. Note that the benefit increased by a factor of 5/30.

$$\text{Expected } PVFB_{95} = PVFB_{94} \times (1 + i)$$

$$= 900{,}000 \times 1.07 = 963{,}000$$

$$\Delta PVFB_{95} = 963{,}000 \times 5/30 = 160{,}500$$

Step II: Calculate the Temporary Annuity as of 1/1/95.

$$\begin{aligned} TA_{95} &= (TA_{94} - 1)(1 + i) \\ &= (11 - 1)(1.07) = 10.7 \end{aligned}$$

Step III: Calculate the increase in Normal Cost for 1995.

$$\begin{aligned} \Delta NC_{95} &= \Delta PVFB_{95} / TA_{95} \\ &= 160{,}500/10.7 = 15{,}000 \end{aligned}$$

Step IV: Calculate the Normal Cost for 1994.

$$\begin{aligned} NC_{94} &= (PVFB_{94} - Assets_{94})/TA_{94} \\ &= (900{,}000 - 500{,}000)/11 = 36{,}364 \end{aligned}$$

Step V: Calculate the total Normal Cost for 1995.

$$\begin{aligned} NC_{95} &= NC_{94} + \Delta NC_{95} \\ &= 36{,}364 + 15{,}000 = 51{,}364 \end{aligned}$$

Answer is D.

**Problem 2 - 41 (Alternative Solution)**

The total Normal Cost for 1995 could have been calculated directly, as follows.

The total PVFB for 1995 is:

$$\begin{aligned} PVFB_{95} &= \text{Expected } PVFB_{95} \times 35/30 \quad \text{(due to amendment)} \\ &= 963{,}000 \times 35/30 = 1{,}123{,}500 \end{aligned}$$

The Assets as of 1/1/95 are:

$$\begin{aligned} Assets_{95} &= (Assets_{94} + Contribution_{94})(1 + i) \\ &= (500{,}000 + 36{,}364)(1.07) = 573{,}909 \end{aligned}$$

And,

$$NC_{95} = (PVFB_{95} - Assets_{95})/TA_{95}$$
$$= (1{,}123{,}500 - 573{,}909)/10.7 = 51{,}364$$

Answer is D.

**Problem 2 - 42**

Step I: Calculate the 1995 Normal Cost.

| | | |
|---|---|---|
| Present Value of Future Benefits | | \$149,000 |
| Less Value of Assets | | (20,000) |
| Equals Present Value of Future Normal Costs | | 129,000 |
| Divided by Present Value of Future Salary | | ÷1,200,000 |
| Equals Normal Cost Rate | | 10.75% |
| Multiplied by Annual Salary | × 150,000 | |
| Equals Normal Cost | | 16,125 |

Step II: Calculate the expected 1995 Normal Cost. Since compensation increased by 10% instead of the expected 3%, the Present Value of Future Benefits increased by a ratio of 1.1/1.03.

$$PVFB = e\text{PVFB} \times (1.1/1.03)$$
$$149{,}000 = e\text{PVFB} \times (1.1/1.03)$$
$$e\text{PVFB} = 139{,}518$$

$$e\text{Assets} = 20{,}000 - 1{,}600 = 18{,}400$$

$$\text{Expected NC} = (e\text{PVFB} - e\text{Assets})/(\text{PVFS/Salary})$$
$$= (139{,}518 - 18{,}400)/(1{,}200{,}000/150{,}000) = 15{,}140$$

$$\text{Increase in NC} = 16{,}125 - 15{,}140 = 985$$

Answer is A.

Note: The ratio of PVFS/Salary is unchanged due to the 10% salary increase even though the individual amounts (PVFS and Salary) have changed since both have increased by the ratio 1.1/1.03.

**Problem 2 - 42 (Alternative Solution)**

The solution can be found more quickly by direct calculation of the experience gain/loss.

$$\text{Expected Compensation} = (150{,}000/1.1) \times 1.03 = 140{,}455$$

$$\begin{aligned} e\text{PVFB} &= (\text{PVFB}) \times (\text{Expected Compensation/Actual Compensation}) \\ &= (149{,}000) \times (140{,}455/150{,}000) = 139{,}519 \end{aligned}$$

The loss due to the 10% compensation increase can be calculated as:

$$\begin{aligned} \text{Loss} &= \text{Actual PVFB} - e\text{PVFB} \\ &= 149{,}000 - 139{,}519 = 9{,}481 \end{aligned}$$

The total experience loss is

$$\begin{aligned} \text{Total Loss} &= \text{Compensation loss} - \text{Investment gain} \\ &= 9{,}481 - 1{,}600 = 7{,}881 \end{aligned}$$

The difference in the Normal Cost is

$$\begin{aligned} \Delta\text{NC} &= (\text{Experience loss})/(\text{PVFS/Salary}) \\ &= (7{,}881)/(1{,}200{,}000/150{,}000) = 985 \end{aligned}$$

Answer is A.

**Problem 2 - 43**

The increased Normal Cost also increased the Present Value of Future Normal Costs by

$$100 \times (\text{PVFS/Salary}) = 100 \times (9{,}500{,}000/750{,}000) = 1{,}267$$

This amount also represents the asset loss. Since the assets were expected to earn 7% and only earned 4%, the loss amounted to 3% of the 1/1/94 assets.

$$\begin{aligned} 1{,}267 &= (.03)(\text{Assets}_{1/1/94}) \\ \text{Assets}_{1/1/94} &= 42{,}233 \end{aligned}$$

Therefore,

$$\text{Assets}_{1/1/95} = 42{,}233 \times 1.04 = 43{,}922$$

Answer is E.

# Chapter Three

# Individual Aggregate Cost Method

# Chapter 3

# Individual Aggregate Cost Method

## 3.3 Problems

**Problem 3 - 16**

Lump sum death benefit: $100,000, payable at end of year of death.

Actuarial cost method for all benefits: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Probability of mortality at each age from 50 through 64: .005
Preretirement terminations other than deaths: None.
Retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/45 |
| Projected monthly retirement benefit | $5,000 |

Value of assets as of 1/1/95: $71,500.

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1995 as of 1/1/95?

(A) Less than $11,500
(B) $11,500 but less than $11,550
(C) $11,550 but less than $11,600
(D) $11,600 but less than $11,650
(E) $11,650 or more

**Problem 3 - 17**

Normal retirement benefit: 60% of final year's compensation.

Actuarial cost method: Individual aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 3% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Valuation data for participant Smith (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/40 |
| 1994 compensation | $90,000 |
| Allocated assets as of 1/1/95 | 18,000 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.74$$

In what range is the normal cost attributable to Smith for 1995 as of 1/1/95?

(A) Less than $35,000
(B) $35,000 but less than $36,500
(C) $36,500 but less than $38,000
(D) $38,000 but less than $39,500
(E) $39,500 or more

## 3.4 Solutions to Problems

**Problem 3 - 16**

Key Concept: The Present Value of Future Benefits is equal to the Present Value of the retirement benefits plus the Present Value of the death benefits.

Step I: Calculate the Present Value of the Retirement Benefit:

$$\text{PV of Retirement Benefit} = (5{,}000)(12\ddot{a}_{65}^{(12)})(v^{15})({}_{15}p_{50})$$
$$= (5{,}000)(12)(8.74)(.3624)(.9276) = 176{,}283$$

Note that since $q_x = .005$ at each age from 50 through 64,
$p_x = .995$ at each age from 50 through 64.

Step II: Calculate the Present Value of the death benefit.

$$\text{PV of death benefit} = (100{,}000)\left[(q_{50})(v) + ({}_{1|}q_{50})(v^2) + \ldots + ({}_{14|}q_{50})(v^{15})\right]$$
$$= (100{,}000)\left[(.005)(v) + (.995)(.005)(v^2) + \ldots + (.995)^{14}(.005)(v^{15})\right]$$
$$= (100{,}000)(.005)/(.995)\left[.995v + (.995v)^2 + \ldots + (.995v)^{15}\right]$$
$$= 502.51(a_{\overline{15}|j}) \qquad \text{where } j = 1.07/.995 - 1 = .07538$$
$$= (502.51)(8.8063) = 4{,}425$$

Step III: Calculate the Normal Cost.

$$\text{PVFNC} = \text{PVFB} - \text{Assets}$$

$$\text{PVFB} - \text{Assets} = \text{NC}(1 + vp_{50} + v^2{}_2p_{50} + \ldots + v^{14}{}_{14}p_{50})$$

$$176{,}283 + 4{,}425 - 71{,}500 = \text{NC}(1 + .995v + (.995v)^2 + \ldots + (.995v)^{14})$$

$$(\text{NC})(\ddot{a}_{\overline{15}|j}) = 109{,}208$$
$$\text{NC} = 109{,}208/9.4701 = 11{,}532$$

Answer is B.

**Problem 3 - 16 (additional information)**

The correct numerical solution is also obtained if the death benefit is funded using a one-year term cost.

The Normal Cost associated with the death benefit would be

$$\begin{aligned} NC_{\text{Death Benefit}} &= 100{,}000 v q_{50} \\ &= (100{,}000)(.9346)(.005) = 467 \end{aligned}$$

The Normal Cost associated with the retirement benefit would be

$$\begin{aligned} NC_{RB} &= (PVFB_{RB} - \text{Assets})/\ddot{a}_{\overline{13}|j} \\ &= (176{,}283 - 71{,}500)/9.4701 = 11{,}065 \end{aligned}$$

The total Normal Cost would be

$$NC = 11{,}065 + 467 = 11{,}532$$

Note that while coincidentally this produces the same result as the previous solution, the methodology is incorrect since the problem states that Individual Aggregate is the cost method for <u>all</u> benefits.

**Problem 3 - 17**

Step I: Calculate the final Salary.

$$\text{Salary} = 90{,}000 \times (1.03)^{10} = 120{,}952$$

Step II: Calculate the Present Value of Future Benefits.

$$\begin{aligned} PVFB &= (120{,}952)(.6)(\ddot{a}_{65}^{(12)})(v^{10}) \\ &= (120{,}952)(.6)(8.74)(.5083) = 322{,}401 \end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned} NC &= (PVFB - \text{Assets})/\ \ddot{a}_{\overline{10}|j} \qquad \text{where } j = 1.07/1.03 - 1 = .038835 \\ &= (322{,}401 - 18{,}000)/8.475 = 35{,}918 \end{aligned}$$

Answer is B.

# Chapter Four

# Entry Age Normal Cost Method

# Chapter 4

# Entry Age Normal Cost Method

## 4.5 Problems

**Problem 4 - 58**

Normal retirement benefit:

Before 1995: $150 per year for each year of service.
After 1994: $200 per year for each year of service.

Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

Interest rate: 7 % per year.
Compensation increases: None.
Preretirement deaths and terminations: None.
Retirement age: 65.

Valuation data and selected valuation results for only participants (both active as of 1/1/95):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/65 | 1/1/40 |
| Date of hire | 1/1/90 | 1/1/80 |
| Normal cost per $1,000 of projected annual benefit | $43.04 | $135.85 |

In what range is the increase in the accrued liability as of 1/1/95 due to the plan amendment?

(A) Less than $4,500
(B) $4,500 but less than $5,000
(C) $5,000 but less than $5,500
(D) $5,500 but less than $6,000
(E) $6,000 or more

**Problem 4 - 59**

Normal retirement benefit:

Before 1995: 30.0% of final year's compensation.
After 1994: 37.5% of final year's compensation.

Actuarial cost method: Individual entry age normal.
Assumed compensation increases: None.

Selected valuation results for sole participant (active as of 1/1/95), before plan amendment:

| | |
|---|---|
| Present value of future benefits | $41,000 |
| Unfunded accrued liability | 8,000 |
| Value of assets | 10,000 |
| Present value of future compensation | 460,000 |

Increase in annual projected retirement benefit due to plan amendment: $3,000.

In what range is the normal cost for 1995 as of 1/1/95 after the plan amendment?

(A) Less than $2,200
(B) $2,200 but less than $2,700
(C) $2,700 but less than $3,200
(D) $3,200 but less than $3,700
(E) $3,700 or more

**Problem 4 - 60**

Actuarial cost method: Entry age normal (level dollar amount).
Assumed retirement age: 65.

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/50 |
| Date of hire | 1/1/80 |

Projected monthly benefit as of 1/1/95: $2,500.

Selected commutation functions and annuity value:

| Age x | $D_x$ | $N_x$ |
|---|---|---|
| 30 | 1,261,611 | 17,887,840 |
| 45 | 445,008 | 5,690,850 |
| 65 | 94,414 | 868,052 |

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability for retirement benefits as of 1/1/95?

(A) Less than $35,000
(B) $35,000 but less than $37,000
(C) $37,000 but less than $39,000
(D) $39,000 but less than $41,000
(E) $41,000 or more

**Problem 4 - 61**

| | |
|---|---|
| Normal retirement benefit: | $10 per month for each year of service. |
| Early retirement benefit: | Accrued benefit, reduced by 0.5 % for each month by which the benefit commencement date precedes the normal retirement date. |
| Normal form of payment: | Fully subsidized 100% joint and survivor annuity for married participants; life annuity for unmarried participants. |
| Actuarial cost method: | Individual entry age normal. |

Actuarial assumptions:

Interest rate: 7% per year.
Preretirement deaths and terminations: None.
Retirement age: 62.
Marital characteristics: 80% of participants at the assumed retirement age are married with a spouse the same age.

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/35 |
| Date of hire | 1/1/85 |
| Date of retirement | 12/31/94 |
| Date of benefit commencement | 1/1/95 |
| Spouse's date of birth | 1/1/35 |

Selected annuity values:

$\ddot{a}_{60}^{(12)} = 10.0$ $\ddot{a}_{62}^{(12)} = 9.0$ $\ddot{a}_{65}^{(12)} = 8.0$

$\ddot{a}_{60:60}^{(12)} = 8.0$ $\ddot{a}_{62:62}^{(12)} = 7.5$ $\ddot{a}_{65:65}^{(12)} = 7.0$

In what range is the absolute value of the experience gain or loss as of 1/1/95 due to Smith's early retirement?

(A) Less than \$450
(B) \$450 but less than \$900
(C) \$900 but less than \$1,350
(D) \$1,350 but less than \$1,800
(E) \$1,800 or more

## 4.6 Solutions to Problems

### Problem 4 - 58

Step I: Calculate the Accrued Liability per \$1,000 of projected annual benefit for each participant.

$$\text{Smith: } 43.04\,\ddot{s}_{\overline{5}|} = (43.04)(6.1533) = 264.84$$
$$\text{Brown: } 135.85\,\ddot{s}_{\overline{15}|} = (135.85)(26.8881) = 3{,}652.75$$

Step II: Calculate the increase in projected annual benefit for each participant. Note that the benefit increases by \$50 per year for each year of service.

$$\text{Smith: } 50 \times 40 \text{ years of service} = 2{,}000$$
$$\text{Brown: } 50 \times 25 \text{ years of service} = 1{,}250$$

Step III: Calculate the increase in the Accrued Liability.

$$\begin{aligned}\text{Increase in AL} &= (264.84)(2{,}000/1{,}000) + (3{,}652.75)(1{,}250/1{,}000)\\ &= 529.68 + 4{,}565.94 = 5{,}095.62\end{aligned}$$

Answer is C.

### Problem 4 - 59

Key Concept: Since the Entry Age Normal Normal Cost is independent of the plan assets, the Normal Cost after the amendment is proportionally increased over the Normal Cost before the amendment.

We can use the relationship PVFB = AL + PVFNC to determine the Normal Cost.

Since PVFNC = NC × (PVFS/Salary), we need to find the salary for the 1995 valuation. The benefit formula increased by 25%, so the benefit before the increase was 12,000 (3,000 × 4).

$$\text{Salary}_{95} = 12{,}000/.3 = 40{,}000$$

So, $$\text{PVFNC} = \text{NC} \times (460{,}000/40{,}000)$$

Also $$\text{AL} = \text{UAL} + \text{Assets}$$

Therefore, $PVFB = AL + PVFNC$

$$41{,}000 = (8{,}000 + 10{,}000) + NC \times (460{,}000/40{,}000)$$

NC (before amendment) = 2,000
and NC (after amendment) = $2{,}000 \times 37.5/30 = 2{,}500$

Answer is B.

**Problem 4 - 60**

Step I: Calculate the Normal Cost.

$$\begin{aligned} NC &= (2{,}500)(12\ddot{a}_{65}^{(12)})/\ddot{s}_{30:\overline{35}|} \\ &= (2{,}500)(12\ddot{a}_{65}^{(12)})/[(N_{30} - N_{65})/D_{65}] \\ &= (2{,}500)(12)(8.74)/[(17{,}887{,}840 - 868{,}052)/94{,}414] = 1{,}454.50 \end{aligned}$$

Step II: Calculate the Accrued Liability.

$$\begin{aligned} AL &= NC \times \ddot{s}_{30:\overline{15}|} \\ &= 1454.50 \times [(N_{30} - N_{45})/D_{45}] \\ &= 1454.50 \times [(17{,}887{,}840 - 5{,}690{,}850)/445{,}008] = 39{,}866 \end{aligned}$$

Answer is D.

**Problem 4 - 60 (Alternative Solution)**

Use the relationship PVFB = AL + PVFNC, or alternatively, AL = PVFB - PVFNC.

$$\begin{aligned} PVFB &= (2{,}500)(12\ddot{a}_{65}^{(12)})(D_{65}/D_{45}) \\ &= (2{,}500)(12)(8.74)(94{,}414/445{,}008) = 55{,}629 \end{aligned}$$

$$\begin{aligned} PVFNC &= NC \times \ddot{a}_{45:\overline{20}|} \\ &= 1{,}454.50 \times [(N_{45} - N_{65})/D_{45}] \\ &= 1{,}454.50 \times [(5{,}690{,}850 - 868{,}052)/445{,}008] = 15{,}763 \end{aligned}$$

$$\begin{aligned} AL &= PVFB - PVFNC \\ &= 55{,}629 - 15763 = 39{,}866 \end{aligned}$$

Answer is D.

**Problem 4 - 61**

Key Concept: The experience gain (loss) is determined by calculating the difference between the Accrued Liability under the funding method to the actual liability due to early retirement.

Step I: Calculate the Entry Age Normal Accrued Liability.

Projected Retirement Benefit = (10)(12 years of service)(1 - (.005)(36)) = 98.40

Value at Retirement of the
Projected Retirement Benefit = $(98.40)(12)[.8(\ddot{a}_{62}^{(12)} + \ddot{a}_{62}^{(12)} - \ddot{a}_{62:62}^{(12)}) + .2(\ddot{a}_{62}^{(12)})] = 12{,}044$

The value of the Projected Retirement Benefit is determined using the assumption that 80% of the participants receive a joint and survivor annuity and 20% receive a life annuity.

$$NC = 12{,}044/\ddot{s}_{\overline{12}|}$$
$$= 12{,}044/19.1406 = 629.24$$

$$AL = 629.24\ \ddot{s}_{\overline{10}|}$$
$$= (629.24)(14.7836) = 9{,}302$$

Step II: Calculate the actual liability.

Early Retirement Benefit = (10)(10 years of service)(1 - (.005)(60)) = 70

$$\text{Value of ERB} = (70)(12)(\ddot{a}_{60}^{(12)} + \ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}) = 10{,}080$$

Step III: Calculate the experience loss.

$$\text{Loss} = \text{Value of ERB} - AL$$
$$= 10{,}080 - 9{,}302 = 778$$

Answer is B.

# Chapter Five

# Individual Level Premium Cost Method

# Chapter 5

# Individual Level Premium Cost Method

## 5.3 Problems

**Problem 5 - 26**

Plan effective date: 1/1/95.

Funding medium: Individual whole life insurance policies and a side fund.

Actuarial cost method: Individual level premium with normal cost for side fund determined for benefits not provided by cash values of insurance policies at age 65.

The amount of whole life insurance is 60 times the expected monthly retirement benefit.

Actuarial assumptions for side fund:

Interest rate: 6% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

Date of birth for sole participant (active as of 1/1/95): 1/1/56.

Expected monthly retirement benefit: \$400.
Cash value at age 65 per \$1,000 of insurance: \$550.

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 10$$

In what range is the normal cost for 1995 as of 12/31/95 for the side fund?

(A) Less than $480
(B) $480 but less than $510
(C) $510 but less than $540
(D) $540 but less than $570
(E) $570 or more

## 5.4 Solutions to Problems

### Problem 5 - 26

Key Concept: The cash value at retirement of the life insurance policy must be subtracted from the value of the benefit at retirement in order to determine the normal cost for the side fund. Alternatively, the present value of the cash value at retirement can be subtracted from the present value of future benefits.

$$\text{Face amount of insurance} = 60 \times \text{Monthly retirement benefit}$$
$$= 60 \times 400 = 24{,}000$$

$$\text{Cash Value at 65} = 24 \times 550 \text{ (per thousand of face amount)} = 13{,}200$$

$$\text{PVFB} = [(400)(12\ddot{a}_{65}^{(12)}) - 13{,}200](v^{26}) = 7{,}649$$

$$\text{Side Fund NC}_{1/1/95} = \text{PVFB}/(\ddot{a}_{\overline{26}|})$$
$$= 7{,}649/13.7834 = 555$$

$$\text{Side Fund NC}_{12/31/95} = 555 \times 1.06 = 588$$

Answer is E.

### Problem 5 - 26 (Alternative Solution)

Since it is the first year of the plan and there are no assets, the normal cost can be calculated using the Present Value of benefits at retirement.

$$\text{Present Value of benefits at retirement} = (400)(12\ddot{a}_{65}^{(12)}) - 13{,}200 = 34{,}800$$

$$\text{Side Fund NC}_{12/31/95} = 34{,}800/s_{\overline{26}|}$$
$$= 34{,}800/59.1564 = 588$$

Note that $s_{\overline{26}|}$ was used instead of $s_{\overline{26}|}$ due to the end of year valuation date.

Answer is E.

# Chapter Six

# Frozen Initial Liability Cost Method

# Chapter 6

# Frozen Initial Liability Cost Method

## 6.3 Problems

**Problem 6 - 31**

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Retirement age: 65.

As of 1/1/94, all participants were active and under age 63.

Selected valuation results as of 1/1/94:

| | |
|---|---|
| Normal cost as of 1/1 | $ 150,000 |
| Present value of future benefits | 2,500,000 |
| Unfunded liability | 750,000 |
| Value of assets | 250,000 |
| Total valuation compensation for 1994 | 4,000,000 |

Contribution for 1994: $225,000 paid on 12/31/94.

There were no deaths, terminations, or retirements during 1994. There was one new entrant as of 1/1/95 who was age 50 with valuation compensation for 1995 of $50,000.

Value of assets as of 1/1/95: $510,000.

Experience gains or losses due to compensation increases during 1994: $0.

In what range is the absolute value of the change in the normal cost for 1995 as of 1/1/95 due to investment experience gains or losses during 1994?

(A) Less than $1,900
(B) $1,900 but less than $1,910
(C) $1,910 but less than $1,920
(D) $1,920 but less than $1,930
(E) $1,930 or more

**Problem 6 - 32**

Plan effective date: 1/1/90.
Actuarial cost method: Frozen initial liability.
Initial accrued liability: $150,000.
Assumed interest rate: 7 % per year.

Selected valuation results as of 1/1/95:

| | | |
|---|---|---|
| Present value of future benefits | | $300,000 |
| Value of assets | | 200,000 |
| Present value of future compensation | | 600,000 |
| Annual compensation | 60,000 | |

The contribution for 1995 is paid on 12/31/95 in an amount equal to the normal cost for 1995 as of 12/31/95 plus a 10-year amortization payment as of 12/31/95 of the initial accrued liability.

Contribution for 1995: $25,000.

In what range is the unfunded liability as of 1/1/95?

(A) Less than $50,000
(B) $50,000 but less than $55,000
(C) $55,000 but less than $60,000
(D) $60,000 but less than $65,000
(E) $65,000 or more

**Problem 6 - 33**

Plan effective date: 1/1/92.
Actuarial cost method: Frozen initial liability.
Assumed interest rate: 6% per year.
Initial accrued liability: $10,000,000.

Normal cost for 1992 as of 1/1/92: $3,000,000.
Normal cost for 1993 as of 1/1/93: $3,200,000.
Normal cost for 1994 as of 1/1/94 (after plan amendment): $3,500,000.
Increase in unfunded liability as of 1/1/94 due to plan amendment: $5,000,000.

Investment fund activity for 1992 through 1994:

| Year | Contribution | Date of Contribution | Actual Investment Return |
|---|---|---|---|
| 1992 | $4,500,000 | 1/1/92 | 8.0% |
| 1993 | 5,000,000 | 4/1/93 | 7.5% |
| 1994 | 5,500,000 | 4/1/94 | 5.5% |

In what range is the unfunded liability as of 1/1/95?

(A) Less than $11,050,000
(B) $11,050,000 but less than $11,200,000
(C) $11,200,000 but less than $11,350,000
(D) $11,350,000 but less than $11,500,000
(E) $11,500,000 or more

## 6.4 Solutions to Problems

**Problem 6 - 31**

Key Concept: The increase (decrease) in Normal Cost due to investment loss (gain) can be found by multiplying the amount of the investment loss (gain) by the ratio of Present Value of Future Salary to current Salary.

Step I: Calculate the investment gain or loss.

$$\text{Expected Assets}_{1/1/95} = (250{,}000 \times 1.07) + 225{,}000$$
$$= 267{,}500 + 225{,}000 = 492{,}500$$

$$\text{Actual Assets}_{1/1/95} = 510{,}000$$

$$\text{Asset Gain} = 510{,}000 - 492{,}500 = 17{,}500$$

Step II: Calculate the Present Value of Future Salary as of 1/1/95. First, use the 1/1/94 valuation results to determine the PVFS as of 1/1/94.

$$\text{NC} = (\text{PVFB} - \text{UL} - \text{Assets})/(\text{PVFS}/\text{Salary})$$
$$150{,}000 = (2{,}500{,}000 - 750{,}000 - 250{,}000)/(\text{PVFS}_{94}/4{,}000{,}000)$$
$$\text{PVFS}_{94} = 40{,}000{,}000$$

The expected PVFS for 1995 is

$$\text{Expected PVFS}_{95} = (\text{PVFS}_{94} - \text{Salary}_{94}) \times 1.07$$
$$= (40{,}000{,}000 - 4{,}000{,}000) \times 1.07 = 38{,}520{,}000$$

The PVFS for the new entrant must be added to the expected PVFS$_{95}$ to get the actual PVFS$_{95}$.

$$\text{New Entrant PVFS} = 50{,}000\ddot{a}_{\overline{15}|j} \quad \text{where } j = 1.07/1.05 - 1 = .0190476$$
$$= (50{,}000)(13.1878) = 659{,}390$$

$$\text{PVFS}_{95} = 38{,}520{,}000 + 659{,}390 = 39{,}179{,}390$$

Step III: Calculate the total salary at of 1/1/95.

$$\text{Expected Salary}_{95} = \text{Salary}_{94} \times 1.05$$
$$= 4{,}000{,}000 \times 1.05 = 4{,}200{,}000$$

The salary for the new participant must be added.

$$\text{Actual Salary}_{95} = 4{,}200{,}000 + 50{,}000 = 4{,}250{,}000$$

Step IV: Calculate the decrease in the Normal Cost due to the investment gain.

$$\Delta NC = \text{Gain}/(PVFS_{95}/\text{Salary}_{95})$$
$$= 17{,}500/(39{,}179{,}390/4{,}250{,}000) = 1{,}898$$

Answer is A.

**Problem 6 - 32**

The 10-year amortization payment as of 12/31/95 is

$$150{,}000/a_{\overline{10}|} = 21{,}357$$

Note the use of $a_{\overline{10}|}$ instead of $\ddot{a}_{\overline{10}|}$, because the payment is made at the end of the year.

Since the total contribution paid 12/31/95 was 25,000, the Normal Cost as of 12/31/95 is:

$$25{,}000 - 21{,}357 = 3{,}643$$

and the Normal Cost as of 1/1/95 is:

$$3{,}643/1.07 = 3{,}405$$

Now we can use the formula for developing Normal Cost:

$$NC = (PVFB - UL - \text{Assets})/(PVFS/\text{Salary})$$
$$3{,}405 = (300{,}000 - UL - 200{,}000)/(600{,}000/60{,}000)$$
$$UL = 65{,}950$$

Answer is E.

**Problem 6 - 33**

Key Concept: The unfunded liability is developed by increasing the sum of the unfunded liability and Normal Cost as of the prior valuation date with interest to the current valuation date and subtracting the contribution made for the prior plan year with interest from the date the contribution was made.

Step I: Calculate the Unfunded Liability as of 1/1/93.

$$UL_{93} = (UL_{92} + NC_{92}) \times (1 + i) - \text{Contribution}_{92} \times (1 + i)$$
$$= (10{,}000{,}000 + 3{,}000{,}000) \times 1.06 - 4{,}500{,}000 \times 1.06 = 9{,}010{,}000$$

Step II: Calculate the Unfunded Liability as of 1/1/94.

$$UL_{94} = (UL_{93} + NC_{93}) \times (1 + i) - \text{Contribution}_{93} \times (1 + 3/4i)$$
$$= (9{,}010{,}000 + 3{,}200{,}000) \times 1.06 - 5{,}000{,}000 \times 1.045 = 7{,}717{,}600$$

Because the contribution was paid on April 1, 1993, the contribution was increased with interest for only 9 months. The interest accumulation used here was based upon simple interest, but could have reflected compound interest by using $(1.06)^{3/4}$ as the interest factor.

Step III: Calculate the Unfunded Liability as of 1/1/95.

$$UL_{95} = (UL_{94} + UL_{\text{Plan Am.}} + NC_{94}) \times (1 + i) - \text{Contribution}_{94} \times (1 + 3/4i)$$
$$= (7{,}717{,}600 + 3{,}500{,}000 + 5{,}000{,}000) \times 1.06 - 5{,}500{,}000 \times 1.045$$
$$= 11{,}443{,}156$$

Answer is D.

# Chapter Seven

# Attained Age Normal Cost Method

# Chapter 7

# Attained Age Normal Cost Method

## 7.3 Problems

There were no problems on this cost method on the 1995 exam.

## 7.4 Solutions to Problems

There were no problems on this cost method on the 1995 exam.

# Chapter Eight

# Miscellaneous

# Chapter 8

# Miscellaneous

## 8.2 Problems

### Problem 8 - 33

| | |
|---|---|
| Normal retirement benefit: | 2% of final year's compensation for each year of service. |
| Early retirement eligibility: | Age 55. |
| Early retirement benefit: | Accrued benefit, reduced by 5% for each year by which the benefit commencement date precedes the normal retirement date. |

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Preretirement deaths and terminations: None.
Probabilities of retirement (assumed to occur at beginning of year):

| | |
|---|---|
| At age 62 | 25% |
| At age 63 | 50% |
| At age 64 | 75% |
| At age 65 | 100% |

Valuation data for sole participant (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/33 |
| Date of hire | 1/1/75 |
| 1994 compensation | $50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.18$ $\ddot{a}_{64}^{(12)} = 8.74$

$\ddot{a}_{63}^{(12)} = 8.96$ $\ddot{a}_{65}^{(12)} = 8.51$

In what range is the present value of future benefits as of 1/1/95?

(A) Less than $155,000
(B) $155,000 but less than $165,000
(C) $165,000 but less than $175,000
(D) $175,000 but less than $185,000
(E) $185,000 or more

**Problem 8 - 34**

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service. |
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |

All optional forms of payment are actuarially equivalent to the normal form of payment.

Valuation data for participant Smith:

| | |
|---|---|
| Date of birth | 1/1/36 |
| Date of hire | 1/1/72 |
| Date of retirement | 12/31/95 |
| Date of benefit commencement | 1/1/96 |

Smith originally elected an optional form of payment under which, for each $100 of monthly early retirement single life annuity benefit, he will receive a monthly benefit of $80 for his lifetime with one-half of this amount continuing after his death to his surviving spouse for her remaining lifetime.

Under an early retirement program, Smith's retirement benefits are enhanced by determining his normal and early retirement benefits with his service increased by one year, and his early retirement benefits reduced now by 3 % for each year by which commencement of payments precedes age 63.

Due to the early retirement program, Smith revises his election to an optional form of payment under which he will receive a monthly benefit of $80 for his lifetime with X% of this amount continuing after his death to his surviving spouse for her remaining lifetime.

In what range is the value of X%?

(A) Less than 75%
(B) 75% but less than 81%
(C) 81% but less than 87%
(D) 87% but less than 93%
(E) 93% or more

**Problem 8 - 35**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.

Postponed retirement benefit: Greater of (a) the normal retirement benefit determined as of the date of postponed retirement based on compensation and service up to the date of postponed retirement, or (b) the actuarial equivalent of the normal retirement benefit determined at age 65.

Normal form of payment: Life annuity.

Optional form of payment for married participants: Joint and 100% survivor annuity which is actuarially equivalent to the normal form of payment.

Preretirement death benefit: None.

Data for participant Smith (active as of 12/31/94):

| | |
|---|---|
| Date of birth | 1/1/29 |
| Date of hire | 1/1/64 |
| Date of retirement | 1/1/95 |
| Spouse's date of birth | 1/1/29 |
| Annual compensation: | |
| 1991 | \$30,000 |
| 1992 | 42,000 |
| 1993 | 45,000 |
| 1994 | 32,000 |

Selected commutation functions and annuity values:

| Age x | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 94,414 | 824,779 |
| 66 | 86,246 | 734,109 |

$\ddot{a}^{(12)}_{65:65} = 6.5$ $\ddot{a}^{(12)}_{66:66} = 6.3$

In what range is Smith's annual benefit under the joint and 100% survivor annuity option?

(A) Less than \$20,000
(B) \$20,000 but less than \$21,000
(C) \$21,000 but less than \$22,000
(D) \$22,000 but less than \$23,000
(E) \$23,000 or more

## 8.3 Solutions to Problems

**Problem 8 - 33**

Key Concept: The total Present Value of Future Benefits is equal to the sum of the Present Value of Future Benefits at each possible retirement age multiplied by the probability of retirement at that age.

Step I: Calculate the Present Value of Future Benefits if retirement occurs at age 62.

Final Compensation = \$50,000

Accrued Benefit at age 62 = (.02)(\$50,000)(20 years of service)(1 - (.05)(3)) = 17,000

$$PVFB_{62} = (17{,}000)(\ddot{a}_{62}^{(12)}) = 156{,}060$$

Step II: Calculate the Present Value of Future Benefits if retirement occurs at age 63.

Final Compensation = \$50,000 × 1.05 = \$52,500

Accrued Benefit at age 63 = (.02)(\$52,500)(21 years of service)(1 - (.05)(2)) = 19,845

$$PVFB_{63} = (19{,}845)(\ddot{a}_{63}^{(12)})(v) = 166{,}179$$

Step III: Calculate the Present Value of Future Benefits if retirement occurs at age 64.

Final Compensation = \$50,000 × $(1.05)^2$ = 55,125

Accrued Benefit at age 64 = (.02)(\$55,125)(22 years of service)(1 - .05) = 23,042

$$PVFB_{64} = (23{,}042)(\ddot{a}_{64}^{(12)})(v^2) = 175{,}899$$

Step IV: Calculate the Present Value of Future Benefits if retirement occurs at age 65.

Final Compensation = \$50,000 × $(1.05)^3$ = 57,881

Accrued Benefit at age 65 = (.02)(\$57,881)(23 years of service) = 26,625

$$PVFB_{65} = (26{,}625)(\ddot{a}_{65}^{(12)})(v^3) = 184{,}956$$

Step V: Calculate the probability of retirement at each age.

$$q_{62}^{(r)} = .25$$
$$q_{63}^{(r)} = (1 - q_{62}^{(r)})(q_{63}^{(r)}) = (.75)(.5) = .375$$
$$q_{64}^{(r)} = (1 - q_{62}^{(r)})(1 - q_{63}^{(r)})(q_{64}^{(r)}) = (.75)(.5)(.75) = .28125$$
$$q_{65}^{(r)} = (1 - q_{62}^{(r)})(1 - q_{63}^{(r)})(1 - q_{64}^{(r)})(q_{65}^{(r)}) = (.75)(.5)(.25)(1) = .09375$$

Step VI: Calculate the total Present Value of Future Benefits.

$$PVFB_{Total} = (156{,}060)(.25) + (166{,}179)(.375) + (175{,}899)(.28125) + (184{,}956)(.09375) = 168{,}143$$

Answer is C.

**Problem 8 - 34**

Under the original option,

$$100\,(12\ddot{a}_{60}^{(12)}) = 80\,(12\ddot{a}_{60}^{(12)}) + 40\,(12\ddot{a}_{y}^{(12)} - 12\ddot{a}_{60:y}^{(12)})$$ where y is the age of the spouse.

$$20\ddot{a}_{60}^{(12)} = 40(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})$$
$$.5 = (\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})/(\ddot{a}_{60}^{(12)})$$

We must determine the amount of benefit Smith is due under the early retirement program.

Amount of benefit under the old program = (.02)(Salary)(24 years of service)(1- (.03)(5))
= .408(Salary)

Amount of benefit under the new program = (.02)(Salary)(25 years of service)(1- (.03)(3))
= .455(Salary)

So, the life annuity benefit under the early retirement program is

$$100 \times (.455/.408) = 111.52$$

And,

$$111.52\,(12\ddot{a}_{60}^{(12)}) = 80(12\ddot{a}_{60}^{(12)}) + 80(X)(12\ddot{a}_{y}^{(12)} - 12\ddot{a}_{60:y}^{(12)})$$
$$31.52\,\ddot{a}_{60}^{(12)} = 80(X)(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})$$
$$31.52 = 80(X)(\ddot{a}_{y}^{(12)} - \ddot{a}_{60:y}^{(12)})/\ \ddot{a}_{60}^{(12)} = 40X$$
$$X = .788, \text{ or } 78.8\%$$

Answer is B.

**Problem 8 - 35**

Step I: Calculate the benefit on the postponed retirement date of 1/1/95.

Final 3-year average salary = (42,000 + 45,000 + 32,000)/3 = 39,667

Benefit = (39,667)(.02)(31 years of service) = 24,594

Step II: Calculate the actuarial equivalent of the normal retirement benefit.

Final 3-year average salary on the normal retirement date of 1/1/94 = (42,000 + 45,000 + 30,000)/3 = 39,000

$$\text{Benefit} = (39{,}000)(.03)(30 \text{ years of service})(N_{65}^{(12)}/N_{66}^{(12)})$$
$$= (23{,}400)(824{,}779/734{,}109) = 26{,}290$$

Step III: Calculate the postponed retirement benefit. This is the greater of the benefits calculated in Steps I and II, which is $26,290.

Step IV: Calculate the 100% joint and survivor annuity.

$$\ddot{a}_{66}^{(12)} = N_{66}^{(12)}/D_{66} = 734{,}109/86{,}246 = 8.5118$$

$$26{,}290 \times \ddot{a}_{66}^{(12)} = X\,(\ddot{a}_{66}^{(12)} + \ddot{a}_{66}^{(12)} - \ddot{a}_{66:66}^{(12)})$$
$$X = 20{,}868$$

Answer is B.

# 1997 Supplement to

# ACTUARIAL COST METHODS

# A REVIEW

# 1997 Supplement to

# ACTUARIAL COST METHODS A REVIEW

By

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**American Society of Pension Actuaries**
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**Chapter Seven - Attained Age Normal Cost Method**



**Chapter Eight - Miscellaneous**

## Guide to Problems

The problems herein were taken from the May, 1996 EA-1(B) Examination. For those students wishing to use this material in conjunction with this examination, the following cross reference list should be helpful. Problems and solutions for the May, 1994 and May, 1995 EA-1(B) Examinations are contained in the 1995 and 1996 Supplements, respectively. The cross reference to those problems is provided here for your convenience.

| | **1995 Supplement** | **1996 Supplement** | **1997 Supplement** |
|---|---|---|---|
| **Problem** | **May, 1994 Examination** | **May, 1995 Examination** | **May, 1996 Examination** |
| 1 | 6 - 28 | 2 - 39 | 1 - 49 |
| 2 | 4 - 53 | 8 - 33 | 2 - 44 |
| 3 | 2 - 37 | 3 - 16 | 4 - 62 |
| 4 | 1 - 42 | 1 - 47 | 3 - 18 |
| 5 | 1 - 43 | 5 - 26 | 5 - 27 |
| 6 | 1 - 44 | 2 - 40 | 7 - 15 |
| 7 | 4 - 54 | 8 - 34 | 2 - 45 |
| 8 | 4 - 55 | 4 - 58 | 8 - 36 |
| 9 | 4 - 56 | 3 - 17 | 8 - 37 |
| 10 | 1 - 45 | 6 - 31 | 4 - 63 |
| 11 | 8 - 31 | 2 - 41 | 6 - 34 |
| 12 | 4 - 57 | 4 - 59 | 1 - 50 |
| 13 | 7 - 14 | 2 - 42 | 5 - 28 |
| 14 | 6 - 29 | 6 - 32 | 4 - 64 |
| 15 | 3 - 15 | 1 - 48 | 8 - 38 |
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| 17 | 2 - 38 | 8 - 35 | 2 - 46 |
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| 20 | 1 - 46 | 4 - 61 | 4 - 66 |

# Glossary of Abbreviations

Since many actuarial terms are lengthy, we will use abbreviations in equations and in the text for the sake of brevity. Listed below are many of the abbreviations which will be seen here:

| | |
|---|---|
| $\ddot{a}_x$ | Present value of an annuity due at age x |
| AA, aa or x | Attained Age |
| Assets | Actuarial Value of Assets |
| AL | Accrued Liability |
| ATA | Average Temporary Annuity |
| B(x) | Benefit at age x |
| BP | Benefit Payments |
| $C_t$ | Contributions for year t |
| CV | Cash Value |
| DB | Death Benefit |
| *e*Assets | Expected Value of Assets |
| *e*AL | Expected Accrued Liability |
| *e*PVFB | Expected Present Value of Future Benefits |
| *e*PVFNC | Expected Present Value of Future Normal Cost |
| *e*UAL | Expected Unfunded Accrued Liability |
| *e*UL | Expected Unfunded Liability |
| EA or ea | Entry Age |
| EAN | Entry Age Normal |
| ERB | Early Retirement Benefit |
| FIL | Frozen Initial Liability |
| $I_{BP}$ | Interest on Benefit Payments |
| $I_c$ | Interest on contributions |
| ILP | Individual Level Premium |
| J&S or J+S | Joint and Survivor |
| NC | Normal Cost |
| NRA | Normal Retirement Age |
| NRD | Normal Retirement Date |
| PVAB | Present Value of Accrued Benefits |
| PVFB | Present Value of Future Benefits |
| PVFEC | Present Value of Future Employee Contributions |
| PVFNC | Present Value of Future Normal Costs |
| PVFS | Present Value of Future Salary |
| PVFY | Present Value of Future Years |
| RA, ra or y | Retirement Age |
| S, Salary | Salary or Compensation |
| TA | Temporary Annuity |
| UAL | Unfunded Accrued Liability |
| UC | Unit Credit |
| UL | Unfunded Liability |

# Introduction

This Supplement is designed to assist students who are preparing for the EA-1(B) Examination, Basic Pension Mathematics, which is offered jointly by the American Society of Pension Actuaries, the Society of Actuaries and the Joint Board for the Enrollment of Actuaries. The problems and solutions in this Supplement are from the May, 1996, EA-1(B) Examination. Material is arranged according to the recognized actuarial cost methods, with the last section dealing with those principles common to all cost methods.

**Actuarial Cost Methods, A Review**, in conjunction with the 1995, 1996 and 1997 supplements, provides a complete, up to date review of the principles of the various pension cost methods and related actuarial topics.

# Chapter One

# Unit Credit Cost Method (Accrued Benefit)

# Chapter 1

# Unit Credit Cost Method (Accrued Benefit)

## 1.4 Problems

### Problem 1 - 49

| | |
|---|---|
| Normal retirement benefit: | 2% of final 3-year average compensation for each year of service up to 25 years. |
| Early retirement eligibility: | Age 62. |
| Early retirement benefit: | Accrued benefit, reduced by 3% for each year by which the benefit commencement date precedes the normal retirement date. |
| Actuarial cost method: | Projected unit credit (based upon actual accrual percentages as of the valuation date). |

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |

Probability of retirement (assumed to occur at beginning of the year):

| | |
|---|---|
| At age 62 | 20% |
| At age 63 | 0% |
| At age 64 | 0% |
| At age 65 | 100% |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/46 |
| Date of hire | 1/1/84 |
| 1996 valuation compensation | \$50,000 |

Selected annuity values:

$\ddot{a}_{62}^{(12)} = 9.39$ $\ddot{a}_{65}^{(12)} = 8.74$

In what range is the accrued liability as of 1/1/96?

(A) Less than \$60,000
(B) \$60,000 but less than \$62,500
(C) \$62,500 but less than \$65,000
(D) \$65,000 but less than \$67,500
(E) \$67,500 or more

**Problem 1 - 50**

Normal retirement benefit: 2/12% of final 3-year average compensation for each month of service up to 120 months plus 1/12% of final 3-year average compensation for each additional month of service.

Actuarial cost method: Projected unit credit (based upon actual accrual percentages as of the valuation date).

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/43 |
| Date of hire | 10/1/86 |
| 1996 valuation compensation | \$50,000 |

Selected annuity value:

$\ddot{a}_{65}^{(12)} = 8.74$

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $4,000
(B) $4,000 but less than $4,500
(C) $4,500 but less than $5,000
(D) $5,006 but less than $5,500
(E) $5,500 or more

**Problem 1 - 51**

Normal retirement benefit: 2% of final 3-year average compensation for each year of service.
Termination benefit: Accrued benefit payable at normal retirement date.
Vesting: Full and immediate.

Actuarial cost method: Projected unit credit.

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 4% per year.
Pre-retirement decrements other than withdrawals: None.
Probability of withdrawal (assumed to occur at beginning of year):

| | |
|---|---|
| 1/1/95 valuation: | 40% at age 50 |
| | 25% at age 55 |
| | 20% at age 60 |
| | 0% at all other ages |
| 1/1/96 valuation: | 50% at age 50 |
| | 20% at age 55 |
| | 0% at all other ages |

Retirement age: 65.

As of 1/1/96, all participants were active and under age 45.

In what range is the ratio of the accrued liability as of 1/1/96 under the revised withdrawal assumptions to the accrued liability as of 1/1/96 under the original withdrawal assumptions?

(A) Less than 0.990
(B) 0.990 but less than 1.000
(C) 1.000 but less than 1.010
(D) 1.010 but less than 1.020
(E) 1.020 or more

## 1.5 Solutions to Problems

**Problem 1 - 49**

Step I: Calculate the Final Average Salary at each retirement age.

Final Average Salary RA = 65: $(50{,}000)[(1.04)^{14} + (1.04)^{13} + (1.04)^{12}]/3 = 83{,}296$

Final Average Salary RA = 62: $(50{,}000)[(1.04)^{11} + (1.04)^{10} + (1.04)^{9}]/3 = 74{,}050$

Step II: Calculate the Accrued Liability at retirement age 65.

Projected Accrued Benefit = (83,296)(.02)(12 years of service) = 19,991

$$\text{AL (RA 65)} = \text{PVAB} = (19{,}991)(\ddot{a}_{65}^{(12)})(v^{15})$$
$$= (19{,}991)(8.74)(.362446) = 63{,}327$$

Step III: Calculate the Accrued Liability at retirement age 62.

Projected Accrued Benefit = (74,050)(.02)(12 years of service)(1 - (.03)(3 years))
= 16,173

$$\text{AL (RA 62)} = \text{PVAB} = (16{,}173)(\ddot{a}_{62}^{(12)})(v^{12})$$
$$= (16{,}173)(9.39)(.444012) = 67{,}430$$

Step IV: Calculate the Accrued Liability under the actual retirement age assumptions (20% probability at age 62, 100% probability at age 65)

$$\text{AL} = (67{,}430)(.2) + (63{,}327)(.8) = 64{,}148$$

Answer is C.

**Problem 1 - 50**

Step I: Calculate the projected Final Average Salary.

Final Average Salary = $(50{,}000)[(1.04)^{11} + (1.04)^{10} + (1.04)^{9}]/3 = 74{,}050$

Step II: Calculate the projected 1996 accrual.

The participant has worked 111 months prior to 1996. So, the benefit to be accrued in 1996 will be based upon 9 months of service at 2/12% of final average salary plus 3 months of service at 1/12% of final average salary.

$$\begin{aligned}\text{Projected 1996 Accrual} &= [(2/12\% \times 9 \text{ months}) + (1/12\% \times 3 \text{ months})](74{,}050) \\ &= 1{,}295.88\end{aligned}$$

Step III: Calculate the Normal Cost.

$$\begin{aligned}NC &= (1{,}295.88)(\ddot{a}_{65}^{(12)})(v^{12}) \\ &= 5{,}029\end{aligned}$$

Answer is D.

**Problem 1 - 51**

Key Concept: The Accrued Liability is equal to the sum of the liability attributable to retirement benefits plus the liability attributable to withdrawal benefits.

Step I: Calculate the Accrued Liability attributable to retirement benefits.

Under the 1/1/95 valuation assumptions,

$$\begin{aligned}\text{Retirement Benefit } AL_{Old} &= PV \times p_{50}^{(w)} \times p_{55}^{(w)} \times p_{60}^{(w)} \\ &= (PV)(.6)(.75)(.8) \\ &= .36PV\end{aligned}$$

where PV represents the present value of the projected accrued benefit as of 1/1/96, discounting with interest only.

Under the 1/1/96 valuation assumptions,

$$\begin{aligned}\text{Retirement Benefit } AL_{New} &= PV \times p_{50}^{(w)} \times p_{55}^{(w)} \\ &= (PV)(.5)(.8) \\ &= .4PV\end{aligned}$$

Step II: Calculate the Accrued Liability attributable to withdrawal benefits.

Under the 1/1/95 valuation assumptions,

$$\begin{aligned}\text{Withdrawal Benefit } AL_{Old} &= (PV)(1/1.04)^{15}(q_{50}^{(w)}) + (PV)(1/1.04)^{10}(p_{50}^{(w)})(q_{55}^{(w)}) \\ &\quad + (PV)(1/1.04)^{5}(p_{50}^{(w)})(p_{55}^{(w)})(q_{60}^{(w)}) \\ &= (PV)(1/1.04)^{15}(.4) + (PV)(1/1.04)^{10}(.6)(.25) \\ &\quad + (PV)(1/1.04)^{5}(.6)(.75)(.2) \\ &= .3974PV\end{aligned}$$

Note that since termination would occur, the participant would not have the impact of future salary increases from termination to retirement. This explains the multiplication of the present value by 1/1.04 for the number of years from termination to retirement.

Under the 1/1/96 valuation assumptions,

$$\begin{aligned}\text{Withdrawal Benefit } AL_{New} &= (PV)(1/1.04)^{15}(q_{50}^{(w)}) + (PV)(1/1.04)^{10}(p_{50}^{(w)})(q_{55}^{(w)}) \\ &= (PV)(1/1.04)^{15}(.5) + (PV)(1/1.04)^{10}(.5)(.2) \\ &= .3452PV\end{aligned}$$

Step III: Calculate the ratio of the revised Accrued Liability to the original Accrued Liability.

$$\begin{aligned}\text{Ratio} &= (.4PV + .3452PV)/(.36PV + .3974PV) \\ &= .7452/.7574 \\ &= .9839\end{aligned}$$

Answer is A.

# Chapter Two

# Aggregate Cost Method

# Chapter 2

# Aggregate Cost Method

## 2.4 Problems

**Problem 2 - 44**

Normal retirement benefit:

| | |
|---|---|
| Before 1996: | 60% of final 5-year average compensation. |
| After 1995: | 70% of final 5-year average compensation. |

Actuarial cost method: Aggregate.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Compensation increases: | 4% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Selected valuation results as of 1/1/95:

| | |
|---|---|
| Present value of future benefits | $1,500,000 |
| Value of assets | 300,000 |
| Normal cost as of 1/1 | 96,000 |

As of 1/1/95, all participants were active and under age 55.

There were no deaths, terminations, retirements, or new participants during 1995, and there are no new participants as of 1/1/96.

For each participant, 1996 valuation compensation is 6% higher than 1995 valuation compensation.

Normal cost for 1996 as of 1/1/96: \$122,650.

In what range is the value of assets as of 1/1/96?

(A) Less than \$200,000
(B) \$200,000 but less than \$300,000
(C) \$300,000 but less than \$400,000
(D) \$400,000 but less than \$500,000
(E) \$500,000 or more

**Problem 2 - 45**

Normal retirement benefit: \$50 per month for each year of service.

Normal form of payment: Life annuity with 120 months certain.

Actuarial cost method: Aggregate.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Value of assets as of 1/1/96: \$110,000.

Valuation data for all participants as of 1/1/96:

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/29 | 1/1/51 |
| Date of hire | 1/1/74 | 1/1/86 |
| Status | Retired on 1/1/94 | Active |

Selected commutation functions:

| $x$ | $D_x$ | $N_x^{(12)}$ |
|---|---|---|
| 65 | 94,414 | 824,780 |
| 67 | 78,601 | 651,367 |
| 75 | 33,855 | 217,236 |
| 77 | 25,541 | 153,955 |

After preparing the 1/1/96 valuation, the actuary was informed that Smith died on 12/15/95. The actuary prepared a revised 1/1/96 valuation based upon this information.

In what range is the difference in the normal cost for 1996 as of 1/1/96 under the two valuations?

(A) Less than $2,450
(B) $2,450 but less than $2,600
(C) $2,600 but less than $2,750
(D) $2,750 but less than $2,900
(E) $2,900 or more

**Problem 2 - 46**

Normal retirement benefit: 1% of final year's compensation for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 6% per year.
Compensation increases: 5% per year.
Pre-retirement decrements other than deaths: None.
Retirement age: 65.

Value of assets as of 1/1/96: $60,000.

Valuation data for all participants as of 1/1/96:

| | Smith | Brown | Green |
|---|---|---|---|
| Status | Active | Terminated Vested | Retired |
| Age at hire | 25 | - | - |
| Attained age | 35 | 50 | 65 |
| 1996 valuation compensation | $48,000 | - | - |
| Monthly accrued benefit | $375 | $200 | $300 |

Selected commutation functions and annuity values:

| x | $\dfrac{{}^{s}N_x - {}^{s}N_{65}}{{}^{s}D_x}$ | ${}_{65-x\mid}\ddot{a}_x^{(12)}$ |
|---|---|---|
| 25 | 16.7 | 0.4 |
| 30 | 14.3 | 0.7 |
| 35 | 12.5 | 1.0 |
| 50 | 11.1 | 4.0 |
| 65 | 0.0 | 10.0 |

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $5,000
(B) $5,000 but less than $5,200
(C) $5,200 but less than $5,400
(D) $5,400 but less than $5,600
(E) $5,600 or more

**Problem 2 - 47**

Type of plan: Contributory.

Normal retirement benefit: $50 per month for each year of service.

Actuarial cost method: Aggregate.

Actuarial assumptions:

Interest rate: 7% per year.
Pre-retirement decrements: None.
Retirement age: 65.

As of 1/1/95, all participants were active and under age 63.

Selected valuation results:

| | 1/1/95 | 1/1/96 |
|---|---|---|
| Employer's normal cost as of 1/1 | $ 7,200 | |
| Present value of future benefits | 100,000 | |
| Present value of all future employee contributions | 25,000 | $ 24,500 |
| Value of plan assets | 10,000 | 22,000 |

There were no deaths, terminations, retirements, or new participants during 1995, and there are no new participants as of 1/1/96.

In what range is the employer's normal cost for 1996 as of 1/1/96?

(A) Less than $7,000
(B) $7,000 but less than $7,200
(C) $7,200 but less than $7,400
(D) $7,400 but less than $7,600
(E) $7,600 or more

## 2.5 Solutions to Problems

**Problem 2 - 45**

Key Concept: The gain due to Smith's death will be the value of the life portion of the benefit. Under the Aggregate method, the gain will be amortized over the 20 year future working lifetime of Brown, the sole active participant.

Step I: Calculate the present value of the decrease in Smith's benefit.

$$\text{Smith Monthly Benefit} = (50)(20 \text{ years of service}) = 1{,}000$$

$$\text{Decrease in Present Value} = (1{,}000)(12)(N_{75}^{(12)}/\ D_{67}) = 33{,}165$$

Step II: Calculate the decrease in the Normal Cost

$$\text{Decrease in Normal Cost} = 33{,}165/\ \ddot{a}_{\overline{20|}} = 2{,}926$$

Answer is E.

**Problem 2 - 46**

Key Concept: Since Smith is the only active participants, the Present Value of Future Benefits will be amortized over Smith's future working lifetime.

Step I: Calculate the final compensation for Smith.

$$\text{Final Compensation} = 48{,}000 \times (1.05)^{29} = 197{,}575$$

Step II: Calculate the Present Value of Future Benefits for each participant.

$$\begin{aligned} PVFB_{Smith} &= (197{,}575)(.01)(40 \text{ years of service})({}_{30|}\ddot{a}_{35}^{(12)}) \\ &= (197{,}575)(.01)(40)(1) \\ &= 79{,}030 \end{aligned}$$

$$\begin{aligned} PVFB_{Brown} &= (200)(12)({}_{15|}\ddot{a}_{50}^{(12)}) \\ &= (200)(12)(4) \\ &= 9{,}600 \end{aligned}$$

$$PVFB_{Green} = (300)(12)(\ddot{a}_{65}^{(12)}) = (300)(12)(10) = 36{,}000$$

$$PVFB_{Total} = 79{,}030 + 9{,}600 + 36{,}000 = 124{,}630$$

Step III: Calculate the Normal Cost.

$$\text{Normal Cost} = (\text{PVFB} - \text{Assets})/[({}^sN_{35} - {}^sN_{65})/{}^sD_{35}] = (124{,}630 - 60{,}000)/12.5 = 5{,}170$$

Answer is B.

**Problem 2 - 47**

Key Concept: The employer's Normal Cost is the total Normal Cost adjusted by the employees' mandatory contributions. Note that it is assumed that the employee contribution does not change from 1995 to 1996 since the employee base does not change.

Step I: Calculate the employee contribution.

The equation representing successive years Present Value of Future Employee Contributions (PVFEC) is:

$$PVFEC_{96} = (PVFEC_{95} - \text{Employee Contribution})(1 + i)$$
$$24{,}500 = (25{,}000 - \text{Employee Contribution})(1.07)$$

Therefore,

$$\text{Employee Contribution} = 2{,}103$$

Step II: Calculate the average temporary annuity for the 1995 valuation.

The equation representing the 1/1/95 Normal Cost is:

$$NC_{95} = (PVFB_{95} - Assets_{95})/\ \ddot{a}_{\overline{n}|}$$
$$7{,}200 + 2{,}103 = (100{,}000 - 10{,}000)/\ \ddot{a}_{\overline{n}|}$$

Therefore,

$$\ddot{a}_{\overline{n}|} = 9.673$$

Step III: Calculate the average temporary annuity for the 1996 valuation.

$$\ddot{a}_{\overline{n-1|}} = (\ddot{a}_{\overline{n|}} - 1)(1.07) = (9.673 - 1)(1.07) = 9.2815$$

Step IV: Calculate the Normal Cost.

$$\begin{aligned}\text{PVFB}_{96} &= (\text{PVFB}_{95})(1 + i) \\ &= (100{,}000)(1.07) \\ &= 107{,}000\end{aligned}$$

$$\begin{aligned}\text{Total NC}_{96} &= (\text{PVFB}_{96} - \text{Assets}_{96})/\ddot{a}_{\overline{n-1|}} \\ &= (107{,}000 - 22{,}000)/9.2815 \\ &= 9{,}158\end{aligned}$$

$$\text{Employer NC}_{96} = 9{,}158 - 2{,}103 = 7{,}055$$

Answer is B.

**Problem 2 - 47 (Alternative Solution)**

The Present Value of Future Benefits are funded through the existing assets, the future employer contributions (normal cost), and the future employee contributions.

The average temporary annuity for 1995 can be calculated from the equation representing the 1/1/95 employer Normal Cost:

$$\begin{aligned}\text{Employer NC}_{95} &= (\text{PVFB}_{95} - \text{PVFEC}_{95} - \text{Assets}_{95})/\ddot{a}_{\overline{n|}} \\ 7{,}200 &= (100{,}000 - 25{,}000 - 10{,}000)/\ddot{a}_{\overline{n|}}\end{aligned}$$

Therefore,

$$\ddot{a}_{\overline{n|}} = 9.0278$$

The average temporary annuity for the 1996 valuation is:

$$\ddot{a}_{\overline{n-1|}} = (\ddot{a}_{\overline{n|}} - 1)(1.07) = (9.0278 - 1)(1.07) = 8.5897$$

Now calculate the employer Normal Cost for 1996:

$$\begin{aligned}\text{Employer } NC_{96} &= (PVFB_{96} - PVFEC_{96} - Assets_{96})/\ddot{a}_{\overline{n-T|}} \\ &= (107{,}000 - 24{,}500 - 22{,}000)/8.8597 \\ &= 7{,}043\end{aligned}$$

Answer is B.

# Chapter Three

# Individual Aggregate Cost Method

# Chapter 3

# Individual Aggregate Cost Method

## 3.3 Problems

### Problem 3 - 18

Benefit: $20 per month for each year of service.

Actuarial cost method:

| | |
|---|---|
| Before 1996: | Aggregate. |
| After 1995: | Individual aggregate with assets allocated in proportion to the entry age normal accrued liability. |

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Value of assets as of 1/1/96: $20,000.

Valuation data for all participants (both active as of 1/1/96):

| | Smith | Brown |
|---|---|---|
| Date of birth | 1/1/56 | 1/1/41 |
| Date of hire | 1/1/91 | 1/1/81 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 10$$

In what range is the normal cost for 1996 for Brown as of 1/1/96?

(A) Less than \$1,500
(B) \$1,500 but less than \$1,700
(C) \$1,700 but less than \$1,900
(D) \$1,900 but less than \$2,100
(E) \$2,100 or more

## 3.4 Solutions to Problems

**Problem 3 - 18**

Key Concept: Since the Individual Aggregate funding method is to be used for the first time in 1996, a reasonable method of allocating assets must be used. The description of the funding method indicates that the method to be used in this problem is to allocate the assets in proportion to Entry Age Normal Accrued Liability.

Step I: Calculate the Present Value of Future Benefits.

$$\text{Projected Retirement Benefit} = (20)(25 \text{ years of service}) = 500.00$$

$$\text{Value at Retirement} = (500.00)(12)(\ddot{a}_{65}^{(12)}) = 60,000$$

$$\text{PVFB} = (60,000)(v^{10}) = 30,501$$

Step II: Calculate the Entry Age Normal Accrued Liability for each participant.

$$\text{Smith Accrued Liability} = (20)(30 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{5}|} / \ddot{s}_{\overline{30}|}) = 4,383$$

$$\text{Brown Accrued Liability} = (20)(25 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{15}|} / \ddot{s}_{\overline{25}|}) = 23,838$$

$$\text{Total Accrued Liability} = 4,383 + 23,838 = 28,221$$

Step III: Calculate the allocation of assets for Brown.

$$\text{Asset Allocation}_{\text{Brown}} = (\text{Total Assets})(\text{EANAL}_{\text{Brown}} / \text{EANAL}_{\text{Total}}) = (20,000)(23,838/28,221) = 16,894$$

Step IV: Calculate the Normal Cost for Brown.

$$\text{Normal Cost}_{\text{Brown}} = (\text{PVFB - Assets}) / \ddot{a}_{\overline{10}|} = (30,501 - 16,894) / 7.515232 = 1,811$$

Answer is C.

# Chapter Four

# Entry Age Normal Cost Method

# Chapter 4

# Entry Age Normal Cost Method

## 4.5 Problems

### Problem 4 - 62

Normal retirement benefit: $25 per month for each year of service.

Eligibility for early retirement: Age 55.

Early retirement benefit: Accrued benefit, reduced for commencement of payments before age 65.

Early retirement adjustment factor at age 55: 0.412.

Actuarial cost method: Individual entry age normal.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for participant Smith (active as of 1/1/95):

| | |
|---|---|
| Date of birth | 1/1/41 |
| Date of hire | 1/1/70 |
| Date of retirement | 12/31/95 |
| Date of benefit commencement | 1/1/96 |

Selected annuity values:

$\ddot{a}_{55}^{(12)} = 10.78$ $\ddot{a}_{65}^{(12)} = 8.74$

In what range is the decrease in the accrued liability as of 1/1/96 due to Smith's retirement?

(A) Less than \$9,000
(B) \$9,000 but less than \$18,000
(C) \$18,000 but less than \$27,000
(D) \$27,000 but less than \$36,000
(E) \$36,000 or more

**Problem 4 - 63**

Normal retirement benefit: \$20 per month for each year of service up to 25 years.

Actuarial cost method:

Before 1996: Entry age normal.
After 1995: Unit credit.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/36 |
| Date of hire | 1/1/56 |

Selected annuity value:

$\ddot{a}_{62}^{(12)} = 9.24$

In what range is the absolute value of the change in the normal cost for 1996 as of 1/1/96 due to the change in the actuarial cost method?

(A) Less than $500
(B) $500 but less than $1,000
(C) $1,000 but less than $1,500
(D) $1,500 but less than $2,000
(E) $2,000 or more

**Problem 4 - 64**

Vesting eligibility: 0% if less than 5 years of service; 100% if 5 or more years of service.

Actuarial cost method: Entry age normal.

Actuarial assumptions:

Interest rate: 8% per year.
Compensation increases: None.
Pre-retirement decrements other than withdrawals: None.
Selected withdrawal rates:

| x | $q_x^{(w)}$ |
|---|---|
| 35 | .5 |
| 36 | .4 |
| 37 | .3 |
| 38 | .2 |
| 39 | .1 |
| 40 and over | 0 |

Retirement age: 65.

Valuation data for sole participant Smith (active as of 1/1/96):

| | |
|---|---|
| Entry age | 35 |
| Attained age | 38 |

Normal cost for Smith for 1996 as of 1/1/96: $10,000.

In what range is the accrued liability as of 1/1/96?

(A) Less than $30,000
(B) $30,000 but less than $50,000
(C) $50,000 but less than $70,000
(D) $70,000 but less than $90,000
(E) $90,000 or more

**Problem 4 - 65**

Normal retirement benefit:

Before 1996: 1.25% of final 5-year average compensation for each year of service.
After 1995: 1.75% of final 3-year average compensation for each year of service.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Pre-retirement decrements: None.
Retirement age: 65.

Valuation data for sole participant Smith (active as of 1/1/96):

Date of birth 1/1/50
Date of hire 1/1/88

Normal cost for Smith for 1988 as of 1/1/88: $6,500.

In what range is the increase in the accrued liability for Smith as of 1/1/96 due to the plan amendment?

(A) Less than $34,000
(B) $34,000 but less than $36,000
(C) $36,000 but less than $38,000
(D) $38,000 but less than $40,000
(E) $40,000 or more

**Problem 4 - 66**

Normal retirement benefit: 60% of final 3-year average compensation.

Actuarial cost method: Entry age normal (level percentage of compensation).

Actuarial assumptions:

Interest rate: 7% per year.
Compensation increases: 5% per year.
Pre-retirement decrements: None.
Retirement age: 65.

Valuation data for sole participant:

| | |
|---|---|
| Date of birth | 1/1/50 |
| Date of hire | 1/1/80 |
| 1995 valuation compensation for 1/1/95 valuation | $50,000 |
| 1996 valuation compensation for 1/1/96 valuation | 50,000 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.736$$

In what range is the experience gain in 1995 due to a compensation increase other than assumed?

(A) Less than $4,000
(B) $4,000 but less than $4,700
(C) $4,700 but less than $5,400
(D) $5,400 but less than $6,100
(E) $6,100 or more

## 4.6 Solutions to Problems

**Problem 4 - 62**

Key Concept: The decrease in the accrued liability is determined by calculating the difference between the accrued liability under the funding method had early retirement not been elected and the actual liability due to early retirement.

Step I: Calculate the Entry Age Normal Accrued Liability.

$$\text{Projected Retirement Benefit} = (25)(36 \text{ years of service}) = 900.00$$

$$\text{Value at Retirement} = (900.00)(12)(\ddot{a}_{65}^{(12)}) = 94{,}392$$

$$\text{Normal Cost} = 94{,}392/\ \ddot{s}_{\overline{36|}} = 94{,}392/159.3374 = 592$$

$$\text{Accrued Liability} = (592) \times \ddot{s}_{\overline{26|}} = (592)(73.4838) = 43{,}502$$

Step II: Calculate the actual liability.

$$\text{Early Retirement Benefit} = (25)(26 \text{ years of service})(.412) = 267.80$$

$$\text{Value of ERB} = (267.80)(12)(\ddot{a}_{55}^{(12)}) = (267.80)(12)(10.78) = 34{,}643$$

Step III: Calculate the decrease in the accrued liability

$$\begin{aligned}\text{Decrease} &= \text{Accrued Liability - Value of ERB}\\ &= 43{,}502 - 34{,}643\\ &= 8{,}859\end{aligned}$$

Answer is A.

**Problem 4 - 63**

Key concept: The Unit Credit Normal Cost is $0 since the participant has more than 25 years of past service. So, the change in the Normal Cost will just be equal to the Entry Age Normal Normal Cost.

$$\text{Projected Retirement Benefit} = (20)(25 \text{ years of service}) = 500$$

$$\text{EAN Normal Cost} = (500)(12)(\ddot{a}_{65}^{(12)})/\ \ddot{s}_{\overline{45}|}$$
$$= (6{,}000)(9.24)/(305.75176)$$
$$= 181$$

Answer is A.

**Problem 4 - 64**

Key Concept: The Entry Age Normal Accrued Liability is equal to the accumulated value of the past Normal Costs. Since the Normal Cost each year was $10,000, each Normal Cost must be accumulated with interest and by the withdrawal decrement (the probability that withdrawal did not occur).

$$\text{Accrued Liability} = (10{,}000)((1.08)^3(1/p_{35}^{(w)})(1/p_{36}^{(w)})(1/p_{37}^{(w)}) + (1.08)^2(1/p_{36}^{(w)})(1/p_{37}^{(w)}) + (1.08)(1/p_{37}^{(w)}))$$

$$= (10{,}000)((1.08)^3(1/.5)(1/.6)(1/.7) + (1.08)^2(1/.6)(1/.7) + (1.08)(1/.7)$$

$$= (10{,}000)(5.9986 + 2.7771 + 1.5429)$$

$$= 103{,}186$$

Answer is E.

**Problem 4 - 65**

Key Concept: There is only one participant in the plan. Therefore, the Accrued Liability will increase in proportion to the increase in the normal retirement benefit.

Step I: Calculate the benefit per dollar of salary and year of service under the old formula.

$$\text{Unit of Benefit (Old)} = (.0125)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17} + (1.05)^{16} + (1.05)^{15}]/5$$
$$= .0287185$$

Step II: Calculate the benefit per dollar of salary and year of service under the new formula.

$$\text{Unit of Benefit (New)} = (.0175)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17}]/3$$
$$= .0421493$$

Step III: Calculate the Accrued Liability under the old benefit formula. Note that since there have been no gains or losses (this must be assumed since there is no information to the contrary), the normal cost would have increased at the salary scale rate each year.

$$\text{Normal Cost}_{96} = (6{,}500)(1.05)^8 = 9{,}603$$

$$\begin{aligned}\text{Accrued Liability}_{96} &= (9{,}603)(\ddot{s}_{\overline{8}|j}) \quad \text{where } j = (1.07/1.05)\text{-}1 \\ &= 83{,}714\end{aligned}$$

Step IV: Calculate the increase in the Accrued Liability.

$$\begin{aligned}\text{AL Increase} &= (83{,}714)[(.0421493 - .0287185)/.0287185)] \\ &= 39{,}151\end{aligned}$$

Answer is D.

**Problem 4 - 66**

Key Concept: The experience gain can be calculated from the decrease in the projected benefit due to the actual compensation experience.

Step I: Calculate the projected Final Average Compensation (FAC) for each of the 1/1/95 and 1/1/96 valuations.

$$\begin{aligned}FAC_{95} &= (50{,}000)[(1.05)^{19} + (1.05)^{18} + (1.05)^{17}]/3 \\ &= 120{,}426\end{aligned}$$

$$\begin{aligned}FAC_{96} &= (50{,}000)[(1.05)^{18} + (1.05)^{17} + (1.05)^{16}]/3 \\ &= 114{,}692\end{aligned}$$

Note that $FAC_{96}$ could have been calculated by dividing $FAC_{95}$ by 1.05 since the compensation remained the same in 1996 as it was in 1995.

Step II: Calculate the benefit decrease.

$$\text{Benefit Decrease} = (120{,}426 - 114{,}692)(.6) = 3{,}440$$

Step III: Calculate the Accrued Liability decrease.

$$\text{decrease in } PVFB_{30} = (3{,}440)(\ddot{a}_{65}^{(12)})(v^{35}) = 2{,}815$$

$$\text{decrease in } NC_{30} = (2{,}815)/\ddot{a}_{\overline{35}|j} = 109 \qquad \text{where } j = (1.07/1.05) - 1$$

$$\text{decrease in } AL_{46} = (109)(1.05)^{16}(\ddot{s}_{\overline{16}|j}) = 4{,}486$$

Answer is B.

# Chapter Five

# Individual Level Premium Cost Method

# Chapter 5

# Individual Level Premium Cost Method

## 5.3 Problems

**Problem 5 - 27**

Plan effective date: 1/1/87.

Normal retirement benefit:

| | |
|---|---|
| Effective 1/1/87: | \$15 per month for each year of service. |
| Effective 1/1/96: | \$18 per month for each year of service. |

Actuarial cost method: Individual level premium.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/57 |
| Date of hire | 1/1/82 |

Value of assets as of 1/1/96: \$5,000.

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 104.83$$

In what range is the normal cost for 1996 as of 1/1/96?

(A) Less than $500
(B) $500 but less than $550
(C) $550 but less than $600
(D) $600 but less than $650
(E) $650 or more

**Problem 5 - 28**

Plan effective date: 1/1/96.

Normal retirement benefit: $1,000 per month.

Pre-retirement death benefit: $100,000, payable at end of year of death.

Actuarial cost method:

Method A: Individual level premium for all benefits other than death benefits, plus one year term cost for death benefit.

Method B: Aggregate for all benefits other than death benefits and benefits provided by cash value, plus insurance premium (split funded).

Actuarial assumptions:

Interest rate: 7% per year.
Pre-retirement decrements other than deaths: None.
Retirement age: 65.

Date of birth for sole participant (active as of 1/1/96): 1/1/56.

Provisions of life insurance policy (purchased on 1/1/96):

| | |
|---|---|
| Level annual premium | $ 1,585 |
| Projected cash value as of 1/1/2021 | 41,900 |

Selected commutation functions and annuity values:

| $x$ | $D_x$ | $M_x$ | $N_x$ | $\ddot{a}_x^{(12)}$ |
|---|---|---|---|---|
| 40 | 632,275 | 79,292 | 8,452,729 | 12.91 |
| 41 | 589,655 | 78,036 | 7,820,454 | 12.80 |
| 65 | 94,414 | 37,625 | 868,052 | 8.74 |

In what range is the absolute value of the difference in the normal cost for 1996 as of 1/1/96 between Method A and Method B?

(A) Less than \$750
(B) \$750 but less than \$825
(C) \$825 but less than \$900
(D) \$900 but less than \$975
(E) \$975 or more

## 5.4 Solutions to Problems

**Problem 5 - 27**

Key Concept: An increase in the benefit formula results in a normal cost increase associated with the benefit increase.

Step I: Calculate the original Normal Cost.

$$\text{Original Normal Cost} = (15)(40 \text{ years of service})(12)(\ddot{a}_{65}^{(12)}) / \ddot{s}_{\overline{35}|}$$
$$= 425$$

Step II: Calculate the Normal Cost increase.

$$\text{Normal Cost Increase} = (3)(40 \text{ years of service})(12)(\ddot{a}_{65}^{(12)}) / \ddot{s}_{\overline{26}|}$$
$$= 171$$

Note that the benefit increase was \$3 per month per year of service and that the value of the benefit due to the increase was amortized over the remaining 26 years of service.

Step III: Calculate the total Normal Cost.

$$\text{Total Normal Cost} = 425 + 171 = 596$$

Answer is C.

**Problem 5 - 28**

Step I: Calculate the Individual Level Premium Normal Cost using Method A.

$$\text{Normal Cost (other than death)} = (1{,}000)(12)(\ddot{a}_{65}^{(12)}) / \ddot{s}_{40:\overline{25}|}$$
$$= (12{,}000)(8.74)/[(N_{40} - N_{65})/D_{65}]$$
$$= (12{,}000)(8.74)/[(8{,}452{,}729 - 868{,}052)/94{,}414]$$
$$= 1{,}305$$

$$\text{One Year Term Cost} = (100{,}000)(C_{40} / D_{40})$$
$$= (100{,}000)((M_{40} - M_{41}) / D_{40})$$
$$= (100{,}000)((79{,}292 - 78{,}036)/632{,}275)$$
$$= 199$$

Total Normal Cost = 1,305 + 199 = 1,504

Step II: Calculate the Aggregate Normal Cost using Method B.

Value of Benefit at Retirement = (1000)(12)($\ddot{a}_{65}^{(12)}$) = 104,880

Normal Cost = (104,880 - 41,900)/ $\ddot{s}_{40:\overline{25|}}$
= 784

Total Normal Cost = 784 + 1,585 = 2,369

Step III: Calculate the difference in the Normal Costs

Difference = 2,369 - 1,504 = 865

Answer is C.

# Chapter Six

# Frozen Initial Liability Cost Method

# Chapter 6

# Frozen Initial Liability Cost Method

## 6.3 Problems

**Problem 6 - 34**

Plan effective date: 1/1/95.

Normal retirement benefit: $50 per month for each year of service.

Actuarial cost method: Frozen initial liability.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/46 |
| Date of hire | 1/1/66 |

Contribution for 1995: $5,000 paid on 1/1/95.

Value of assets as of 1/1/96: $6,000.

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 9.24$$

In what range is the unfunded liability as of 1/1/96?

(A) Less than $76,000
(B) $76,000 but less than $77,000
(C) $77,000 but less than $78,000
(D) $78,000 but less than $79,000
(E) $79,000 or more

## 6.4 Solutions to Problems

### Problem 6 - 34

Key Concept: The initial unfunded liability determined using the Frozen Initial Liability cost method is equal to the Accrued Liability determined using the Entry Age Normal cost method.

Step I: Calculate the initial unfunded liability as of 1/1/95.

$$\text{Entry Age Normal } AL_{95} = (50)(45 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{29}|} / \ddot{s}_{\overline{45}|})$$
$$= 76{,}260$$

Step II: Calculate the Normal Cost as of 1/1/95.

$$NC_{95} = (50)(45 \text{ years of service})(12)(\ddot{a}_{65}^{(12)}) / \ddot{s}_{\overline{45}|}$$
$$= 816$$

Step III: Calculate the Unfunded Liability as of 1/1/96.

$$UAL_{96} = (AL_{95} + NC_{95} - Contrib_{95})(1.07)$$
$$= (76{,}260 + 816 - 5{,}000)(1.07)$$
$$= 77{,}121$$

Answer is C.

# Chapter Seven

# Attained Age Normal Cost Method

# Chapter 7

# Attained Age Normal Cost Method

## 7.3 Problems

### Problem 7 - 15

Plan effective date: 1/1/96.

Normal retirement benefit: $10 per month for each year of service.

Actuarial assumptions:

| | |
|---|---|
| Interest rate: | 7% per year. |
| Pre-retirement decrements: | None. |
| Retirement age: | 65. |

Valuation data for sole participant (active as of 1/1/96):

| | |
|---|---|
| Date of birth | 1/1/51 |
| Date of hire | 1/1/86 |

Selected annuity value:

$$\ddot{a}_{65}^{(12)} = 8.74$$

In what range is the absolute value of the difference in the unfunded liability as of 1/1/96 under the attained age normal cost method and the entry age normal cost method?

(A) Less than $1,600
(B) $1,600 but less than $1,700
(C) $1,700 but less than $1,800
(D) $1,800 but less than $1,900
(E) $1,900 or more

## 7.4 Solutions to Problems

**Problem 7 - 15**

Step I: Calculate the Attained Age Normal Unfunded Liability.

Note that this is the Unit Credit Accrued Liability, which is just the present value of the accrued benefit.

$$\text{Unfunded Liability} = (10)(10 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(v^{20}) = 2{,}710$$

Step II: Calculate the Entry Age Normal Unfunded Liability.

$$\text{Unfunded Liability} = (10)(30 \text{ years of service})(12)(\ddot{a}_{65}^{(12)})(\ddot{s}_{\overline{10|}} / \ddot{s}_{\overline{30|}}) = 4{,}602$$

Step III: Calculate the difference between the unfunded liabilities.

$$\text{Difference} = 4{,}602 - 2{,}710 = 1{,}892$$

Answer is D.

# Chapter Eight

# Miscellaneous

# Chapter 8

# Miscellaneous

## 8.2 Problems

### Problem 8 - 36

Age of retiree and spouse at date of retirement: 60.

Actuarially-equivalent annuity options available to retiree and spouse:

Option A: Monthly benefit of (\$1,000 + \$X) for first 5 years of lifetime of retiree, and monthly benefit of \$X for remaining lifetime of retiree.

Option B: Monthly benefit of \$1,000 for joint lifetime of retiree and spouse, and monthly benefit of (\$1,000 - \$X) for remaining lifetime of survivor after the first death.

Selected commutation functions and annuity values:

$D_{60} = 144{,}405$ $\quad$ $\ddot{a}_{60}^{(12)} = 9.815$ $\quad$ $\ddot{a}_{60:60}^{(12)} = 8.094$

$D_{65} = 94{,}414$ $\quad$ $\ddot{a}_{65}^{(12)} = 8.736$

In what range is \$X?

(A) Less than \$500
(B) \$500 but less than \$600
(C) \$600 but less than \$700
(D) \$700 but less than \$800
(E) \$800 or more

**Problem 8 - 37**

Assumed interest rate: 7% per year.

Valuation data for all retired participants as of 1/1/95:

| | Smith | Brown | Green |
|---|---|---|---|
| Date of birth | 1/1/35 | 1/1/30 | 1/1/25 |
| Monthly benefit (life annuity) | \$4,000 | \$5,000 | \$6,000 |

Brown died on 12/31/95. There were no other deaths or new retired participants during 1995.

Selected annuity values:

| x | $\ddot{a}_x^{(12)}$ |
|---|---|
| 60 | 9.81 |
| 61 | 9.60 |
| 65 | 8.74 |
| 66 | 8.51 |
| 70 | 7.60 |
| 71 | 7.37 |

In what range is the experience gain during 1995 due to mortality for retired participants?

(A) Less than \$470,000
(B) \$470,000 but less than \$475,000
(C) \$475,000 but less than \$480,000
(D) \$480,000 but less than \$485,000
(E) \$485,000 or more

**Problem 8 - 38**

Assumed interest rate: 7% per year.

Data as of 1/1/96 for a retiree:

| | |
|---|---|
| Age of retiree: | x. |
| Age of spouse: | y. |
| Annual benefit: | \$10,000 payable each 1/1. |

Form of payment: Life annuity for the retiree, with 50% continuing for the life of the spouse if the retiree dies first.

Selected annuity values:

$\ddot{a}_x = 8.157$ $\ddot{a}_y = 10.301$ $\ddot{a}_{xy} = 7.281$

$\ddot{a}_{x+1} = 7.915$ $\ddot{a}_{y+1} = 10.059$

In what range is the experience loss during 1996 due to mortality if both the retiree and the spouse are still alive as of 12/31/96?

(A) Less than \$1,350
(B) \$1,350 but less than \$1,425
(C) \$1,425 but less than \$1,500
(D) \$1,500 but less than \$1,575
(E) \$1,575 or more

## 8.3 Solutions to Problems

**Problem 8 - 36**

Option A can be described as a life annuity of \$X, plus a temporary life annuity of \$1,000 payable from age 60 to age 65:

$$(X)(12\ddot{a}_{60}^{(12)}) + (12)(1{,}000)[\ddot{a}_{60}^{(12)} - (\ddot{a}_{65}^{(12)})(D_{65} / D_{60})]$$

$$= (12X)(9.815) + (12{,}000)[9.815 - (8.716)(94{,}414/144{,}405)]$$

$$= 117.78X + 49{,}239$$

Option B can be described as a life annuity of \$1,000 payable as long as both the retiree and the spouse are alive, plus a reduced pension of \$1,000 - \$X payable to the retiree if the spouse dies or to the spouse if the retiree dies:

$$(1{,}000)(12\ddot{a}_{60:60}^{(12)}) + (2)(12)(1{,}000\text{-}X)[\ddot{a}_{60}^{(12)} - \ddot{a}_{60:60}^{(12)}]$$

$$= (12{,}000)(8.094) + (24{,}000 - 24X)[9.815 - 8.094]$$

$$= 138{,}432 - 41.304X$$

Since the two options are actuarially equivalent,

$$117.78X + 49{,}239 = 138{,}432 - 41.304X$$

$$X = 561$$

Answer is B.

**Problem 8 - 37**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since Brown died at the end of the year, only Smith and Green will continue to receive benefits.

$$\begin{aligned}\text{Actual } AL_{96} &= (4{,}000)(12\ddot{a}_{61}^{(12)}) + (6{,}000)(12\ddot{a}_{71}^{(12)})\\ &= (4{,}000)(12)(9.60) + (6{,}000)(12)(7.37)\\ &= 460{,}800 + 530{,}640 = 991{,}440\end{aligned}$$

Step II: Calculate the expected Accrued Liability. If each retired participant were still alive on 1/1/96, the Accrued Liability would be:

$$\text{AL (all live until 1/1/96)} = (4{,}000)(12\ddot{a}_{61}^{(12)}) + (5{,}000)(12\ddot{a}_{66}^{(12)}) + (6{,}000)(12\ddot{a}_{71}^{(12)})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individual lives from 1/1/95 to 1/1/96. Therefore, the expected Accrued Liability is:

$$eAL_{96} = (4{,}000)(12\ddot{a}_{61}^{(12)})p_{60} + (5{,}000)(12\ddot{a}_{66}^{(12)})p_{65} + (6{,}000)(12\ddot{a}_{71}^{(12)})p_{70}$$

The values of $p_{60}$, $p_{65}$, and $p_{70}$ can be calculated using the formula for successive annuities due.

$$\begin{aligned}\ddot{a}_{60} &= 1 + v\, p_{60}\, \ddot{a}_{61}\\ 10.26833 &= 1 + (1/1.07)(p_{60})(10.05833)\\ p_{60} &= .98596\end{aligned}$$

$$\begin{aligned}\ddot{a}_{65} &= 1 + v\, p_{65}\, \ddot{a}_{66}\\ 9.19833 &= 1 + (1/1.07)(p_{65})(8.96833)\\ p_{65} &= .97813\end{aligned}$$

$$\begin{aligned}\ddot{a}_{70} &= 1 + v\, p_{70}\, \ddot{a}_{71}\\ 8.05833 &= 1 + (1/1.07)(p_{70})(7.82833)\\ p_{70} &= .96475\end{aligned}$$

Note the use of the formula $\ddot{a}_x^{(12)} = \ddot{a}_x - 11/24$.

Therefore,

$$\begin{aligned}eAL_{96} &= (4{,}000)(12)(9.60)(.98596) + (5{,}000)(12)(8.51)(.97813)\\ &\quad + (6{,}000)(12)(7.37)(.96475)\\ &= 454{,}330 + 499{,}433 + 511{,}935 = 1{,}465{,}698\end{aligned}$$

Step III: Calculate the mortality gain. This is the difference between the actual and expected Accrued Liabilities.

$$\begin{aligned}\text{Gain} &= eAL_{96} - \text{Actual } AL_{96}\\ &= 1{,}465{,}698 - 991{,}440\\ &= 474{,}258\end{aligned}$$

Answer is B.

**Problem 8 - 38**

Key Concept: The mortality gain is equal to the difference between the expected and actual liabilities.

Step I: Calculate the actual Accrued Liability. Since both the retiree and the spouse are alive as of 12/31/96, both are entitled to benefits.

$$\text{Actual AL}_{97} = (10{,}000)(\ddot{a}_{x+1}) + (5{,}000)(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$$

The value of $\ddot{a}_{xy}$ can be calculated using the formula for successive annuities due.

$$\begin{aligned}
\ddot{a}_x &= 1 + v\,p_x\,\ddot{a}_{x+1} \\
8.157 &= 1 + (1/1.07)(p_x)(7.915) \\
p_x &= .96753
\end{aligned}$$

$$\begin{aligned}
\ddot{a}_y &= 1 + v\,p_y\,\ddot{a}_{y+1} \\
10.301 &= 1 + (1/1.07)(p_y)(10.059) \\
p_y &= .98937
\end{aligned}$$

$$\begin{aligned}
\ddot{a}_{xy} &= 1 + v\,p_{xy}\,\ddot{a}_{x+1:y+1} \\
7.281 &= 1 + (1/1.07)(.96753)(.98937)(\ddot{a}_{x+1:y+1}) \\
\ddot{a}_{x+1:y+1} &= 7.021
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{Actual AL}_{97} &= (10{,}000)(7.915) + (5{,}000)(10.059 - 7.021) \\
&= 94{,}340
\end{aligned}$$

Step II: Calculate the expected Accrued Liability. If the retiree and the spouse were both still alive on 1/1/97, the Accrued Liability would be:

$$\text{AL (both live until 1/1/97)} = (10{,}000)(\ddot{a}_{x+1}) + (5{,}000)(\ddot{a}_{y+1} - \ddot{a}_{x+1:y+1})$$

To determine the expected Accrued Liability, multiply each annuity due by the probability that the individual lives from 1/1/96 to 1/1/97. Therefore, the expected Accrued Liability is:

$$\begin{aligned}
e\text{AL}_{97} &= (10{,}000)(\ddot{a}_{x+1})p_x + (5{,}000)[(\ddot{a}_{y+1})p_y - (\ddot{a}_{x+1:y+1})p_{xy}] \\
&= (10{,}000)(7.915)(.96753) \\
&\quad + (5{,}000)[(10.059)(.98937) - (7.021)(.96753)(.98937)] \\
&= 92{,}736
\end{aligned}$$

Step III: Calculate the mortality loss. This is the difference between the actual and expected Accrued Liabilities.

$$\begin{aligned} \text{Loss} &= \text{Actual AL}_{97} - e\text{AL}_{97} \\ &= 94{,}340 - 92{,}736 \\ &= 1{,}604 \end{aligned}$$

Answer is E.

**Problem 8 - 38 (Alternative Solution)**

The expected Accrued Liability could have been calculated using a retrospective method.

$$\begin{aligned} e\text{AL}_{97} &= (\text{Prior liability - prior payment}) \times (1+i) \\ &= [10{,}000\ddot{a}_x + 5{,}000(\ddot{a}_y - \ddot{a}_{xy}) - 10{,}000](1.07) \\ &= 92{,}736 \end{aligned}$$

$$\begin{aligned} \text{Loss} &= \text{Actual AL}_{97} - e\text{AL}_{97} \\ &= 94{,}340 - 92{,}736 \\ &= 1{,}604 \end{aligned}$$

Answer is E.